A Review of Portfolio Planning: Models and Systems

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Outline

- Introduction and Overview
- An Information Systems Perspective: Financial Data marts
- Alternative Computational Models
- Solution Algorithms and Computational Experience
- Discussions

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Introduction and Overview

- Modelling Paradigm
  - Markowitz M-V model
  - Risk and return…two objectives
  - Efficient frontier…Pareto optimal
  - Utility function…risk aversion
- Role of Information Systems (IS)
- Risk Metrics
- Computational Solution

Mean-Variance Model

- Markowitz (1952,1959)
- alternative formulations

QP1

$$Min \quad Z_{QP1} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$s.t. \quad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1,\ldots,N$$

QP2 (Arrow-Pratt absolute risk aversion index)

$$Max \quad Z_{QP2}^{R_A} = \frac{R_A}{2}\sum_{i=1}^{N} x_i \mu_i - \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$s.t. \quad \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1,\ldots,N$$

$$R_A \geq 0$$

$i, j = 1,\ldots,N$: denotes the different risky assets

μ_i : expected return of asset i

σ_{ij} : covariance between asset i and asset j

ρ : desired level of return

x_i : the fraction of portfolio value invested in asset i

Mean-Variance Model

- Arrow Pratt absolute Risk Aversion Index

$$R_A = \frac{u''(w)}{u'(w)}$$

where

w is the portfolio wealth

u a Von Neumann-Morgenstern utility function with first and second derivatives

- Portfolios with similar Absolute Risk Aversion index yield in similar portfolios (weight-vector) regardless of functional form and parameters of the utility function (Kallberg and Ziemba 1983)

Mean-Variance Model

QP2 (Lambda-formulation)

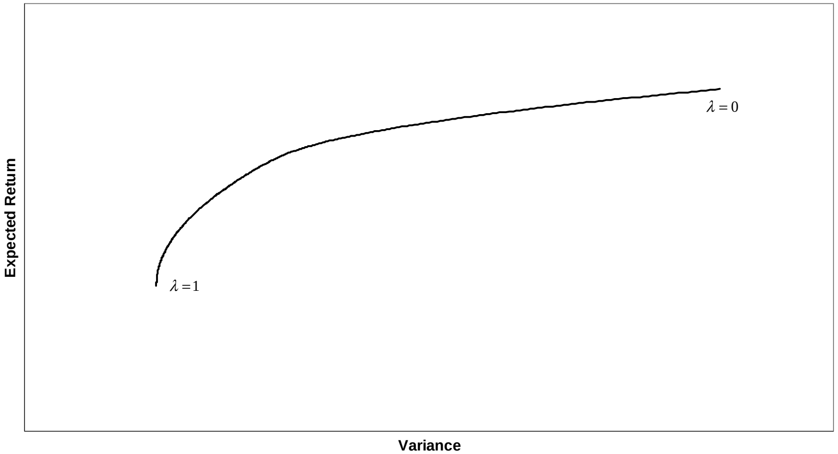
$$Min \qquad Z_{QP2} = \lambda \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \, x_j \sigma_{ij} - (1-\lambda) \sum_{i=1}^{N} x_i \mu_i$$

$$s.t. \qquad \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1, \ldots, N$$

