The 15th Annual Investment Seminar

Alpha Strategies

UBS Warburg

Emmanuel College and the University Arms Hotel, Cambridge,UK Sunday 8th to Wednesday 11th September 2002

CARISMA

OptiRisk SYSTEMS

Solving Large Scale QP and QMIP Portfolio Models: Discussion of Algorithms

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presented to
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Acknowledgments

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  INQUIRE (UK),UBS Warburg, Fidelity Investment Services, MB Risk Management, Advanced Portfolio Technology Inc.

- Public Sector:

  Strategic research initiative Brunel University, EPSRC (UK), EU Framework 5.

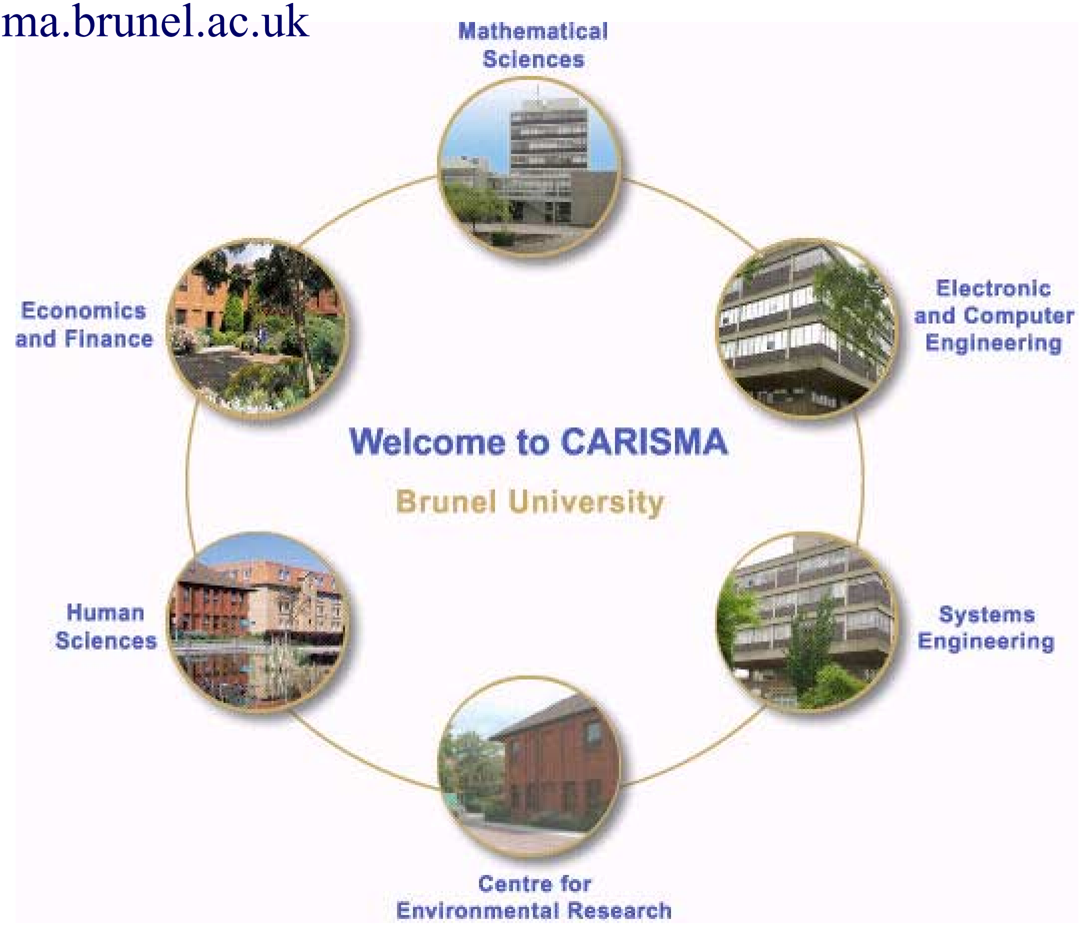
Outline

- Background
- QP portfolio models
- QP solution algorithms
- Computational performance of algorithms
- Solving family of QP models
- Solving QMIP models
- Discussion
- References

CARISMA

The Centre for the Analysis of Risk and Optimisation Modelling Applications

http://carisma.brunel.ac.uk



CARISMA

CARISMA Background

Sponsored by the strategic research initiative of Brunel University.

Set up as an interdisciplinary centre, bringing together diverse groups:

Operational Research within Mathematical Sciences

Economics and Finance

Human Sciences

Electrical and Computer Engineering

Systems Engineering

Centre for Environmental Research (plus Geography)

Faculty of CARISMA

Director: Professor Gautam Mitra

Deputy Director: Professor Christos Ioannidis

Research Lecturers: Paresh Date, Fabio Spagnolo, Chandra Poojari

Faculty members: 7 professors and 5 lecturers

Research Associates : 4

Ph.D. Students: 16

Mission of CARISMA

The mission of CARISMA is to be a **centre** of excellence recognised for its research and scholarship in the following:

- the analysis of risk,
- optimisation modelling,
- the combined paradigm of risk and return quantification.

Industry Focus
Finance Industry - Bank, Insurance, Pension Funds
Large Corporates - FTSE 100, Multinationals, EUROTOP
Public Sector/Utilities, Environment, Food, Agriculture, Health

OptiRisk Systems

- OptiRisk Systems is a UK based company with a global reach, set up to meet the rapidly expanding market of Optimisation and risk modelling applications.
- OptiRisk Systems provides a comprehensive service covering Risk and Optimisation models, algorithms and software components. These services are provided face to face with the client or can be delivered and supported across the internet.
- OptiRisk Systems offers research and development support to its clients in all aspects of risk and optimisation modelling, prototype developments and embedded DSS solutions.

The People & The Advisers

- **The People**

  Gautam Mitra, Cormac Lucas, Nikitas-Spiros Koutsoukis and Triphonas Kyriakis. The company also draws upon research groups within CARISMA and benefits from the support of their many researchers.

- **The Advisers**

  The advisers include:

  Professor Robert Fourer, Northwestern University;
  Professor Stavros Zenios, University of Cyprus and the Wharton Financial Institutions Center;

  Professor Bill Ziemba, University of British Columbia.

- **Product and Services**

  Modelling and Solution Software

  Modelling support

  Training and Workshops

The mission of OptiRisk

- The mission of OptiRisk is to be a global yet a niche player and leader in the provision of optimisation and risk solutions. OptiRisk set out to achieve its mission through

- Development of models, algorithms, software systems
- Acquisition and value added reselling of optimisation software and specialist applications,
- Provision of services covering training and customised applications for a number of sectors including:
  - Supply Chain and Logistics
  - Finance
  - Utilities
  - Transportation
- Utilising leading edge networking, computing and software technologies (e-business).

