Stochastic programming and simulation

A combined paradigm for decision making under uncertainty

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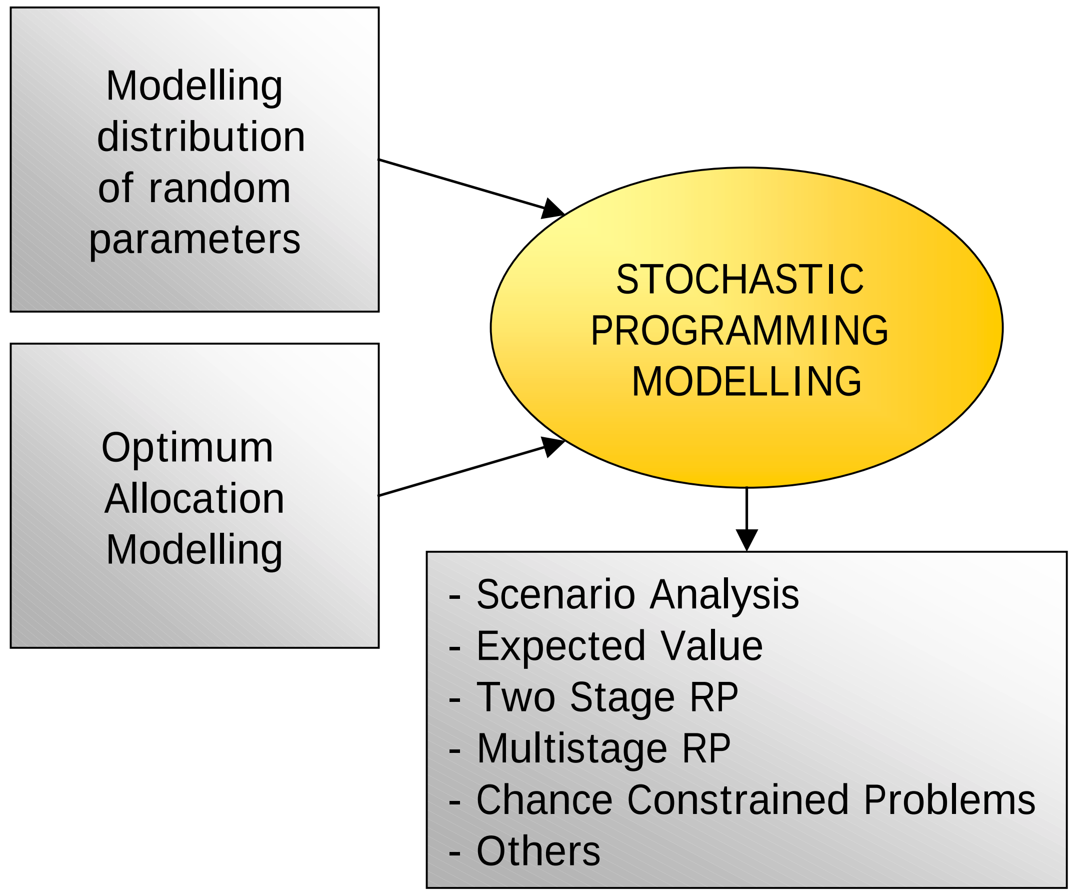
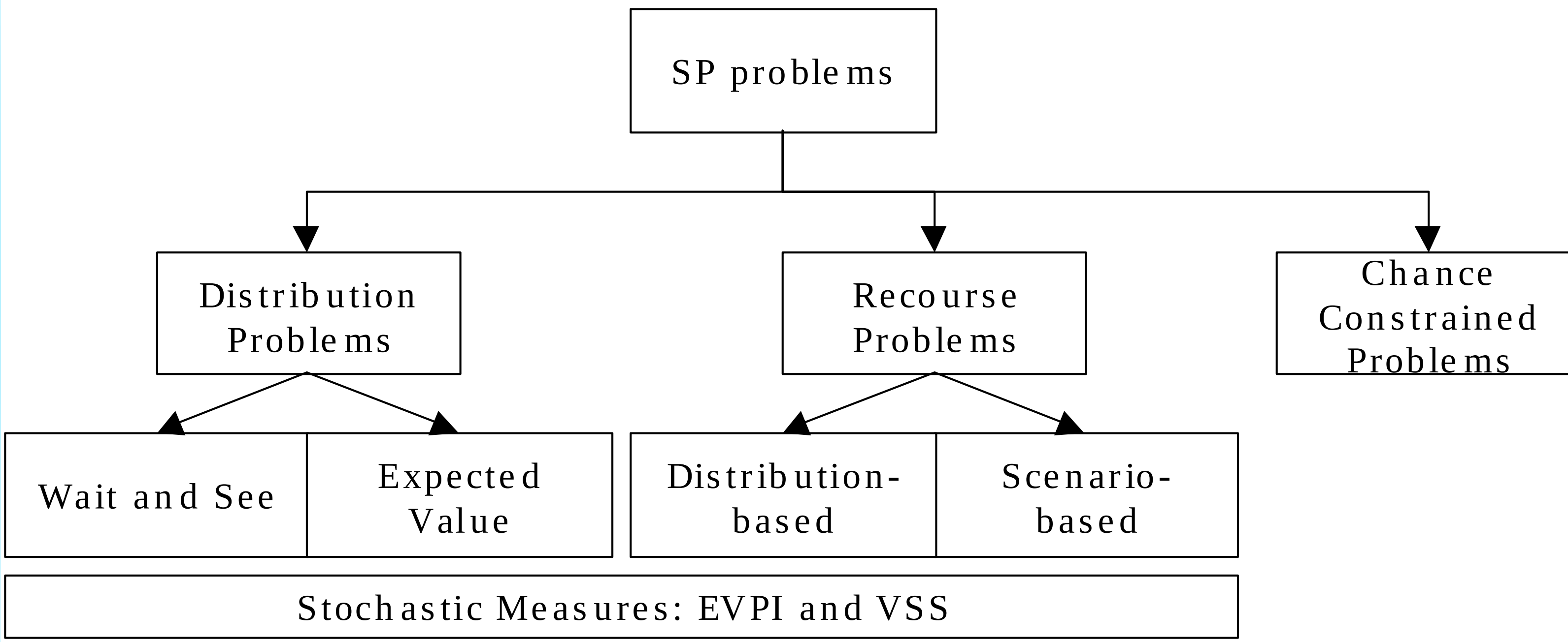
Outline