$$0 \leq \lambda \leq 1$$

Efficient Frontier



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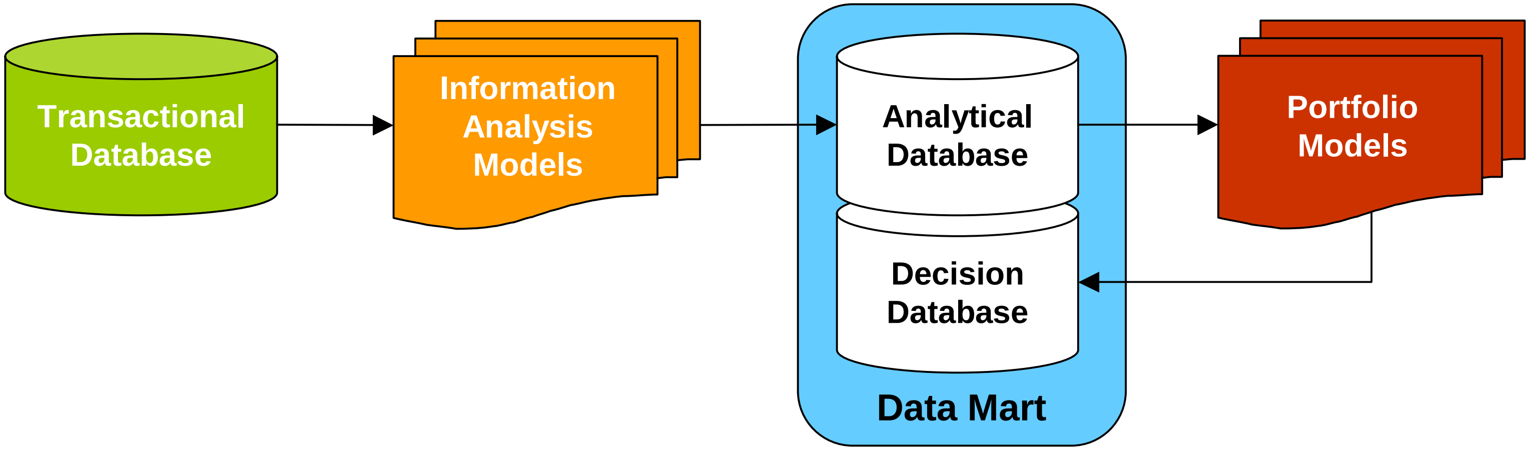
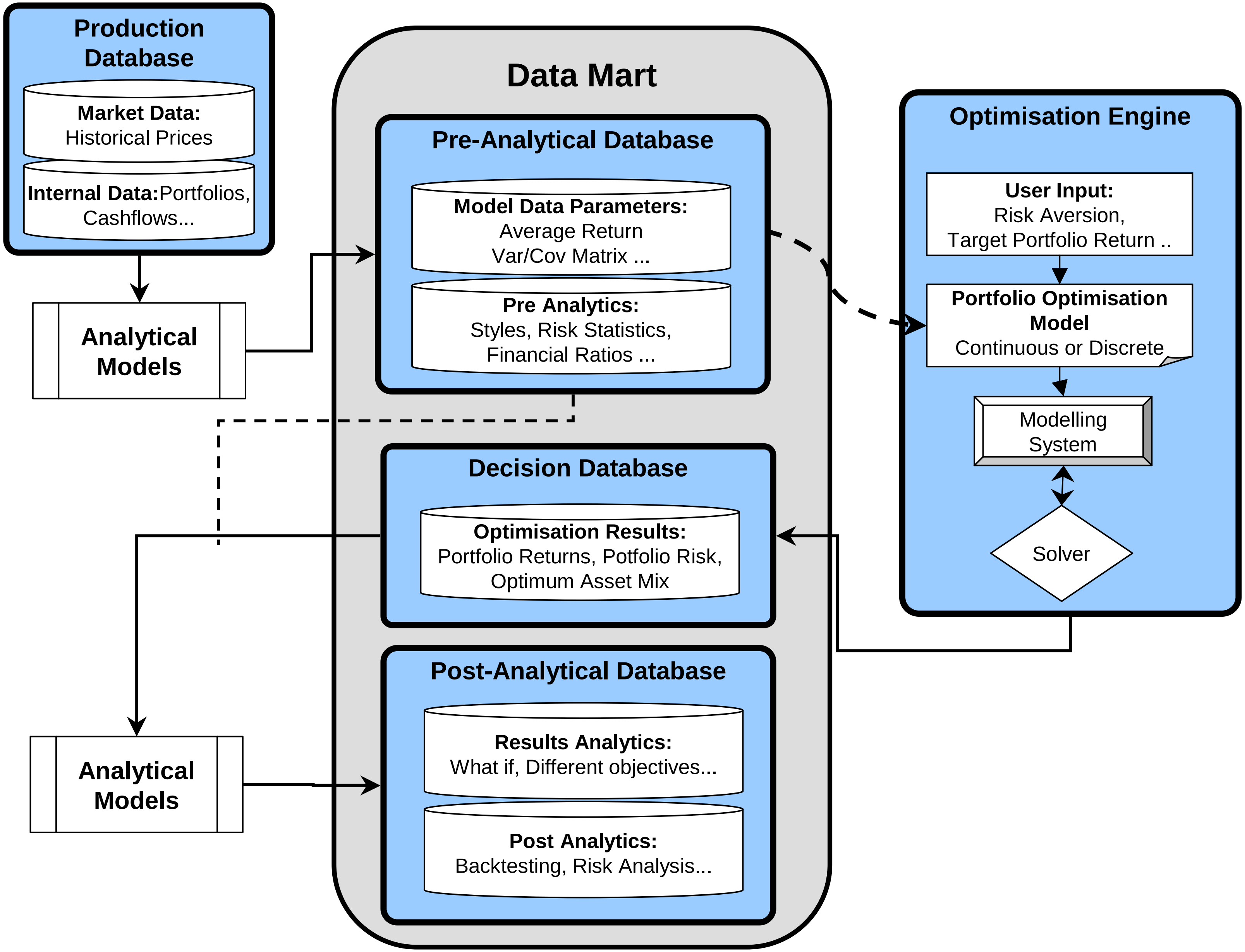
Information Systems…Data marts