OptiRisk: OSP Craft project

- The aim of OSP Craft project: is to develop an optimisation environment and offer it as an Application Service provider (ASP) solution.
- ASPs rent the use of business applications, which they implement, customise, and out-host for multiple end-user organisations and charge on a "pay as you go" basis.
- By adopting the ASP model, and by adding further value to it in the form of optimisation-based analytic engines, OSP Craft defines an enhanced approach which is an Optimisation Service Provider (OSP).

www.OSP-CRAFT.com

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QP Models: Introduction and Overview

- Modelling Paradigm
  - Markowitz M-V model
  - Risk and return…two objectives
  - Efficient frontier…Pareto optimal
  - Utility function…risk aversion

Mean-Variance Model

- Markowitz (1952,1959)
- alternative formulations

QP1

$$\text{Min} \qquad Z_{QP1} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1,\ldots, N$$

$i, j = 1,\ldots, N$: denotes the different risky assets

μ_i : expected return of asset i

σ_{ij} : covariance between asset i and asset j

ρ : desired level of return

x_i : the fraction of portfolio value invested in asset i

QP2

(Arrow-Pratt absolute risk aversion index)

$$\text{Max} \qquad Z_{QP2}^{R_A} = \frac{R_A}{2}\sum_{i=1}^{N} x_i \mu_i - \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$\text{s.t.} \qquad \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1,\ldots, N$$

$$R_A \geq 0$$

Mean-Variance Model

- Arrow Pratt absolute Risk Aversion Index

$$R_A = -\frac{u''(w)}{u'(w)}$$

where w is the portfolio wealth and

u a Von Neumann-Morgenstern utility

function with first and second derivatives

- Portfolios with similar Absolute Risk Aversion index leads to similar portfolios (weight-vector) regardless of functional form and parameters of the utility function (Kallberg and Ziemba 1983)

Mean-Variance Model

QP2 (Lambda-formulation)

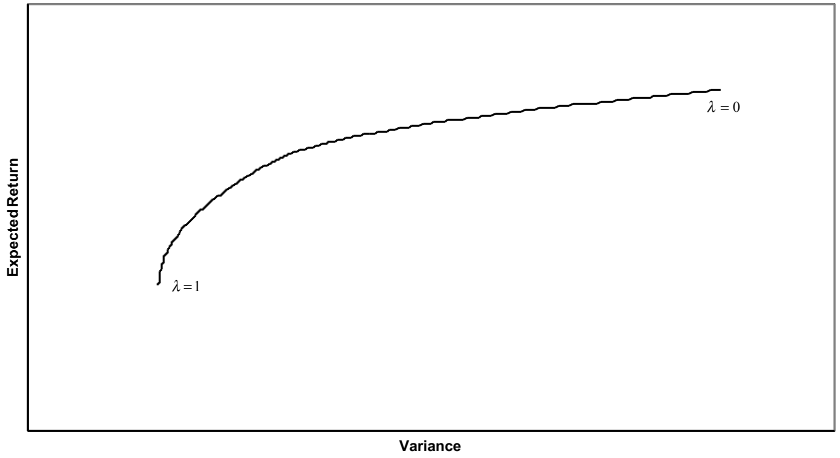
$$Min \qquad Z_{QP2} = \lambda \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \, x_j \sigma_{ij} - (1-\lambda) \sum_{i=1}^{N} x_i \mu_i$$

$$s.t. \qquad \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1, \ldots, N$$

$$0 \leq \lambda \leq 1$$

Efficient Frontier



Alternative Models

- QP models
- Diagonal models
  - Cholesky decomposition
  - Multiplicative form
  - Factor models
- Mean Absolute Deviation (MAD)
- Minimax
- Goal Programming

Cholesky Model

- Decompose covariance matrix: $V = L^T L$
- *N* new variables y_i
- $y_i = \sum_{j=1}^{i} l_{ij} x_j \qquad i = 1,\dots, N$
- $$\min \quad Z_{DIAG\,1} = \sum_{i=1}^{N} y_i^2$$

  $$s.t. \quad y_i = \sum_{j=1}^{i} l_{ij} x_j \qquad i = 1,\dots, N$$

Multiplicative Model

- Return matrix R observed over T periods
- Mean returns: $\overline{R}$
- Define *S…(TxN)*: $S = \frac{1}{\sqrt{N-1}}(R - \overline{R})$
- Covariance matrix: $V = S^T S$
-

$$\min \quad Z_{DIAG\ 2} = \sum_{t=1}^{T} y_t^2$$

$$s.t. \quad y_t = \sum_{i=1}^{i} s_{it} x_i \qquad t = 1, \ldots, T$$

Factor Model

- K … factors
- $f_k, \beta_{ik}, a_i, e_i$
- Return : $r_i = \alpha_i + \sum_{k=1}^{K} \beta_{ik} f_k + e_i$

$$\min \quad Z_{DIAG} = \sum_{k=1}^{K} y_{P,k}^2 + \sum_{i=1}^{N} x_i \sigma_{\varepsilon_i}^2$$

$$s.t. \quad y_{P,k} = \sum_{i=1}^{N} x_i \beta_{iK} \sigma_{f_k} \qquad k = 1,\dots, K$$

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QP Solution Algorithms

- Optimality criteria
- Sparse Simplex (SSX)
- Interior Point Method (IPM)
- Alternative Solution Algorithms

Problem Statement

$$
\begin{aligned}
(PQP) \quad & \min \ c^T x + \frac{1}{2} x^T Q x \\
& s.t. \quad A\,x \ge b, \quad \Rightarrow Ax - w = b, \\
& \qquad\quad x, w \ge 0.
\end{aligned}
$$

$$
\begin{aligned}
(DQP) \quad & \max \ b^T v - \frac{1}{2} x^T Q x \\
& s.t. \quad A^T v + u - Qx = c, \\
& \qquad\quad v, u \ge 0.
\end{aligned}
$$

Where $c, x, u \in R^n; b, w, v \in R^m; Q \in R^{n\times n}; A \in R^{m\times n}$
and Q is symmetric positive semi-definite.

Optimality criteria

- Lagrange multipliers

$$L(x,w,v,u) = c^T x - x^T Qx - v^T (Ax - w - b) - u^T x$$

- Karush-Kuhn-Tucker(KKT) necessary conditions

$$\begin{array}{lcl} \nabla_{(v,x)} L(x,w,v,u) & = & 0, \\ v^T w & = & 0, \\ u^T x & = & 0, \\ w,x,v,u & \geq & 0. \end{array}$$