- Stochastic Programming
- Tools for SP
- Simulation and SP
- Case study: Supply Chain Planning
- Conclusions

Modelling SP



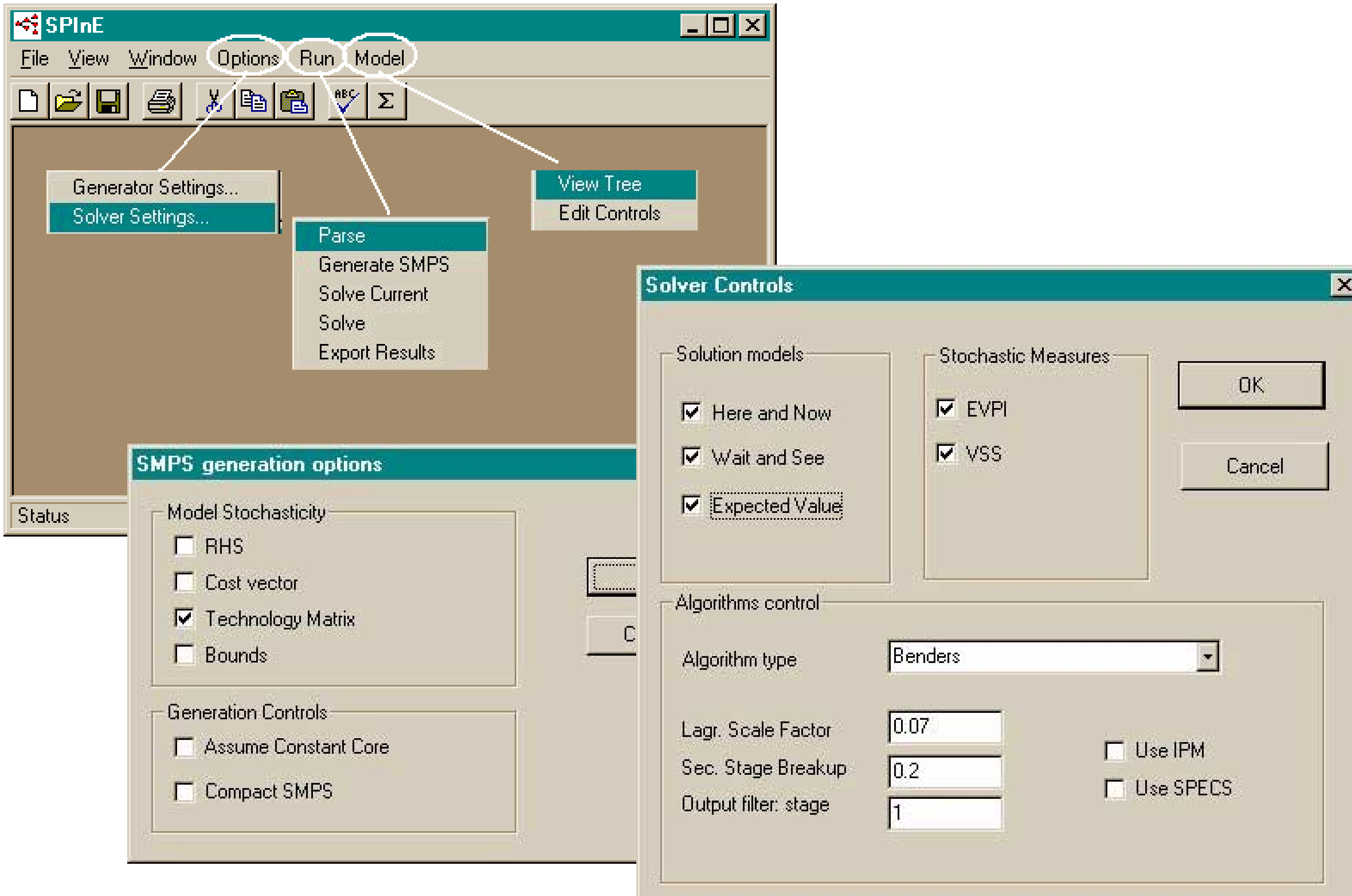
SP models classification



SPInE

- Stochastic Programming Integrated Environment
- Algebraic modelling using SMPL/SAMPL
- Support of scenario based multistage SP and CCP
- EV, HN, WS solutions plus EVPI and VSS
- Integration with libraries of scenario generators
- Solver based on Benders' decomposition and DEQ

SPInE Controls



SP and Simulation

- Simulation of models of randomness (Scenario Generation)
- Simulation in solution algorithms (stochastic approximation)
- Simulation in model validation (robustness)
- Simulation in risk analysis