Information Systems…Datamarts

Information Analysis Models			
Pre-analysis	**Model Data Parameters**	**Solution Analysis**	**Post-analysis**
Performance Indicators Style Analysis Financial Ratios CAPM APT Simulation Models Internal Company Models	Historical data Weighted Moving Average Factor Models Time Series Models ARCH, GARCH,… Neural Networks Genetic Algorithms Kalman Filters Chaos Internal Company Models	What if Analysis Scenario Analysis Simulation Backtesting Internal Company Models	Performance Indicators Risk Statistics and Indices Financial Ratios CAPM APT Simulation Models Risk Metrics Internal Company Models

Information Systems…Data marts



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Alternative Models

- QP models
- Diagonal models
  - Cholesky decomposition
  - Multiplicative form
  - Factor models
- Mean Absolute Deviation (MAD)
- Minimax
- Goal Programming

Diagonal Model 1…Cholesky

- Decompose covariance matrix: $V = L^T L$
- N new variables y_i
- $$y_i = \sum_{j=1}^{i} l_{ij} x_j \quad i = 1,\ldots,N$$
- Min $Z_{DIAG1} = \sum_{i=1}^{N} y_i^2$

Subject to

$$y_i = \sum_{j=1}^{i} l_{ij} x_j \quad i = 1,\ldots,N$$

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Diagonal Model 2…Multiplicative

- Return matrix *R* observed over *T* periods
- Mean returns: $\overline{R}$
- Define *S…(TxN)*: $S = \frac{1}{\sqrt{N-1}}(R-\overline{R})$
- Covariance matrix: $V = S^T S$
- Min $Z_{DIAG2} = \sum_{t=1}^{T} y_t^2$

Subject to

$$y_t = \sum_{i=1}^{N} s_{it} x_i \qquad t=1 \ldots T$$

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Diagonal Model 3…Factor

- K … factors
- f_k , β_{ik} , a_i , e_i
- Return : $r_i = \alpha_i + \sum_{k=1}^{K} \beta_{ik} f_k + e_i$
- Min $\quad Z_{DIAG} = \sum_{k=1}^{K} y_{P,k}^2 + \sum_{i=1}^{N} x_i^2 \sigma_{\varepsilon_i}^2$

Subject to

$$y_{P,k} = \sum_{i=1}^{N} x_i \beta_{ik} \sigma_{f_k} \qquad k=1\ldots K$$

MAD Model

- Konno and Yamazaki introduced MAD model as a credible alternative to M-V model:
  - No requirement for covariance matrix of asset returns
  - Linear program easier to solve than the quadratic