SSX Primal

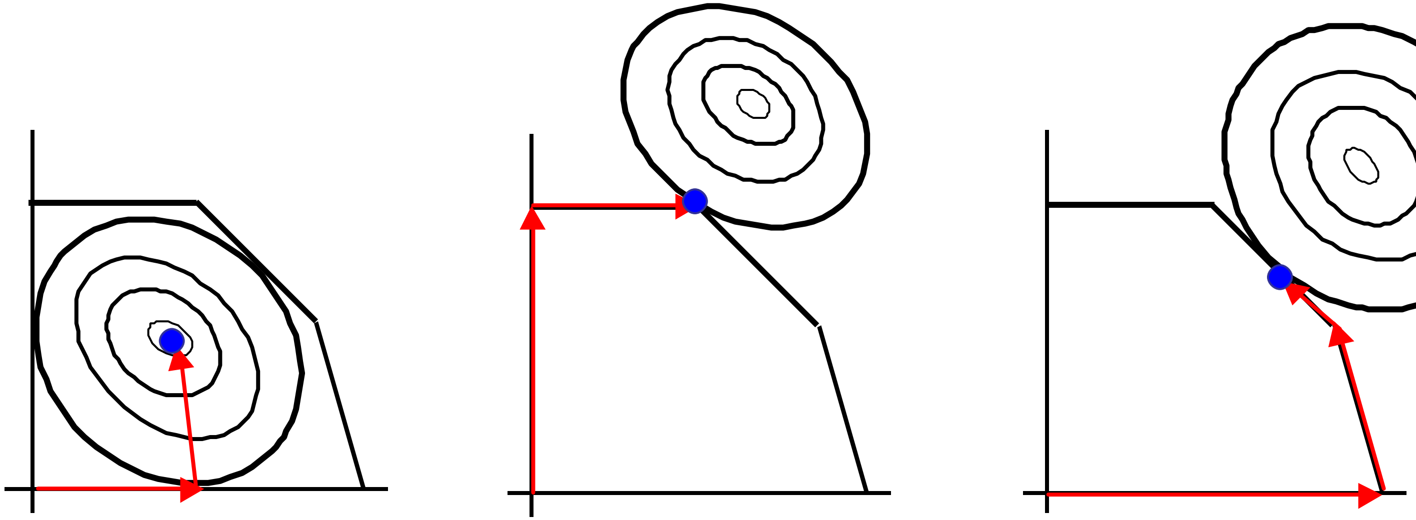
KKT condition

$$\begin{aligned} Ax - w &= b, \\ -Qx - v^T A - u &= c, \\ v^T w &= 0, \\ u^T x &= 0, \\ w, x, v, u &\geq 0. \end{aligned}$$

Initial tableau

Basic variables	Value of basic variables	Non basic variables	
	x_0	-x	-v
u_0	0	c^T	b^T
u	-c	Q	A^T
w	-b	$-A^T$	0

Visualisation of SSX



Salient aspects of SSX

- Solution of a (un/skew) symmetric linear system
- Sparse system … (sparse matrix technology… unstructured sparsity)
- Solved in nonnegative variables
- Iterations … fast sparse transformations and update of basis factors
- Refactorisation of a given "Basis" … a fast procedure … "Warm Restart"
- Number of iterations usually k times the number of equations (appr. 0<k<10)
- Primal/Dual algorithms can be used to process a family of models/problems

IPM optimality criteria

Applying barrier method to *PQP*

$$\begin{aligned} &\min \quad c^T x + \frac{1}{2} x^T Q x - \mu \sum_i \log x_i - \mu \sum_i \log w_i \\ &s.t. \quad A x - w = b, \end{aligned}$$

Introducing Lagrange multipliers, $y \in \Re^m$

$$\min \quad c^T x + \frac{1}{2} x^T Q x - \mu \sum_i \log x_i - \mu \sum_i \log w_i - y^T (A x - w - b)$$

Applying the KKT condition results into

$$\begin{aligned} &A x - w = b, \\ &-Qx + A^T y + z = c, \\ &XZe = \mu e, \\ &YWe = \mu e. \end{aligned}$$

where $z = \mu X^{-1} e$ and

X,Z,Y,W are diagonal matrices

$$X = \begin{bmatrix} x_1 & & \\ & \ddots & \\ & & x_n \end{bmatrix}$$

IPM algorithm

Set $(x+\Delta x; w+\Delta w; y+\Delta y; z+\Delta z)$

$\mu \in \Re$

Solve

$$\begin{bmatrix} -(X^{-1}Z+Q) & A^T \\ A & Y^{-1}W \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} c - A^T y + \mu X^{-1} e + Qx \\ b - Ax - \mu Y^{-1} e \end{bmatrix}$$

Where

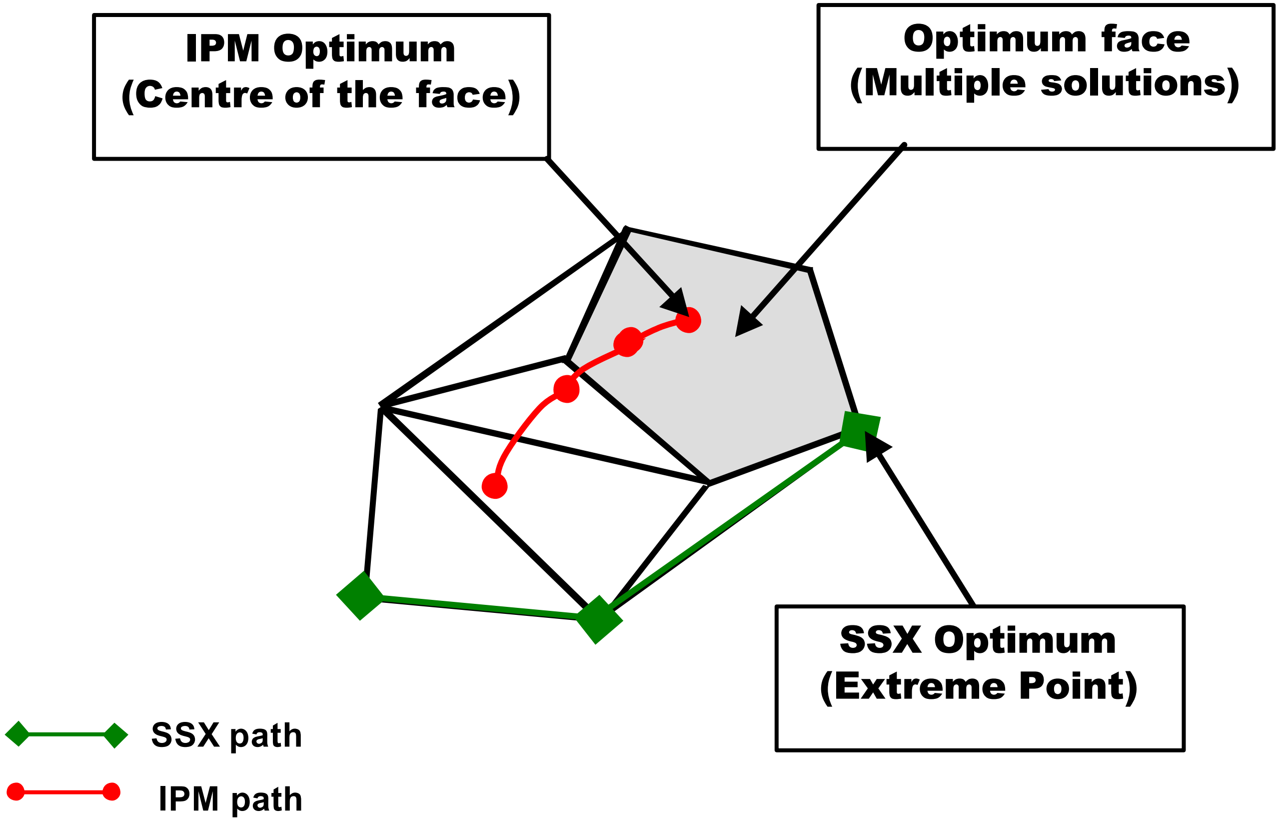
$$\Delta z = X^{-1}(\mu e - XZe - Z\Delta x)$$

$$\Delta w = Y^{-1}(\mu e - YWe - W\Delta y)$$

Salient aspects of IPM

- Repeated application of the Newton step to solve a nonlinear system
- Solution of a symmetric system of linear equations
- Either a system of “normal equations” or an “augmented system” is solved
- Sparse representation is an issue and determines which of the two forms is appropriate
- Number of iterations is “nearly” invariant of model size; usually $0<k<100$…!