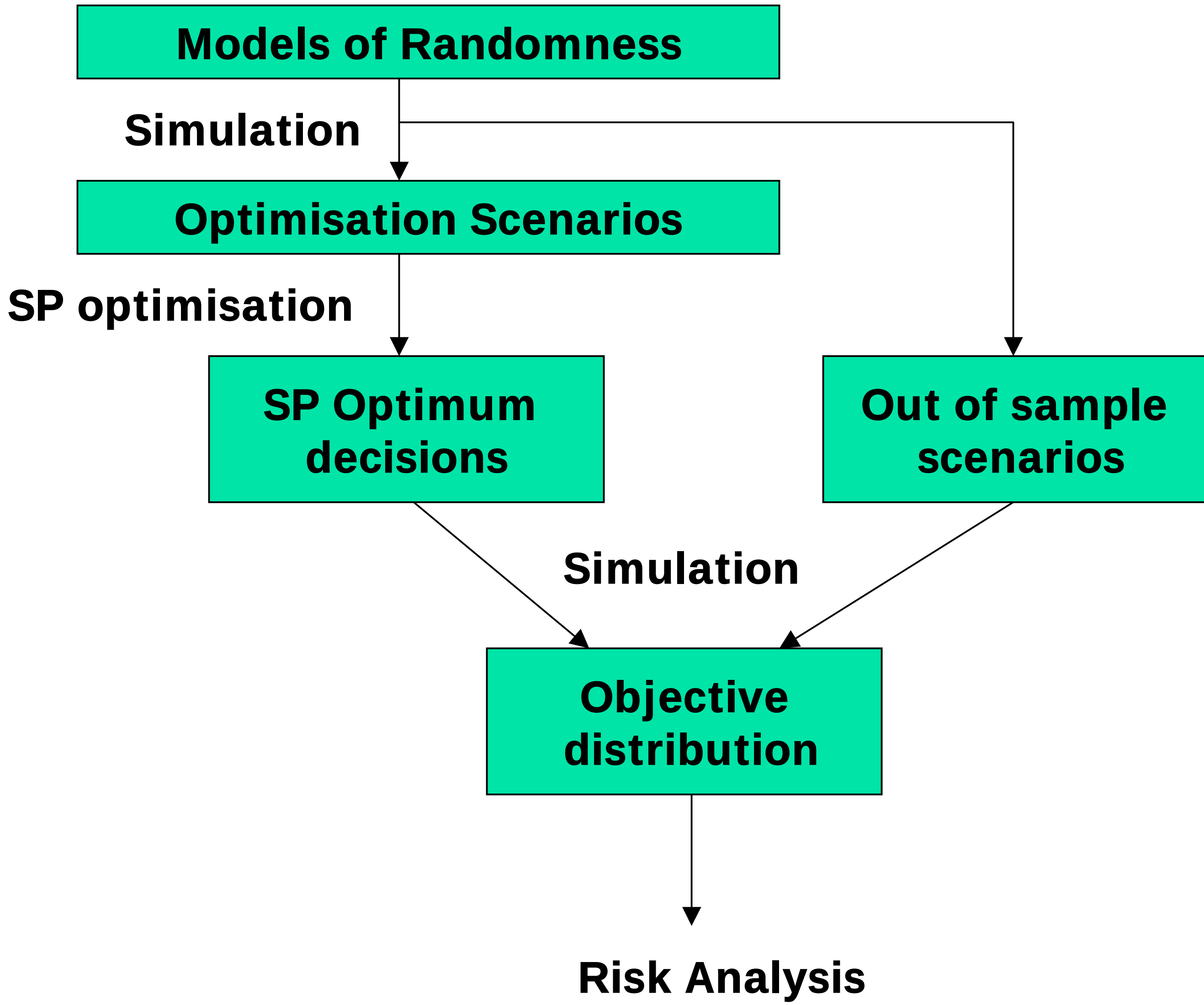
Framework for Risk Analysis

- SP provides „hedged“ decisions against a set of future scenarios
- Risk can be incorporated into the model

or

- Risk can be measured against „out of sample„ scenarios
- Need for an integrated framework for
  - Scenario generation
  - SP modelling and optimisation
  - Simulation of the optimum decisions‘ performance

Framework for Risk Analysis



Scenario Generation

- Time Series Analysis
  - AutoRegressive (AR)
  - Moving Average (MA)
  - ARMA
  - GARCH
- Diffusion processes
  - Ito
  - Wiener
  - Geometric Brownian Motion
- Moment Based Scenario generation

