

# Stochastic programming and simulation

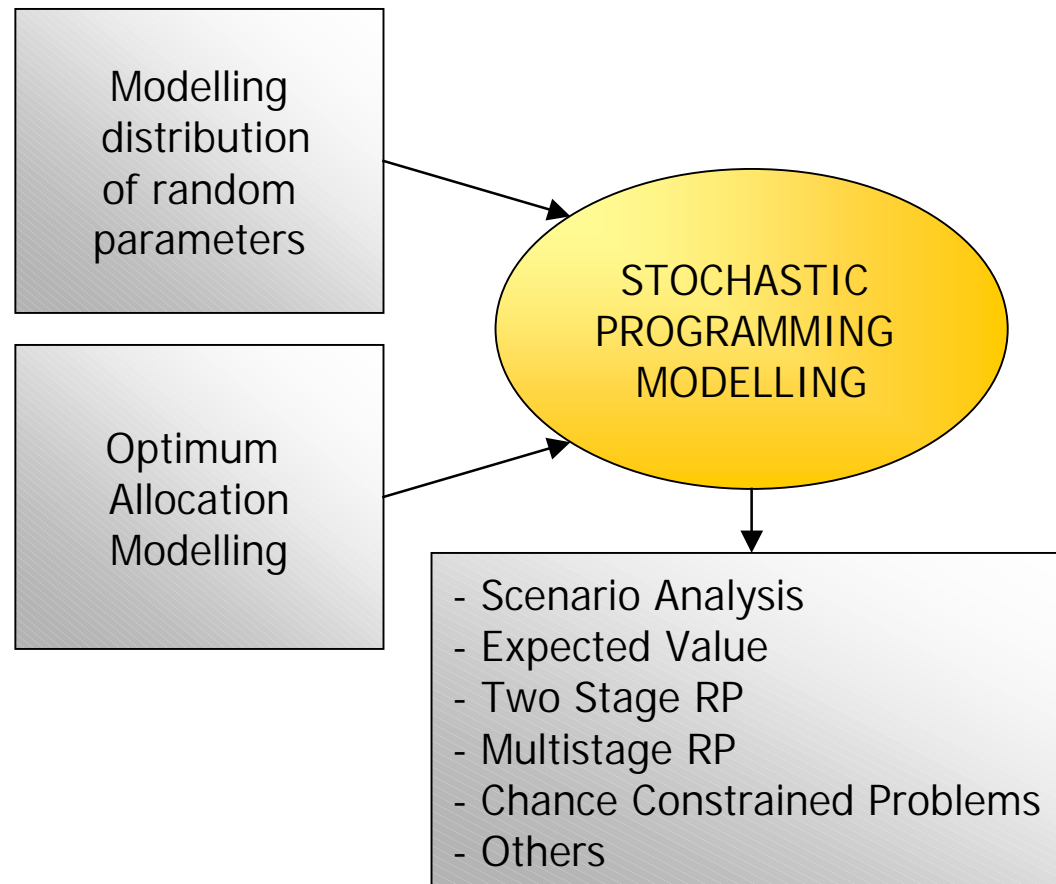
A combined paradigm for decision making under uncertainty