MAD model

- MAD model – special case of piecewise linear risk model

$$\text{Min} \quad Z_{MAD} = \frac{1}{T}\sum_{t=1}^{T} m_t$$

$$\text{Subject to} \quad \sum_{i=1}^{N} (r_{it} - \mu_i)\, x_i \le m_t \qquad t=1,\ldots,T$$

$$\sum_{i=1}^{N} (r_{it} - \mu_i)\, x_i \ge -m_t \qquad t=1,\ldots,T$$

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Practical Extensions

- Discrete restrictions
  - Buy-in thresholds
  - Cardinality
  - Roundlot
- Portfolio dedication
- Portfolio immunisation
- Convexity restriction

Discrete Constraint Efficient Frontier (DCEF) (1)

- Buy-in thresholds
  - min. level below an asset is not traded
  - eliminates unrealistically small trades
- Cardinality Constraints
  - controls the number of stocks in a portfolio
  - monitoring and control issues (management effort)
- Roundlots
  - trades only in multiples of ‘discrete’ numbers of assets possible

Mathematical Representation (1)

Extending QP1 with discrete constraints

- Buy-in thresholds

$$Min \quad Z_{BUY-IN} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$s.t. \quad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$l_i \delta_i \le x_i \le u_i \delta_i, \quad i = 1,\ldots, N$$

$$\delta_i \in \{0,1\}, \quad i = 1,\ldots, N$$

l_i, u_i : lower and upper bound on the stock weight

δ_i : binary variable

- Cardinality Constraints

$$\sum_{i=1}^{N} \delta_i = k$$

k: number of assets

Mathematical Representation (2)

- Transaction Roundlots
  - integer number of blocks y_i
  - a lot can be illustratively expressed as fraction f_i of the portfolio wealth
  - re-express x_i as $\quad x_i = y_i f_i, \qquad i = 1,\dots,N$

$$Min \qquad Z_{LOT} = \sum_{i=1}^{N}\sum_{j=1}^{N} y_i f_i \; y_j f_j \sigma_{ij} + \gamma(\varepsilon^- + \varepsilon^+)$$

$$s.t. \qquad \sum_{i=1}^{N} y_i f_i \mu_i = \rho$$

$\varepsilon^-, \varepsilon^+$: undershoot, overshoot variable

$$\sum_{i=1}^{N} y_i f_i + \varepsilon^- - \varepsilon^+ = 1$$

$$l_i \le y_i f_i \le u_i, \quad i = 1,\dots, N$$

γ : penalty

$$y_i \quad \text{integer}, \quad i = 1,\dots, N$$

$$\varepsilon^-, \varepsilon^+ \ge 0$$

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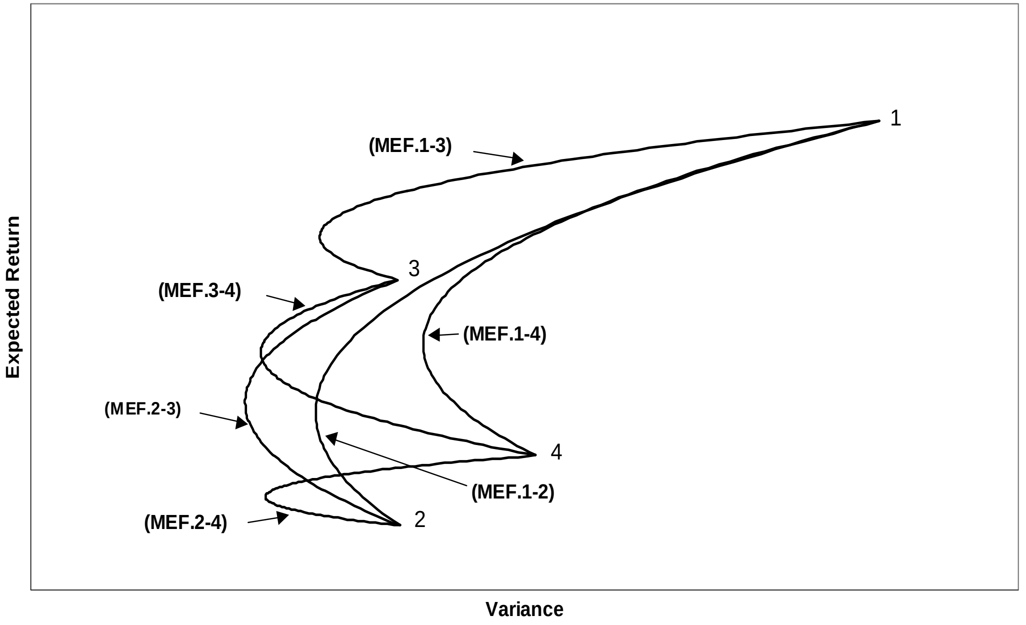
Discrete Constraint Efficient Frontier (DCEF) (2)

Why discontinuities?