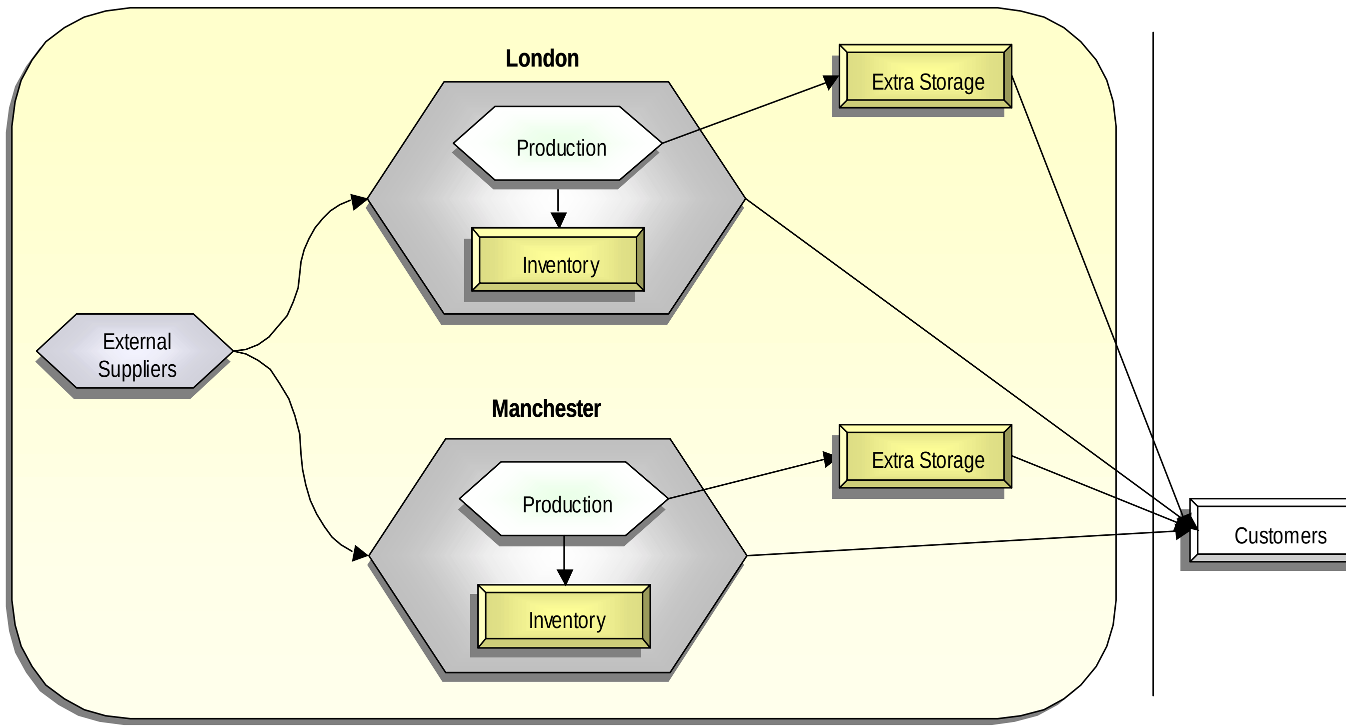
Risk Measures

- Variance, Standard Deviation
- Value at Risk
- Conditional Value at Risk
- Downside Risk

SCM Planning (1)

- Strategic-Tactical Model
- Strategic decisions:
  - Capacity configuration revision
  - Outsourced production capacity
  - Additional inventory capacity
- Tactical decisions:
  - production and inventory holding