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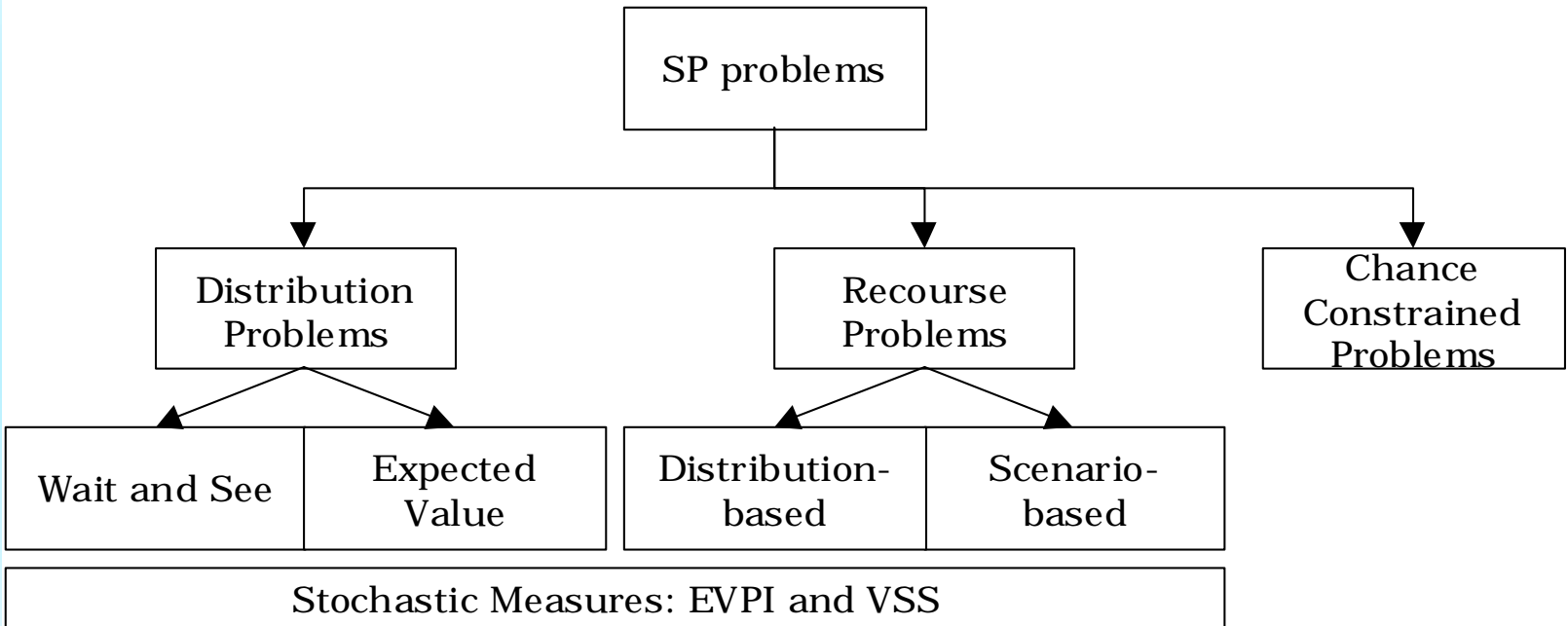
# Outline

- Stochastic Programming
- Tools for SP
- Simulation and SP
- Case study: Supply Chain Planning
- Conclusions

# Modelling SP



# SP models classification



# SPInE

- Stochastic Programming Integrated Environment
- Algebraic modelling using SMPL/SAMPL
- Support of scenario based multistage SP and CCP
- EV, HN, WS solutions plus EVPI and VSS
- Integration with libraries of scenario generators
- Solver based on Benders' decomposition and DEQ

# SPInE Controls

The image displays the SPInE software interface with several key components highlighted:

- SPInE Main Window:** The top menu bar includes File, View, Window, Options, Run, and Model. The Options menu is open, showing a list of actions: Parse, Generate SMPS, Solve Current, Solve, and Export Results. Callouts point from the Options, Run, and Model menu items to this list.
- SMPS generation options Dialog:** This dialog is titled "SMPS generation options" and contains two sections:
  - Model Stochasticity:** Includes checkboxes for RHS, Cost vector, Technology Matrix (checked), and Bounds.
  - Generation Controls:** Includes checkboxes for Assume Constant Core and Compact SMPS.
- Solver Controls Dialog:** This dialog is titled "Solver Controls" and contains:
  - Solution models:** Checkboxes for Here and Now, Wait and See, and Expected Value (checked).
  - Stochastic Measures:** Checkboxes for EVPI and VSS (checked).
  - Algorithms control:** Includes a dropdown for Algorithm type (set to Benders), input fields for Lagr. Scale Factor (0.07), Sec. Stage Breakup (0.2), and Output filter: stage (1). It also has checkboxes for Use IPM and Use SPECS.
  - Buttons for OK and Cancel.