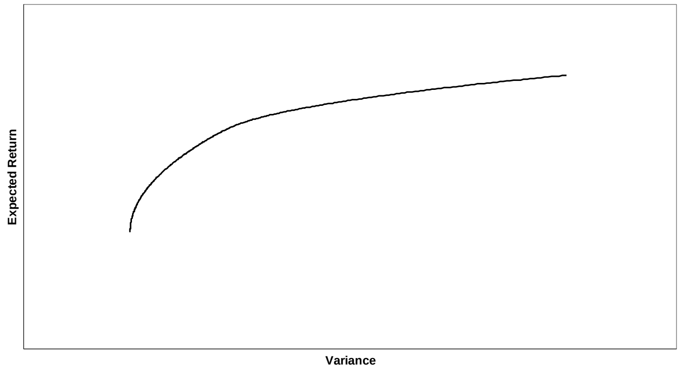
- take investment opportunity set
- delete all inefficient portfolios (dominated points)



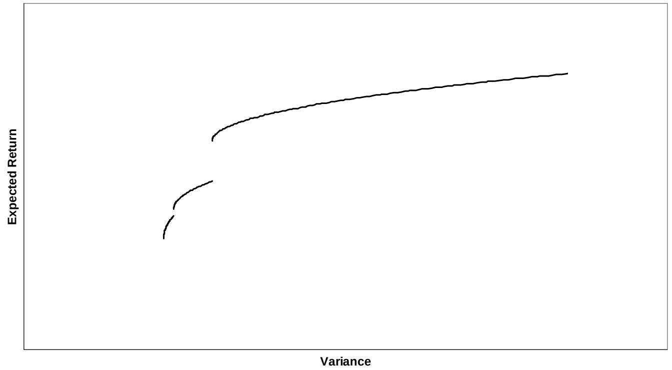
Discrete Constraint Efficient Frontier (DCEF) (3)

Example:

- 4 stock universe, EF



- introduce cardinality constraint, i.e. build a portfolios containing 2 stocks only



Portfolio Dedication (2)

Introduce two variables, v_t^+ and v_t^-, as cash surplus and shortfall respectively in time period t.

Then the restrictions set out below:

$$\sum_{i=1}^{N} F_{i0} x_i + v_0 + v_0^- = v_0^+,$$

$$\sum_{i=1}^{N} F_{it} x_i + (1 + r_t) v_{t-1}^+ + v_t^- = L_t + v_t^+ + (1 + r_t + \Delta) v_{t-1}^-, \quad \text{for all } t=1,\ldots,T.$$

capture portfolio dedication as cashflows matching with borrowing and reinvestment.

Portfolio Immunization

Duration of bond … weighted average of present values of cash flows

Let D_i denote the duration of the i^{th} bond

Then the duration of the portfolio is computed as

$D_P = x_1 D_1 + x_2 D_2 + \ldots + x_N D_{N\ldots}$

If we also compute the duration of all the liabilities, then by balancing the portfolio duration and liability duration

$D_P = x_1 D_1 + \ldots + x_N D_N = D_L$

we immunize the portfolio against interest rate risk.

Convexity Restrictions

- The price-yield relationship of a fixed-income security is a non-linear function for which the second-order condition (differential) is called convexity.
  - Q_i: convexity of the assets. The first derivative of duration (with respect to the interest rate) for asset i, $i=1,\ldots,N$;
  - Q_L: convexity of the liabilities.
- The constraint $Q_1x_1+Q_2x_2+\ldots+Q_Nx_N >= Q_L$, restricts the sensitivity to the shape of the term structure … 'shape risk' of the portfolio.