IPM vs. SSX



IPM Optimum (Centre of the face)

Optimum face (Multiple solutions)

SSX Optimum (Extreme Point)

◆—◆ SSX path

●—● IPM path

Alternative Solution Algorithms

The other favoured approaches are

- the reduced-gradient method
- conjugate directions/gradient method
- method based on trust region

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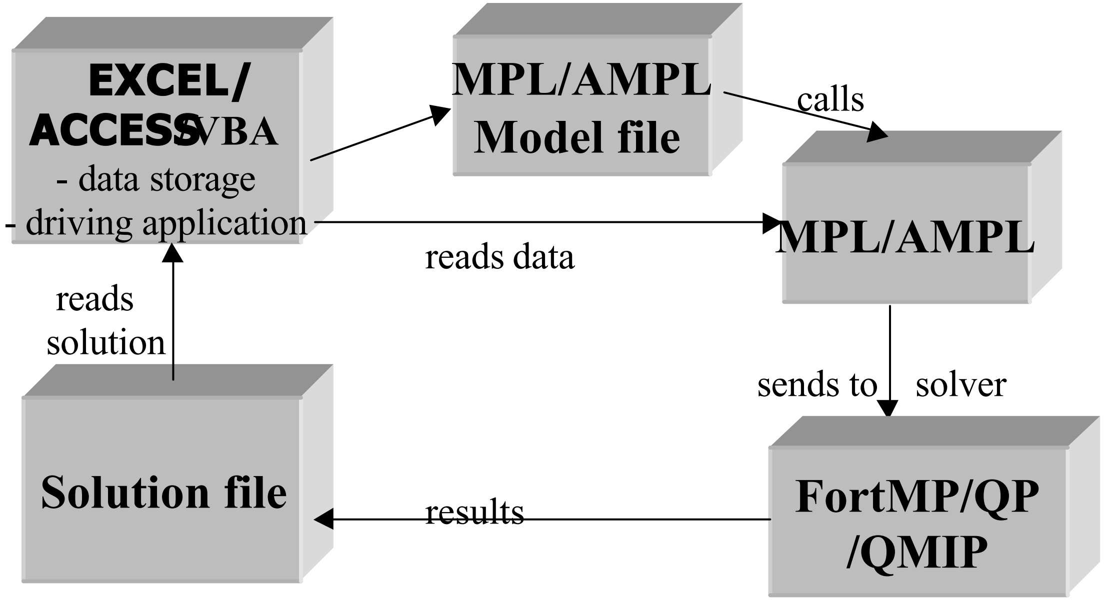
Computational Investigation of QPs

Interested in

- the scale up property of the algorithms
- the solution speed
- identifying the features of the algorithms which support processing of a family of problems
- identifying the impact of model sparsity on processing speed

Tools and Computational Platform

- Solver System (FortMP/QP/QMIP)
- Mathematical Programming Modeling languages (MPL/OptiMax, AMPL/)
- EXCEL/ACCESS,VBA – application



- Hardware:

  Pentium IV, 2 GHz, 1GB RAM

  MS Digital Fortran, C compilers

Choice of Test Data

- *Factor models (22 factors)*

	Number of rows	Number of columns	Number of non-zeros	Number of Q non-zeros
QPF50	25	72	436	228
QPF100	25	122	784	278
QPF500	25	522	3960	678
QPF1000	25	1022	8037	1178
QPF4500	25	4522	35641	4678

- *Var/Covar models*

	Number of rows	Number of columns	Number of non-zeros	Number of Q non-zeros
QPMV50	3	50	100	1275
QPMV100	3	100	200	5050
QPMV500	3	500	1000	125250
QPMV1000	3	1000	2000	500500
QPMV4500	-	-	-	-

Results for QP problems

- *Factor models*

	IPM		SSX	
	Number of iterations	Total time	Number of iterations	Total time
QPF50	9	0.02	72	0.02
QPF100	9	0.02	126	0.03
QPF500	11	0.13	527	0.28
QPF1000	11	0.39	1009	1.23
QPF4500	12	6.59	3280	29.36

- *Var/Covar models*

	IPM		SSX	
	Number of iterations	Total time	Number of iterations	Total time
QPMV50	7	0.01	66	0.05
QPMV100	7	0.06	120	0.08
QPMV500	5	7.41	528	11.05
QPMV1000	10	181.05	1002	162.28
QPMV4500				

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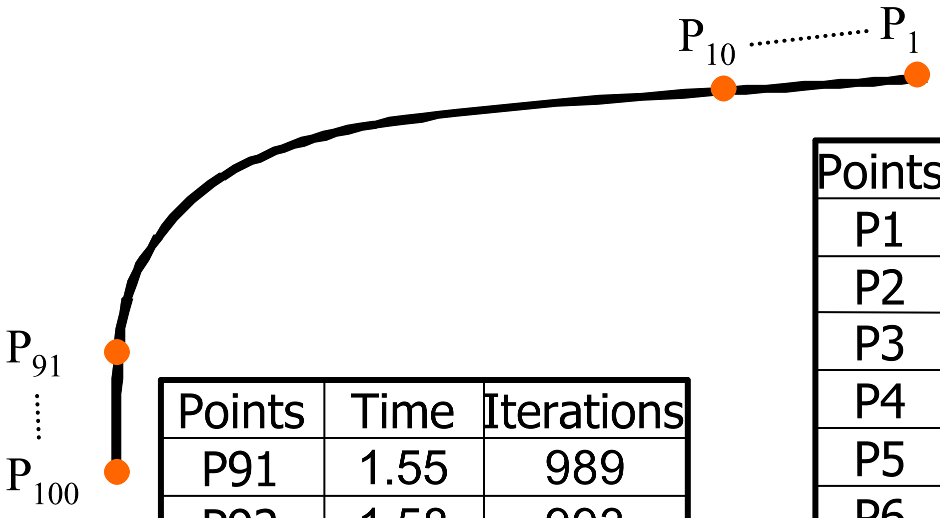
CEF with 1000 stocks