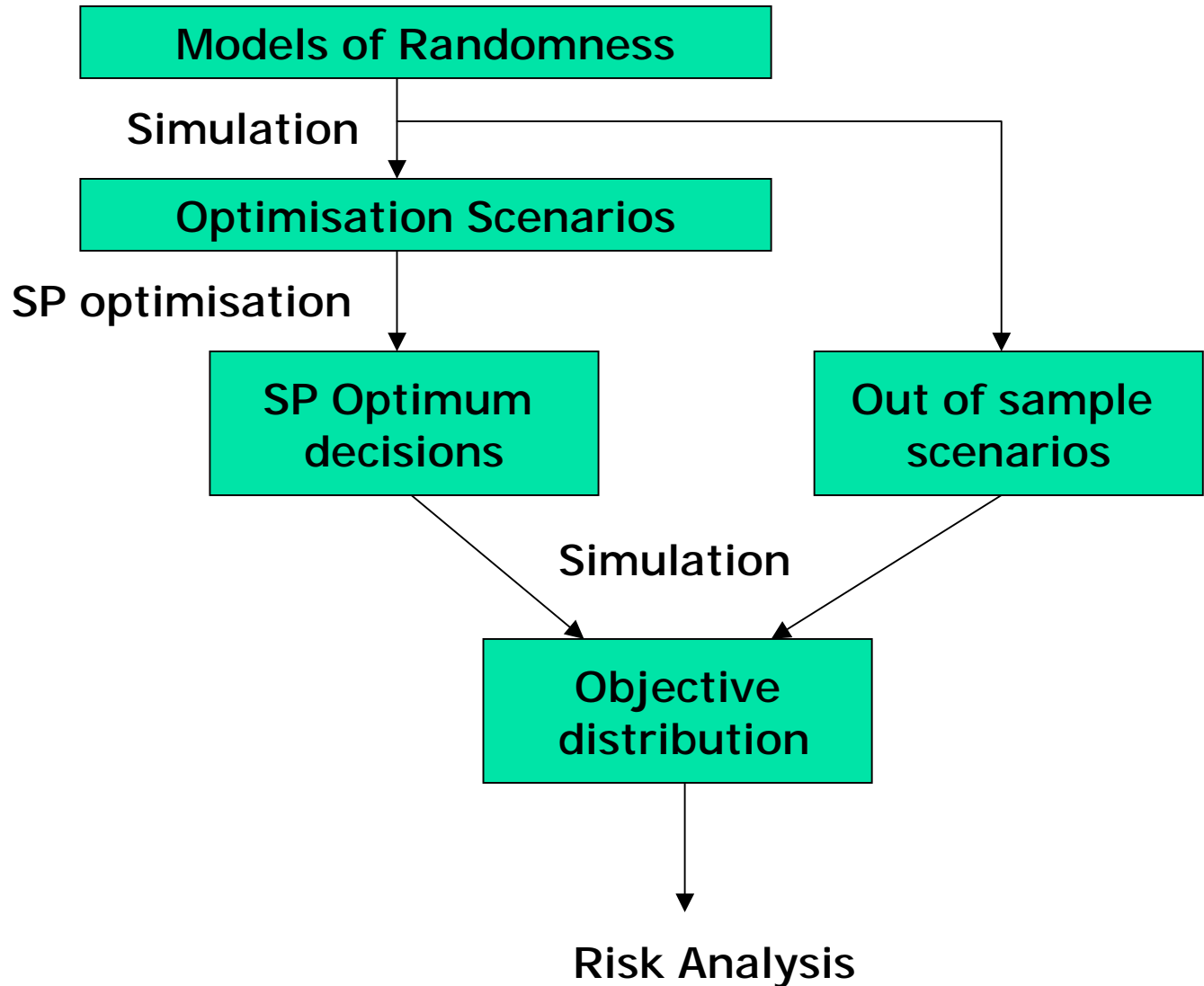
# SP and Simulation

- Simulation of models of randomness (Scenario Generation)
- Simulation in solution algorithms (stochastic approximation)
- Simulation in model validation (robustness)
- Simulation in risk analysis

# Framework for Risk Analysis

- SP provides „hedged“ decisions against a set of future scenarios
- Risk can be incorporated into the model  
or
- Risk can be measured against „out of sample„ scenarios
- Need for an integrated framework for
  - Scenario generation
  - SP modelling and optimisation
  - Simulation of the optimum decisions' performance

# Framework for Risk Analysis



# Scenario Generation

- Time Series Analysis
  - AutoRegressive (AR)
  - Moving Average (MA)
  - ARMA
  - GARCH
- Diffusion processes
  - Ito
  - Wiener
  - Geometric Brownian Motion
- Moment Based Scenario generation