SCM planning (2)



SCM Planning (3)

- Objective Function Terms:
  - Recoverable shortage costs
  - Lost demand (shortage) costs
  - Production costs
  - Outsourced production costs
  - Inventory costs
  - Additional inventory costs
  - Costs of changing configurations
  - Fixed costs for the installed configurations

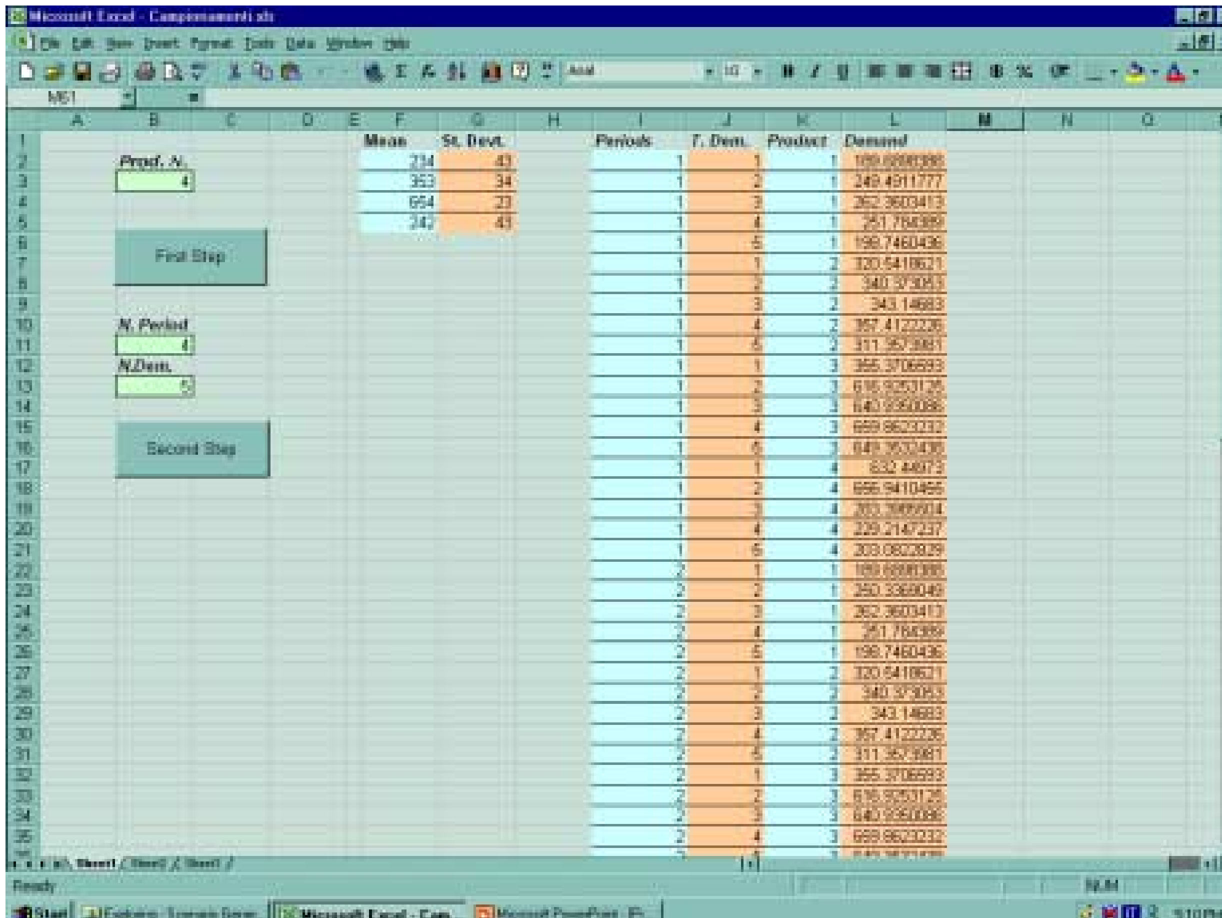
SCM Planning (4)