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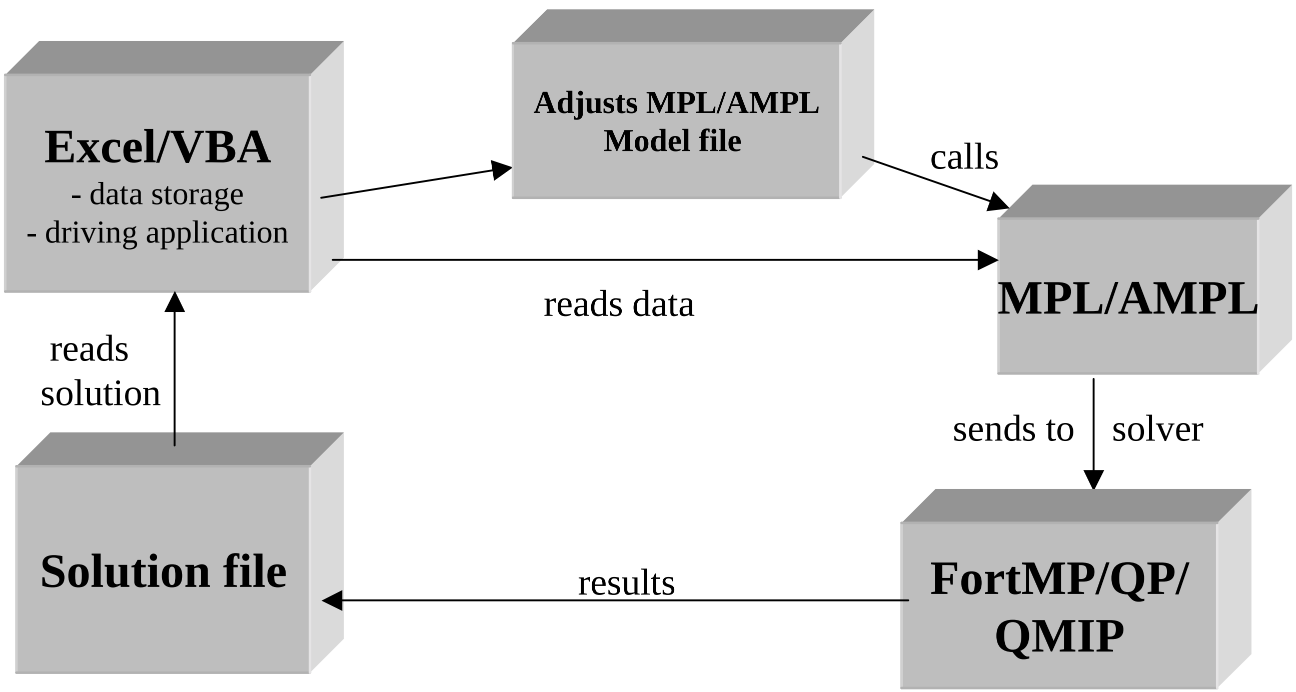
Computational Experience

- Industrial Applications
  - Advanced Portfolio Technology Inc.
  - Fidelity Investment Services
  - UBS Warburg Portfolio Analytics System
- Data sets:
  - -Maros/Meszaros
  - -J.Beasley
  - -Fidelity
  - -UBS Warburg
- Computing Platform: Windows NT/Windows 2000, Pentium III, 500 MHZ processor with 128 MB of RAM

Modelling and Solution Tools

- Solver System (FortMP/QP/QMIP)
- Mathematical Programming Modeling languages (MPL/ AMPL)
- EXCEL/VBA - application



Computational Experience

- Computing the entire DCEF to optimality is computationally challenging
- Integer restart heuristic computes a reasonable number of optimal or near optimal points within a restricted B&B search
- Both methods outperform modern heuristic approaches (reported average errors are about 1%)
- Real application, only a segment of the frontier may be of interest, Integer restart approach can be used to “zoom in” and compute few alternative portfolios exact or at least more accurate

Computational Experience

- Beasley models
- Integer restart heuristic

Index	No. of Stocks	Total no. of DCEF pts	No. of integer optimal pts	Solution time *	Mean Error	Median Error
Hang Seng	31	500	492	57.55	0.01415	0.00997
DAX	85	500	228	8405.33	0.01399	0.01159
FTSE	89	500	244	10978.12	0.01141	0.00860
S & P	98	500	192	15831.97	0.01586	0.01325
Nikkei	225	500	486	18345.56	0.00618	0.00252