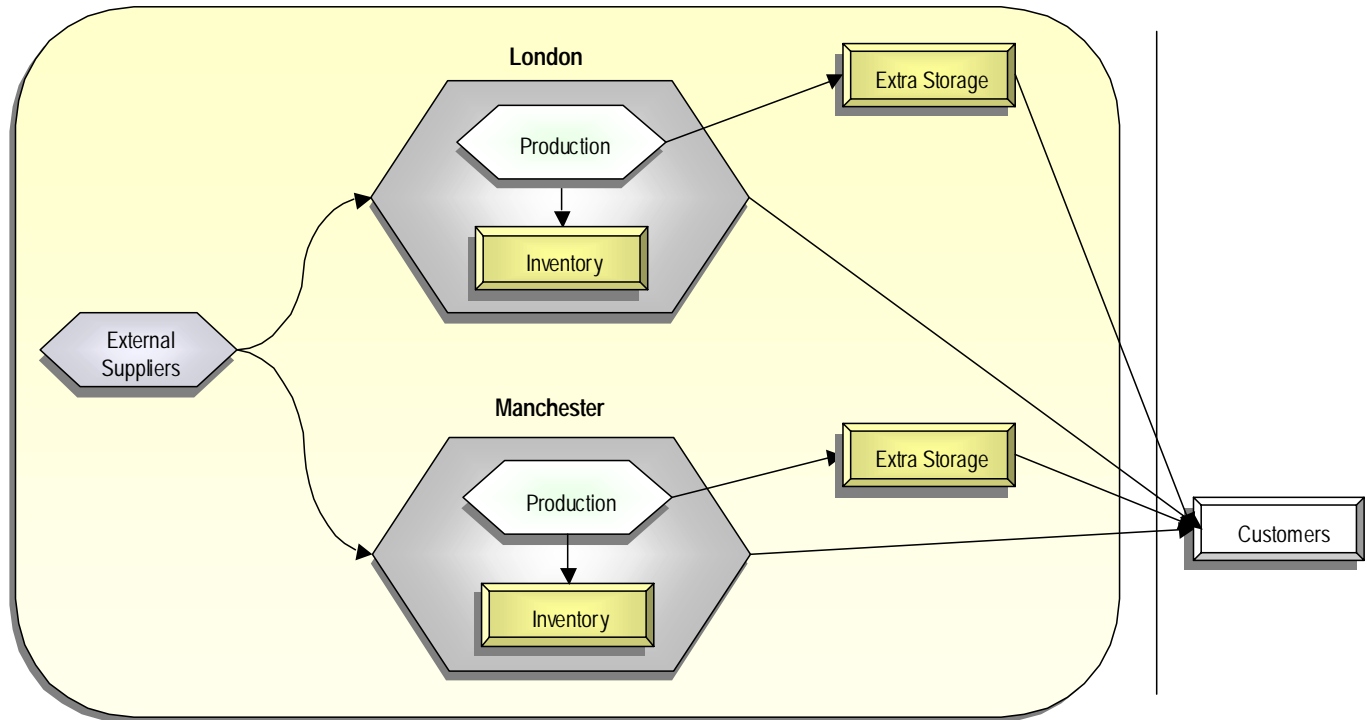
# Risk Measures

- Variance, Standard Deviation
- Value at Risk
- Conditional Value at Risk
- Downside Risk

# SCM Planning (1)

- Strategic-Tactical Model
- Strategic decisions:
  - Capacity configuration revision
  - Outsourced production capacity
  - Additional inventory capacity
- Tactical decisions:
  - production and inventory holding

# SCM planning (2)



# SCM Planning (3)

- Objective Function Terms:
  - Recoverable shortage costs
  - Lost demand (shortage) costs
  - Production costs
  - Outsourced production costs
  - Inventory costs
  - Additional inventory costs
  - Costs of changing configurations
  - Fixed costs for the installed configurations

# SCM Planning (4)

- Constraints:
  - Demand Satisfaction
  - Demand Shortage
  - Production Constraints
  - Outsourced Production
  - Inventory balance
  - Outsourced Inventory
  - Configurations constraints

# Scenario Generation for the Demand

- Mean and Variance for the demand are computed from historical data
- Demand is assumed lognormal
- Scenarios for the demand are sampled from the distributions
- Assumed independency between distributions for different products