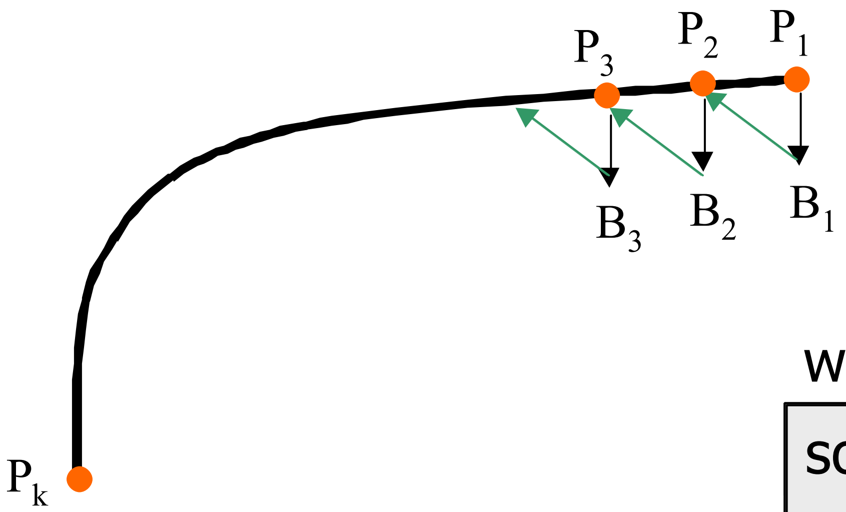
(39809 iterations, 52.05 seconds, 100 points)



Points	Time	Iterations
P1	0.05	6
P2	0.03	11
P3	0.05	11
P4	0.05	18
P5	0.03	17
P6	0.03	13
P7	0.05	22
P8	0.05	22
P9	0.05	28
P10	0.05	25
SUM	*0.44*	*173*

Points	Time	Iterations
P91	1.55	989
P92	1.58	993
P93	1.55	993
P94	1.55	995
P95	1.56	995
P96	1.56	995
P97	1.56	991
P98	1.55	990
P99	1.56	984
P100	1.58	981
SUM	*15.60*	*9906*

CEF solution algorithms

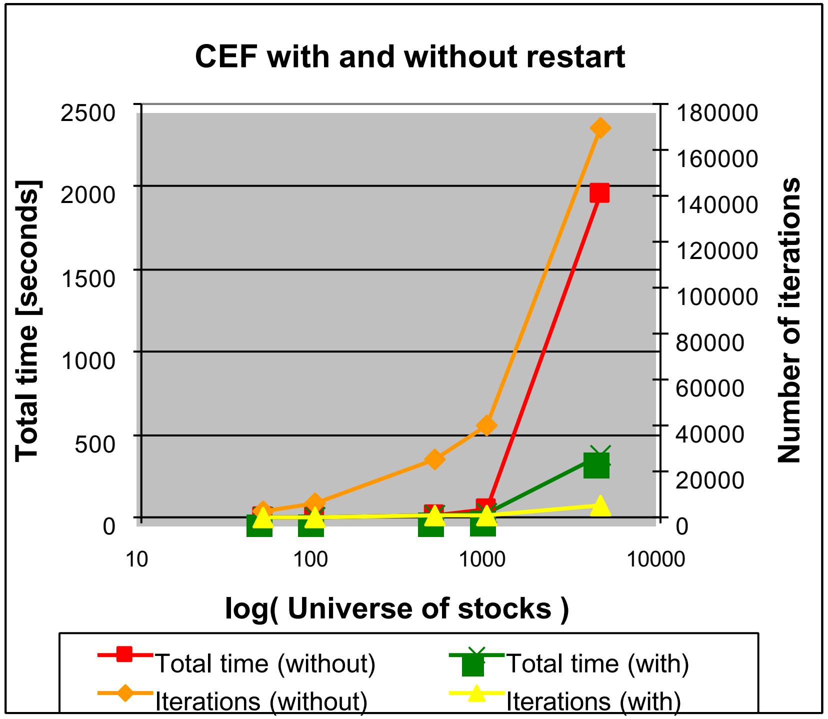


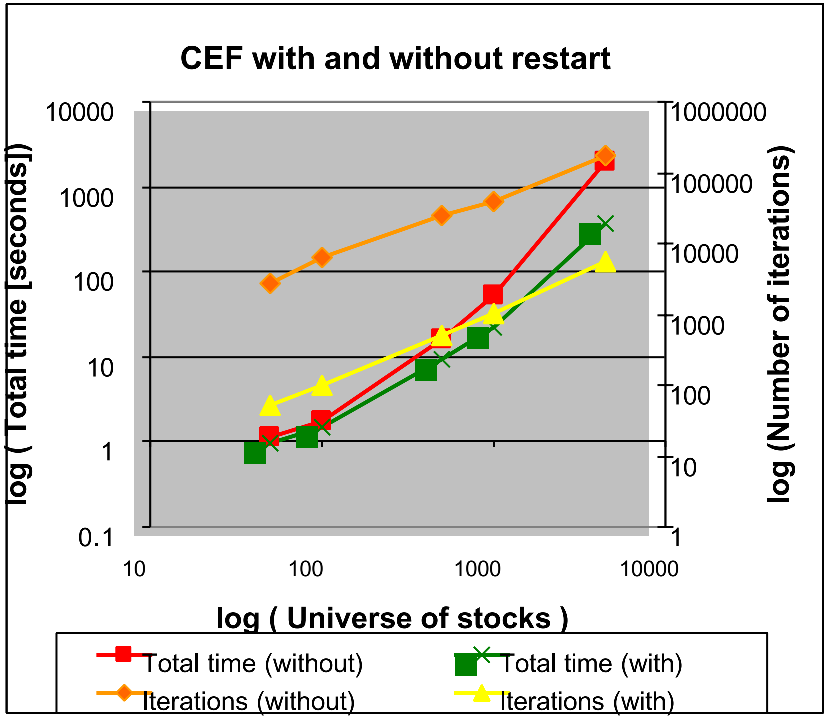
with basis restart:

```
solve $P_1$
save basis $B_1$
for t=2,...,k
        restart with basis $B_{t-1}$
        solve $P_t$
        save basis $B_t$
repeat
```

Computation of CEF

	Without restart		With restart	
	Iterations	Total time	Iterations	Total time
QPF50	2756	1.15	53	0.98
QPF100	6372	1.77	103	1.53
QPF500	25486	15.67	519	9.20
QPF1000	39809	52.05	1041	21.73
QPF4500	169946	1961.29	5541	375.21





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Discrete Constraint Efficient Frontier (DCEF)

- Buy-in thresholds
  - min. level below which an asset is not traded
  - eliminates unrealistically small trades
- Cardinality Constraints
  - controls the number of stocks in a portfolio
  - monitoring and control issues (management effort)
- Roundlots
  - trades only in multiples of ‘discrete’ numbers of assets possible

Mathematical Representation (1)

Extending QP1 with discrete constraints

- Buy-in thresholds

$$Min \qquad Z_{BUY-IN} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$s.t. \qquad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$l_i \delta_i \le x_i \le u_i \delta_i, \quad i = 1,\ldots,N$$

$$\delta_i \in \{0,1\}, \quad i = 1,\ldots,N$$

l_i, u_i : lower and upper bound on the stock weight

δ_i : binary variable

- Cardinality Constraints

$$\sum_{i=1}^{N} \delta_i = k$$

k: number of assets

Mathematical Representation (2)