- Constraints:
  - Demand Satisfaction
  - Demand Shortage
  - Production Constraints
  - Outsourced Production
  - Inventory balance
  - Outsourced Inventory
  - Configurations constraints

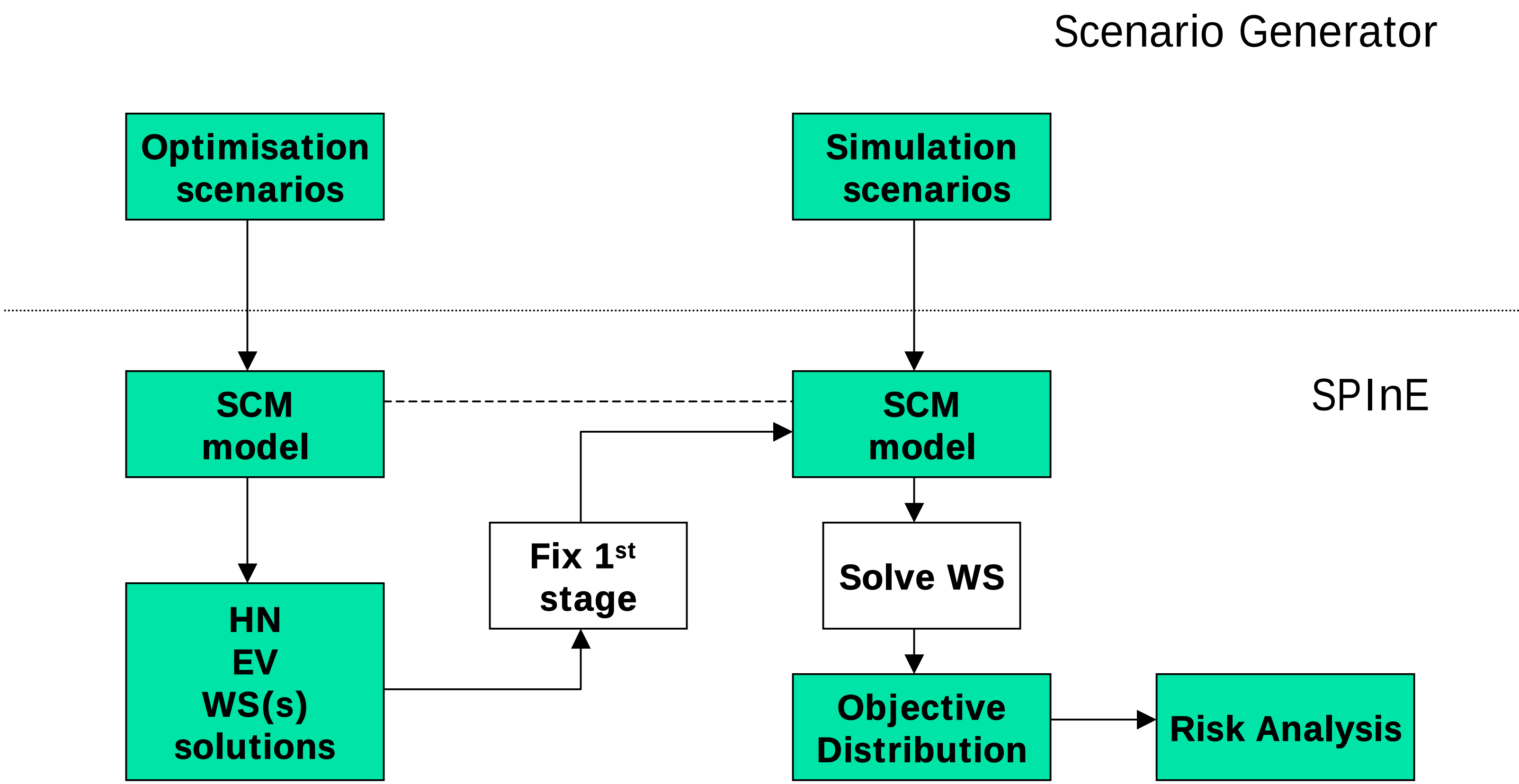
Scenario Generation for the Demand

- Mean and Variance for the demand are computed from historical data
- Demand is assumed lognormal
- Scenarios for the demand are sampled from the distributions
- Assumed independency between distributions for different products