# Scenario Generation for the Demand

The screenshot shows a Microsoft Excel spreadsheet titled "Microsoft Excel - Compensations.xls". The spreadsheet is used for scenario generation for demand. It features several input fields and two buttons:

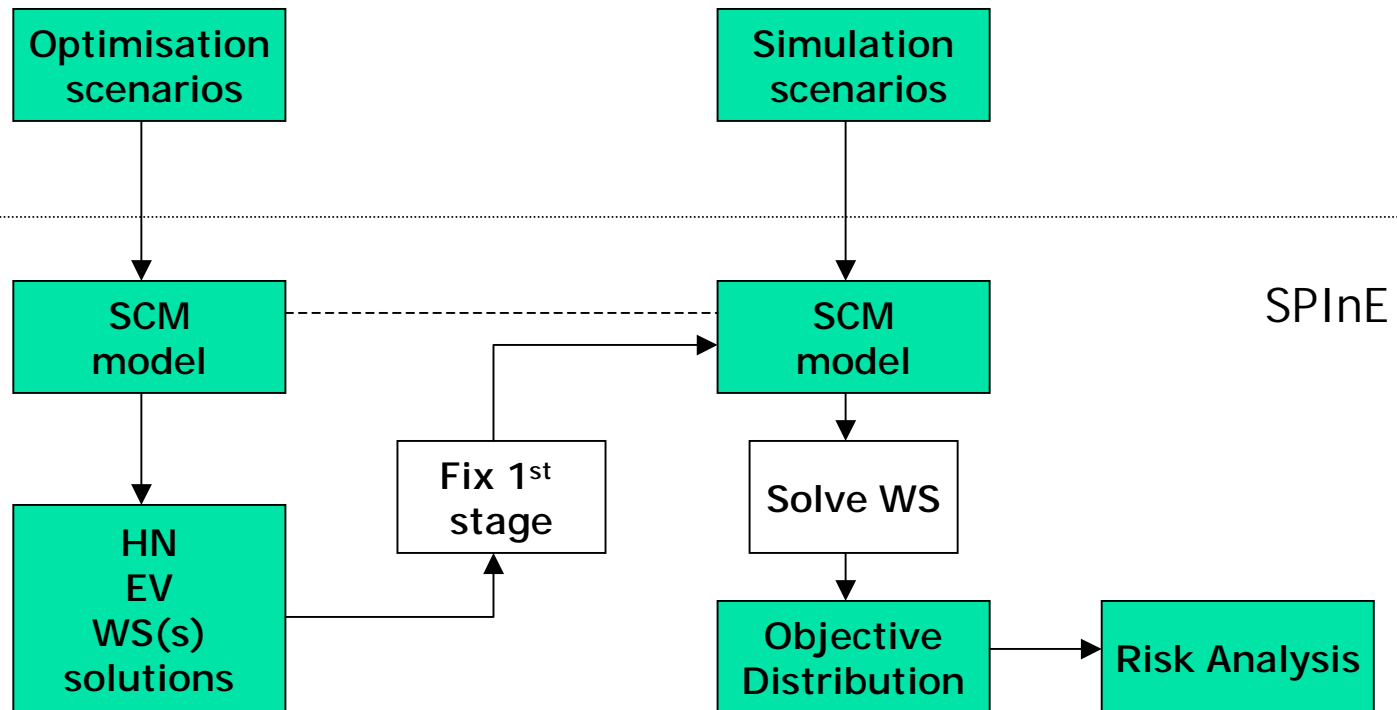
- Prof. N.:** Input field with value 4.
- N. Period:** Input field with value 4.
- N. Dem.:** Input field with value 5.
- Buttons:** "First Step" and "Second Step".