- Transaction Roundlots
  - integer number of blocks y_i
  - a lot can be illustratively expressed as fraction f_i of the portfolio wealth
  - re-express x_i as $x_i = y_i f_i, \qquad i = 1, \dots, N$

$$Min \qquad Z_{LOT} = \sum_{i=1}^{N} \sum_{j=1}^{N} y_i f_i \; y_j f_j \sigma_{ij} + \gamma(\varepsilon^- + \varepsilon^+)$$

$$s.t. \qquad \sum_{i=1}^{N} y_i f_i \mu_i = \rho$$

$$\sum_{i=1}^{N} y_i f_i + \varepsilon^- - \varepsilon^+ = 1$$

$$l_i \leq y_i f_i \leq u_i, \quad i = 1, \dots, N$$

$$y_i \quad \text{integer}, \quad i = 1, \dots, N$$

$$\varepsilon^-, \varepsilon^+ \geq 0$$

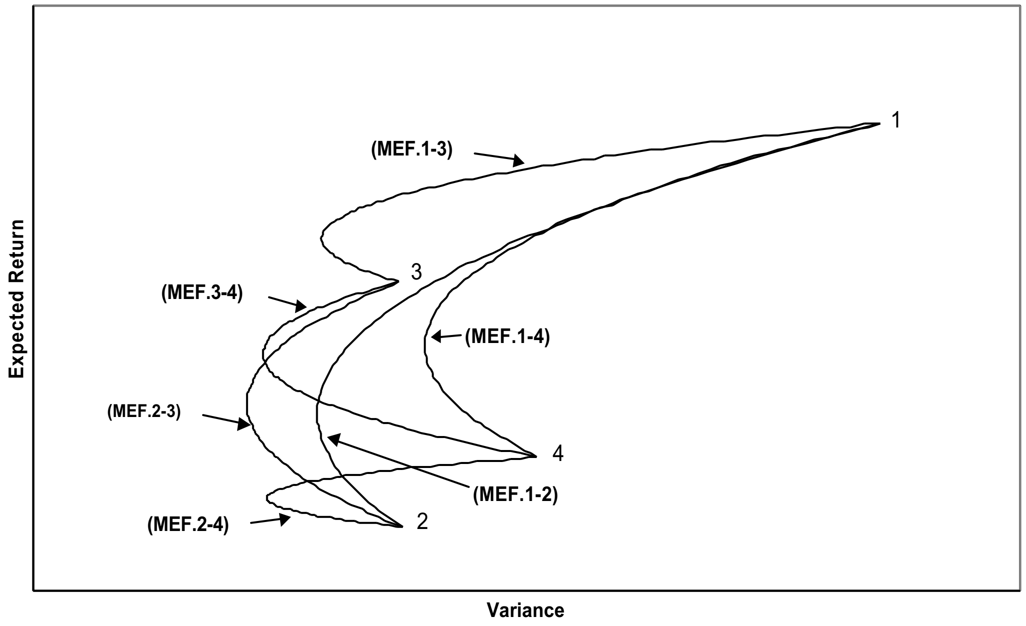
$\varepsilon^-, \varepsilon^+$: undershoot, overshoot variable

γ : penalty

Discrete Constraint Efficient Frontier (DCEF) (2)

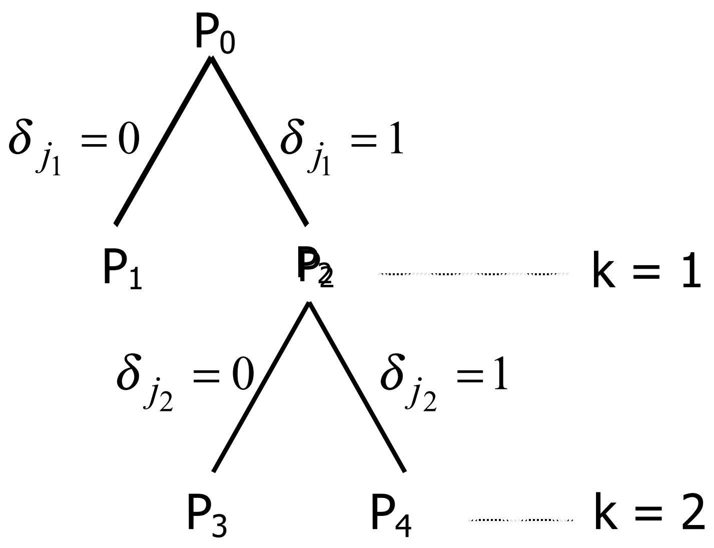
Why discontinuities?

- take investment opportunity set
- delete all inefficient portfolios (dominated points)



Branch & Bound … Tree Search

a pedestrian approach (with great „promise"!)

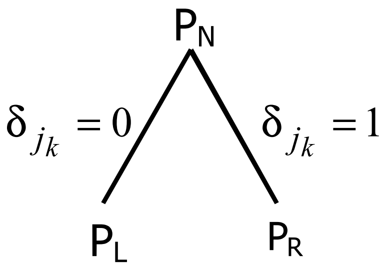


k: tree depth

P_N: parent problem

P_L: Left branch P_N plus $\delta_{j_k} = 0$

P_R: Right branch P_N plus $\delta_{j_k} = 1$



(i) „ Warm Restart" of P_L, P_R using „basis of P_N "

(ii) Dual algorithm to solve P_L, P_R

Together they speed up the search process

Statistics for QMIP factor models

(22 factors)

	Number of rows	Number of columns	Number of non-zeros	Number of Q rows	Number of binaries	Cardinality
QMIPFR50	126	122	686	72	50	3
QMIPFR150	326	322	1878	172	150	20
QMIPFR300	626	622	3775	322	300	30
QMIPFR600	1226	1222	7746	622	600	30
QMIPFR1200	2426	2422	15606	1222	1200	30
QMIPFR1800	3626	3622	23352	1822	1800	40
QMIPFR4500	9026	9022	58141	4522	4500	40

Problems with DCEF

- Discrete constraint efficient frontier (DCEF)
- If the tree-search for each point on the DCEF is not complete, we get anomalous results . . . points computed are no longer efficient
- Remedy:

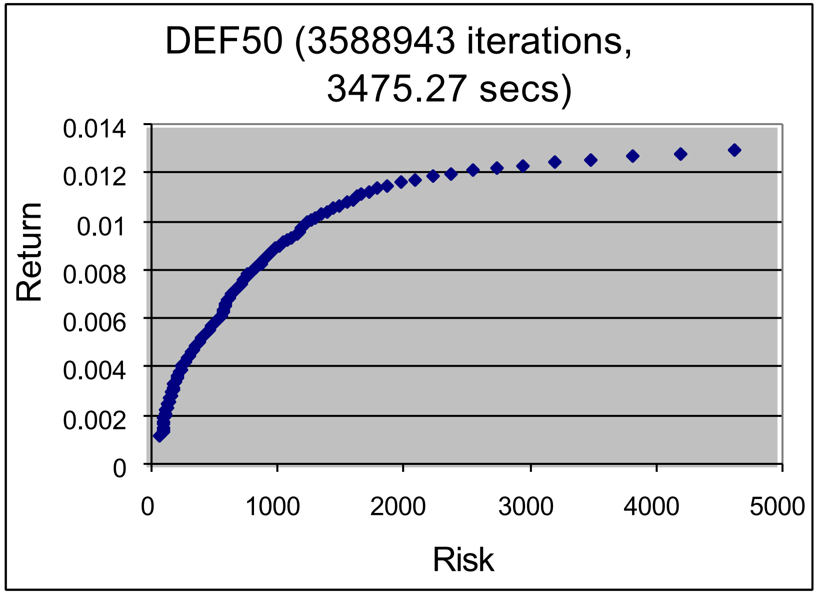
  Reuse the previous "best solution" as the starting point of tree search . . . efficiency is not undermined
- Alternative restarts:

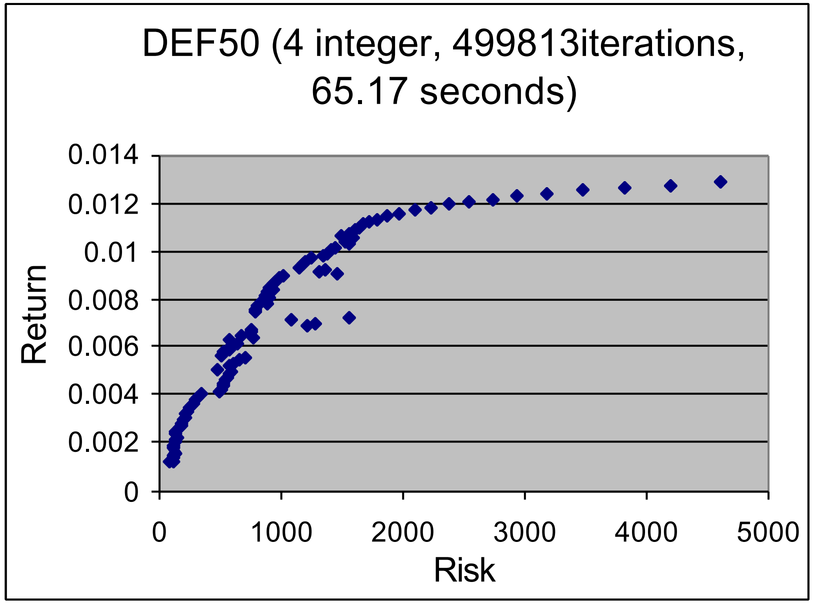
  (1) Use "previous integer solution"

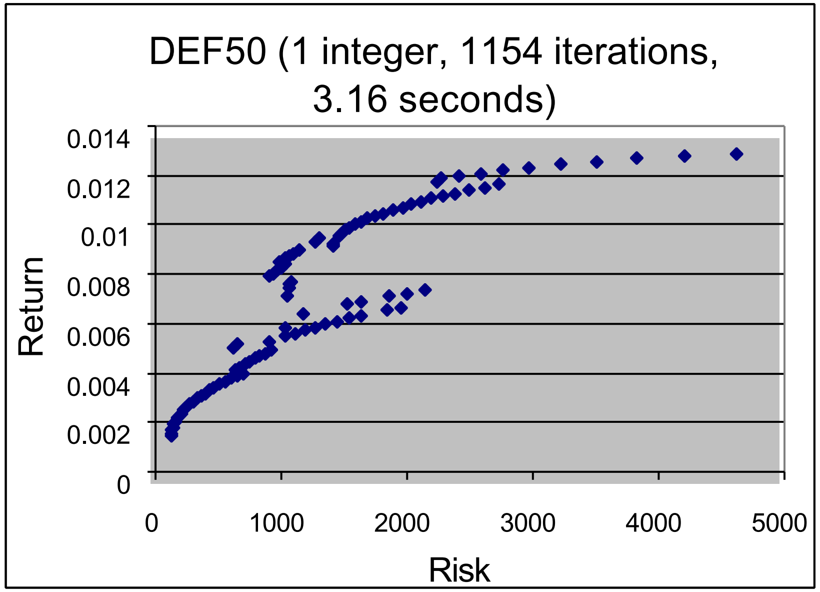
  (2) Use "previous integer solution"

  plus " previous basis"

DCEF without restart

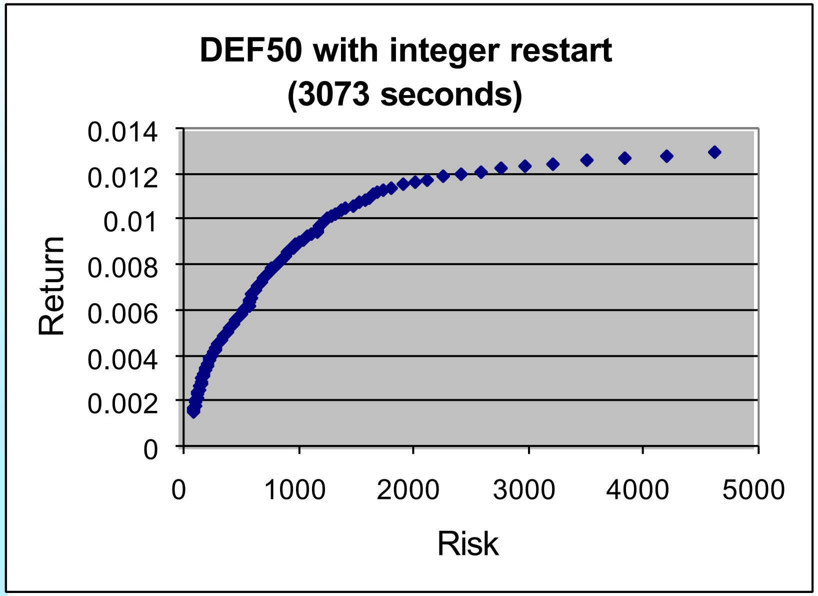


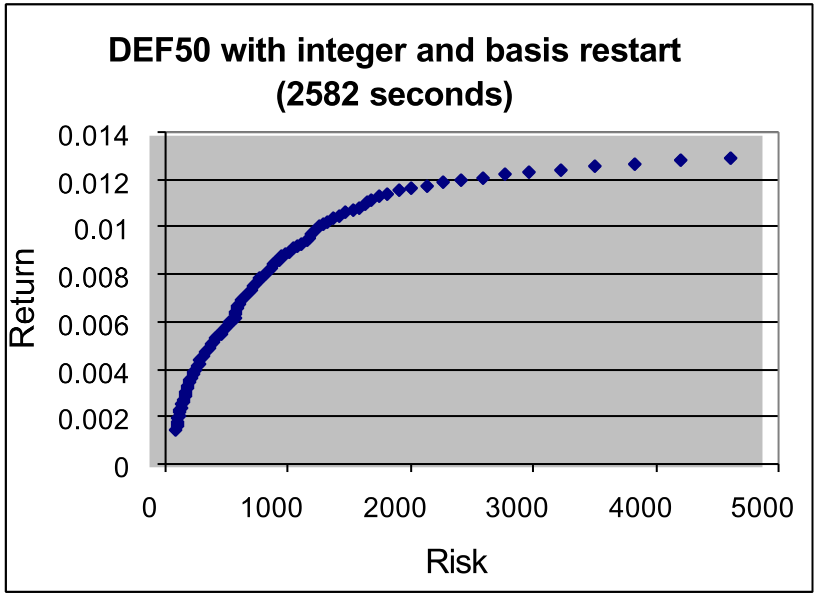




	Number of iterations	Total time
QMIPFR50	3588943	3475.27
QMIPFR150	59883797	37336.11
QMIPFR300	12793358	110560.80