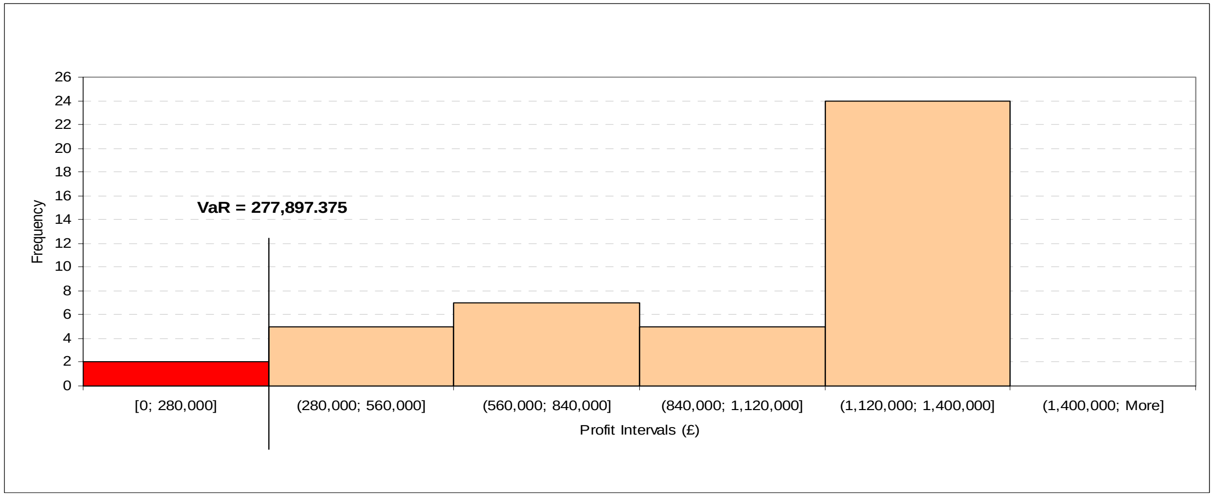
Scenario Generation for the Demand

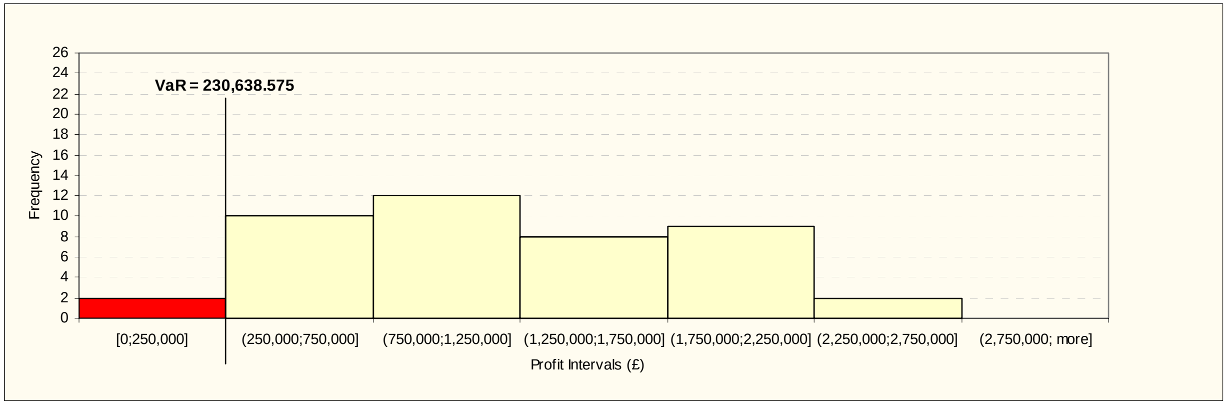


Risk Analysis framework in SPInE



Results: VaR for EV and HN

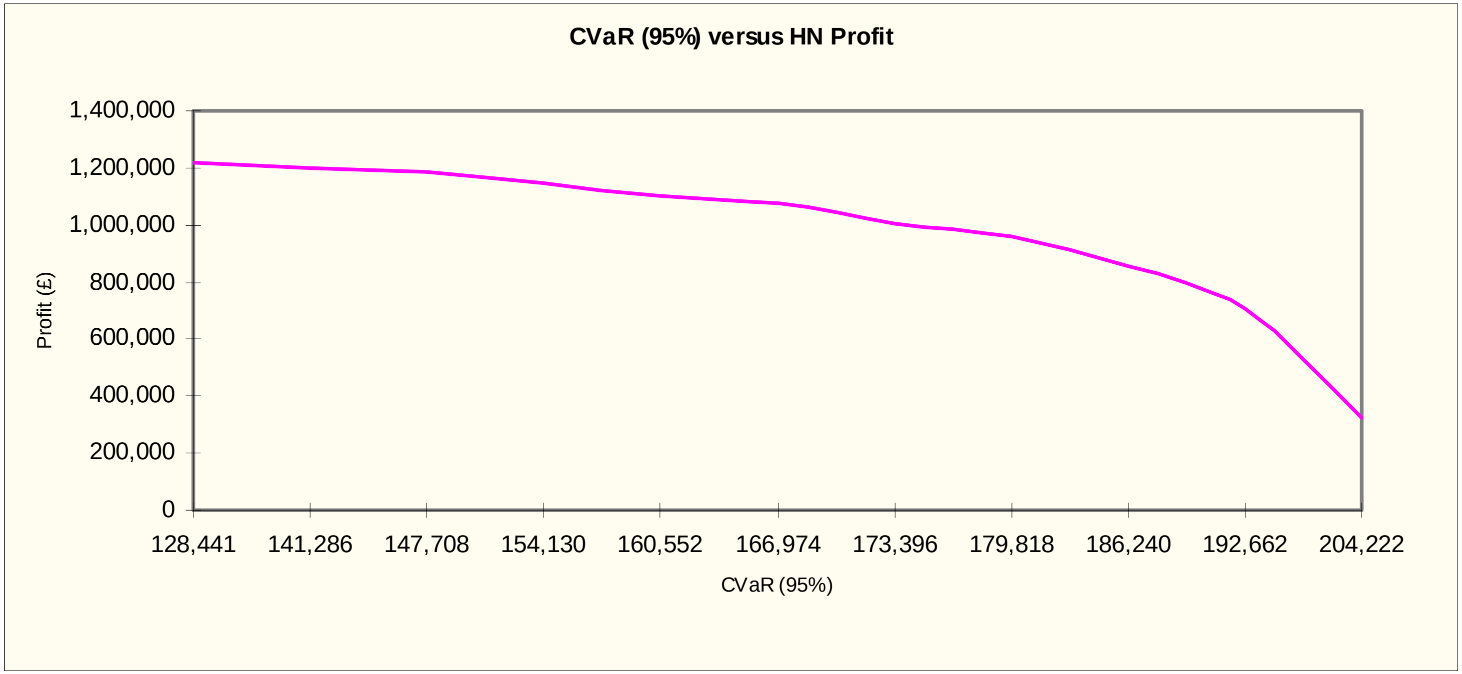




Calculation of CVaR

- Solve the model without CVaR constraints
- Set the fractile level β.
- Identify the β-CVaR for the unconstrained strategy, that is, the β-$CVaR_0$.
- Append the β-CVaR constraint to the model, and solve it.
- Efficient Frontier is computed by repeating the last step with the risk constraint tightened successively.

Results: CVaR Efficient Frontier



Conclusions

- Simulation can be applied to all phases involved in SP optimisation
- SP optimum solution may not be the „best“ in terms of risk measures
- Simulation can be integrated with SP optimisation for risk analysis
- We have implemented a framework for risk analysis based on SP and simulation using SPInE and a library of scenario generators