The main data table has the following columns: Mean, St. Dev., Periods, F. Dem., Product, and Demand. The data is organized into 35 rows, with the first 5 rows representing the first step and the remaining 30 rows representing the second step. The data is as follows:

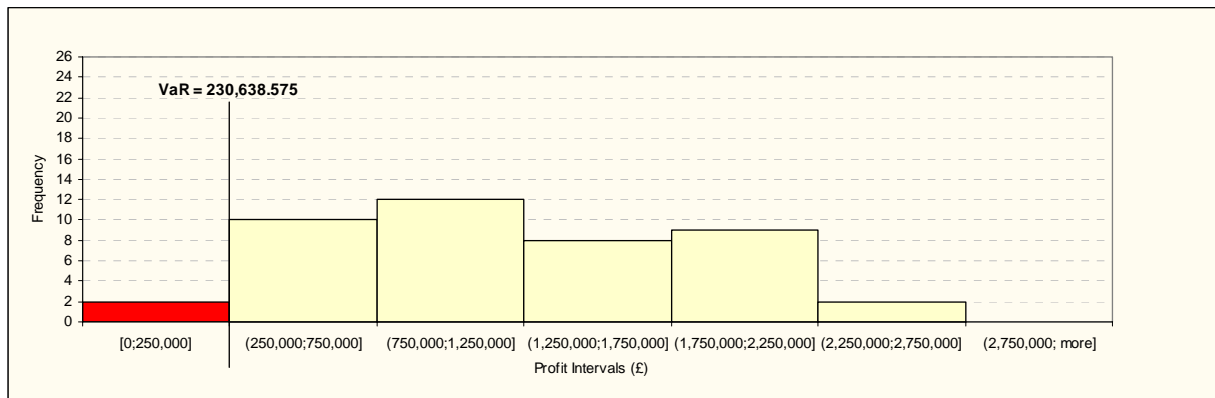
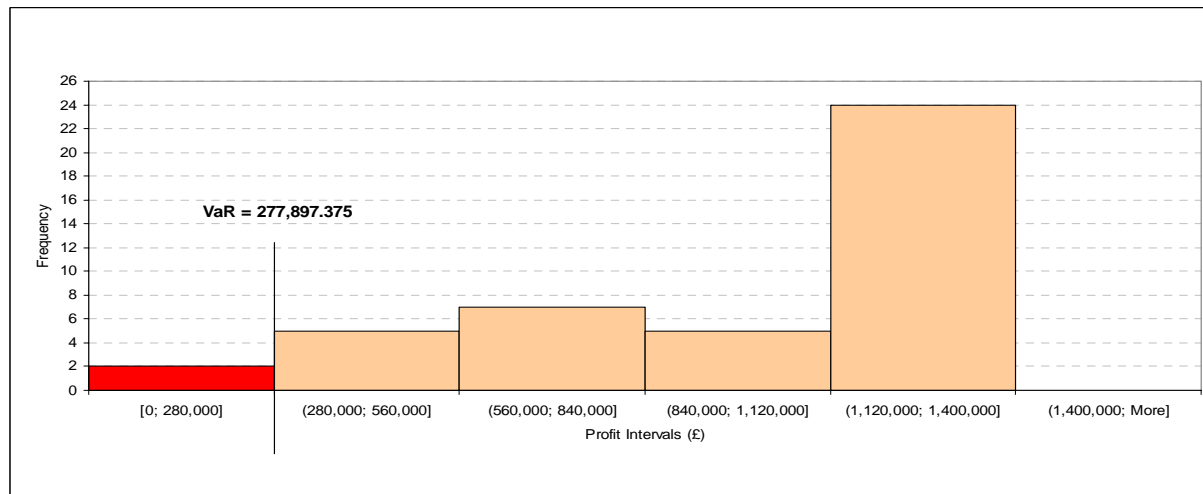
	Mean	St. Dev.	Periods	F. Dem.	Product	Demand
1	234	43	1	1	1	199.6990395
2	353	34	1	2	1	349.4911777
3	654	27	1	3	1	262.3603413
4	347	41	1	4	1	251.756399
5			1	5	1	199.7460436
6			1	1	2	320.5419621
7			1	2	2	340.373053
8			1	3	2	343.14693
9			1	4	2	357.4122235
10			1	5	2	311.3623981
11			1	1	3	355.3706693
12			1	2	3	616.9253125
13			1	3	3	640.9260396
14			1	4	3	659.9623232
15			1	5	3	649.3531439
16			1	1	4	832.44973
17			1	2	4	656.9410455
18			1	3	4	303.3999904
19			1	4	4	229.2147237
20			1	5	4	303.0822829
21			2	1	1	199.6990395
22			2	2	1	350.3369049
23			2	3	1	262.3603413
24			2	4	1	251.756399
25			2	5	1	199.7460436
26			2	1	2	320.5419621
27			2	2	2	340.373053
28			2	3	2	343.14693
29			2	4	2	357.4122235
30			2	5	2	311.3623981
31			2	1	3	355.3706693
32			2	2	3	616.9253125
33			2	3	3	640.9260396
34			2	4	3	659.9623232
35			2	5	3	649.3531439