DCEF with restart





	Total time without restart	Total time with restart (BAS+INT)
QMIPFR50	3475.27	2580.30
QMIPFR150	37336.11	23373.97
QMIPFR300	110560.80	71671.02

QMIP results

- up to 1 integer solution

	Value of cont. obj. function	Value of discrete obj. function	Number of iterations (nodes)	Branch & Bound time	Total time
QMIPFR50	19.678784	118.30733	133(3)	0.03	0.05
QMIPFR150	11.486424	32.913636	405(20)	0.28	0.36
QMIPFR300	6.0598815	32.634988	696 (30)	1.08	1.25
QMIPFR600	2.67354	19.831407	1300 (30)	4.91	5.73
QMIPFR1200	1.5793611	18.945271	2533 (40)	16.41	19.47
QMIPFR1800	1.2640586	16.470851	4043 (40)	39.08	49.55
QMIPFR4500	0.84480798	13.268292	8518 (40)	304.83	440.30

- up to 10 integer solution

	Value of cont. obj. function	Value of discrete obj. function	Number of iterations (nodes)	Branch & Bound time	Total time
QMIPFR50	19.678784	69.852319	22339 (10744)	36.03	36.06
QMIPFR150	11.486424	28.436833	59569 (29178)	175.06	175.13
QMIPFR300	6.0598815	20.51969	11548 (2896)	42.09	42.32
QMIPFR600	3.3194915	19.665274	131240 (50000)	1399.72	1400.19
QMIPFR1200	1.5793611	15.831303	1634129 (50000)	19297.97	19306.33
QMIPFR1800	-	-	-	-	-
QMIPFR4500	-	-	-	-	-

Two-stage Group Branching Heuristic

In this procedure t_i out of k (zero-one variables)

$$k \equiv 0 \bmod(t_i)$$

are fixed at a given heuristic step *i* . For $t_i = 1$ this amounts to Branch & Bound.

(i) Parent basis is reused
(ii) Dual algorithm is also applied

- This leads to much faster fathoming of the search tree.
- Good integer solutions are obtained faster than the ordinary branch and bound procedure of QMIP

UBS Warburg Dataset

	Model 1	Model 2	Model 3	Model 4	Model 5
Universe of stocks	757	1,304	1,305	1,305	1,305
Initial portfolio size	332	251	251	251	251
Target for maximum assets	400	250	250	250	250
Risk acceptance parameter	0.6	0.6	0.6	0.6	0.6

Hardware:

Pentium II, 500 MHz, 128 MB RAM

MS Digital Fortran, C compilers

UBS Warburg- Model 1; Model 2

Model 1		
Relaxed QP	Objective Value	1.8882922E-14
	Time to optimum (secs)	32.42
FortMP (QMIP)	IP Nodes	400
	IP processing time	1,903.94
	IP Objective	2.8018533E-08
Two-stage ...	IP Nodes	129
	Time (secs)	121.48
	Objective function	2.8437663E-08

Model 3		
Relaxed QP	Objective Value	3.2291911E-16
	Time to optimum (secs)	172.32
FortMP (QMIP)	IP Nodes	250
	IP processing time	2,943.12
	IP Objective	1.7839276E-06
Two-stage ...	IP Nodes	84
	Time (secs)	235.45
	Objective function	1.5851747E-06

Compare with modern heuristics

- Computing the entire DCEF to optimality is computationally challenging
- Integer restart heuristic computes a reasonable number of optimal or near optimal points within a restricted B&B search
- Both methods outperform modern heuristic approaches (reported average errors are about 1%)
- Real application, only a segment of the frontier may be of interest, Integer restart approach can be used to “zoom in” and compute few alternative portfolios exact or at least more accurate

Compare with modern heuristics

- Beasley models
- Integer restart heuristic

Index	No. of Stocks	Total no. of DCEF pts	No. of integer optimal pts	Solution time *	Mean Error	Median Error
Hang Seng	31	500	492	57.55	0.01415	0.00997
DAX	85	500	228	8405.33	0.01399	0.01159
FTSE	89	500	244	10978.12	0.01141	0.00860
S & P	98	500	192	15831.97	0.01586	0.01325
Nikkei	225	500	486	18345.56	0.00618	0.00252

* Pentium 500, 128 MB RAM

Compare with modern heuristics

- Comparison to modern heuristic approaches

Index	No. of Stocks	Solution Method	No. of efficient points	Mean Error	Median Error
Hang Seng	31	Integer restart heuristic	500	0.01415	0.00997
			3000	0.00826	0.00628
		Rounding heuristic	103	0.00021	0.00051
		GA heuristic	1317	0.94570	1.18190
		TS heuristic	1268	0.99080	1.19920
		SA heuristic	1003	0.98920	1.20820
		pooled (GA, TS, SA)	2491	0.93320	1.18990
DAX	85	Integer restart heuristic	500	0.01399	0.01159
		Rounding heuristic	349	0.01444	0.01155
		GA heuristic	1270	1.95150	2.12620
		TS heuristic	1467	3.06350	2.53830
		SA heuristic	1135	2.42990	2.46750
		pooled (GA, TS, SA)	2703	2.19270	2.46260

No direct comparison possible as results for modern heuristics are not available to the same detail !

*** Beasley, Chang et al. (2000)**

Outline

- Background
- QP portfolio models
- QP solution algorithms
- Computational performance of algorithms
- Solving family of QP models
- Solving QMIP models
- Discussion
- References

Discussion

- All large scale models are sparse
- Sparsity of a model is important in determining choice of algorithms
- Algorithms which may be efficient for a single QP model may not be the best to process a family of models
- Typical family of models arise in

  (i) Continuous efficient frontier

  (ii) Discrete constraint QMIP models

  (iii) Discrete efficient frontier

  (iv) Resampled efficient frontier (?)

  Speed up and scale up are challenging issues in real applications
- Almost all the above cases (i)...(iv) provide scope of scale up and speed up through parallelisation
- Exploitation of Mathematical properties of primal and dual representation lead to superior processing and better results than a ‘simplistic’ application of modern heuristics

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Thank You

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