# Risk Analysis framework in SPInE

Scenario Generator



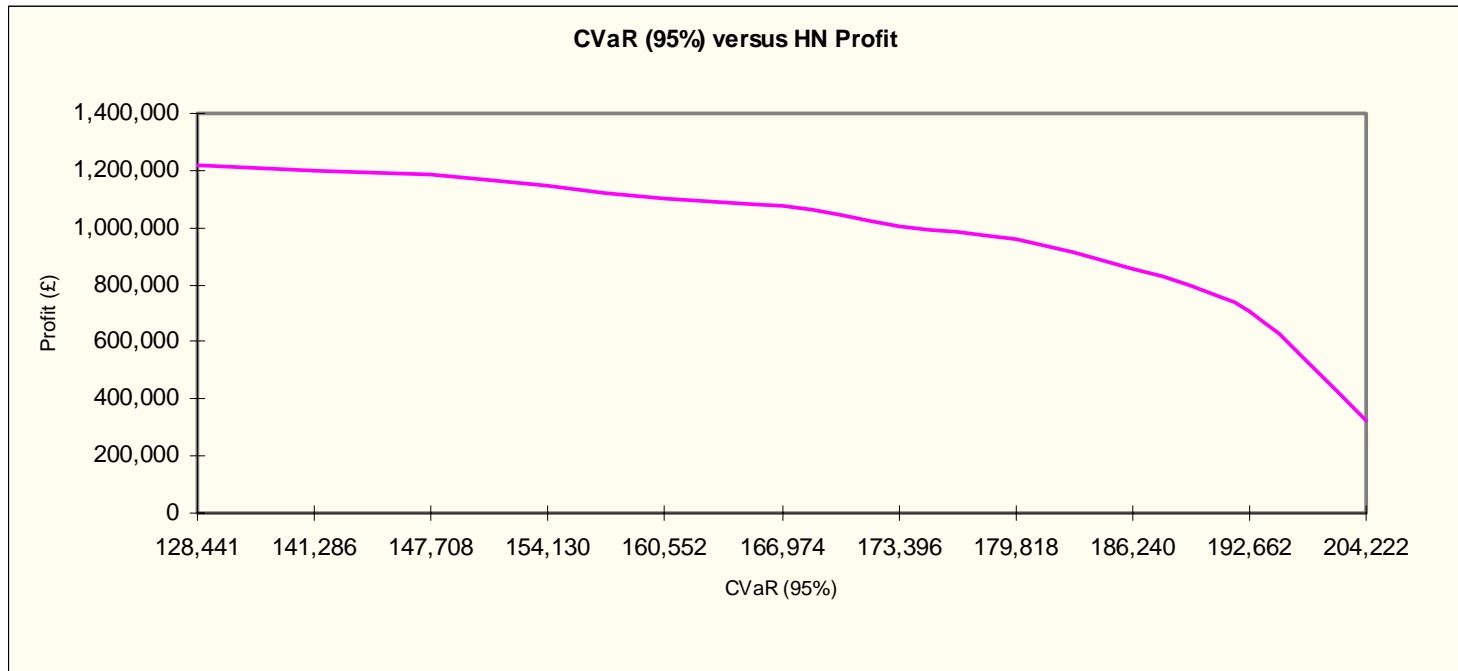
# Results: VaR for EV and HN



# Calculation of CVaR

- Solve the model without CVaR constraints
- Set the fractile level  $\beta$ .
- Identify the  $\beta$ -CVaR for the unconstrained strategy, that is, the  $\beta$ -CVaR<sub>0</sub>.
- Append the  $\beta$ -CVaR constraint to the model, and solve it.
- Efficient Frontier is computed by repeating the last step with the risk constraint tightened successively.

# Results: CVaR Efficient Frontier



# Conclusions

- Simulation can be applied to all phases involved in SP optimisation
- SP optimum solution may not be the „best“ in terms of risk measures
- Simulation can be integrated with SP optimisation for risk analysis
- We have implemented a framework for risk analysis based on SP and simulation using SPInE and a library of scenario generators