* Pentium 500, 128 MB RAM

Computational Experience

- Comparison to modern heuristic approaches

Index	No. of Stocks	Solution Method	No. of efficient points	Mean Error	Median Error
Hang Seng	31	Integer restart heuristic	500	0.01415	0.00997
			3000	0.00826	0.00628
		Rounding heuristic	103	0.00021	0.00051
		GA heuristic	1317	0.94570	1.18190
		TS heuristic	1268	0.99080	1.19920
		SA heuristic	1003	0.98920	1.20820
		pooled (GA, TS, SA)	2491	0.93320	1.18990
DAX	85	Integer restart heuristic	500	0.01399	0.01159
		Rounding heuristic	349	0.01444	0.01155
		GA heuristic	1270	1.95150	2.12620
		TS heuristic	1467	3.06350	2.53830
		SA heuristic	1135	2.42990	2.46750
		pooled (GA, TS, SA)	2703	2.19270	2.46260

No direct comparison possible as results for modern heuristics are not available to the same detail !

*** Beasley, Chang et al. (1999)**

UBS Warburg Dataset

	Model 1	Model 2	Model 3	Model 4	Model 5
Stock Universe	757	1,304	1,305	1,305	1,305
Initial Portfolio Size	332	251	251	251	251
Target for Maximum Assets	400	250	250	250	250
Risk Acceptance Parameter	0.6	0.6	0.6	0.6	0.6

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UBS Warburg- Model 1

Model 1		
Relaxed QP	Objective Value	0.18882922E-13
	Time to optimum (secs)	32.42
FortMP (QMIP)	IP Nodes	400
	IP processing time	1,903.94
	IP Objective	0.28018533E-07
Two-stage Heuristics	IP Nodes	129
	Time (secs)	121.48
	Objective function	0.28437663E-07

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UBS Warburg- Model 3

		Model 3
Relaxed QP	Objective Value	0.32291911E-15
	Time to optimum (secs)	172.32
FortMP (QMIP)	IP Nodes	250
	IP processing time	2,943.12
	IP Objective	0.17839276E-05
Two-stage Heuristics	IP Nodes	84
	Time (secs)	235.45
	Objective function	0.15851747E-05

Discussions

- Portfolio planning
  - good examples of OR applied in Finance
- Convergence of skills:
  - Information engineering
  - Decision engineering
  - Computational algorithms
- There are some challenging problems of:
  - risk modelling
  - optimisation modelling
  - computational solutions

Thank you for your attention

Any questions?

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Variance

- The risk of the portfolio or investment return is measured by the variance:

$$\sigma_p^{\,2}(x) = \mathrm{E}\left[R(x,\tilde{r}) - R(x,\bar{r})\right]^2 = \sum_{i=1}^{m}\sum_{j=1}^{m}\rho_{ij}x_i x_j$$

- ρ_{ij} : the covariance between the returns of assets i and j.
- Using scenarios the covariance can be estimated by:

$$\rho_{ij} = \sum_{s\in S}\pi_s(r_i^{\,s} - \bar{r}_i)(r_j^{\,s} - \bar{r}_j)$$

- $\rho_{ii} = \sigma_i^2$

Semi-Variance

- Variance as a measure of risk is symmetric.
- Left Semi-Variance focuses only on downside deviations and ignores upside deviations

$$\sigma_L^{\;2} = \mathrm{E}[\left(\max\left\{0, R(x,\bar{r}) - R(x,\tilde{r}\right\}\right)^2]$$

- We can also define the Right Semi-Variance that measures upside deviations

$$\sigma_R^{\;2} = \mathrm{E}[\left(\max\left\{0, R(x,\tilde{r}) - R(x,\bar{r}\right\}\right)^2]$$

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Mean Absolute Deviation

- Konno (1988) introduced Mean Absolute Deviation as a Linear Measure of Risk
- Absolute deviation is defined as:

$$\delta_p(x) = \mathrm{E}\left[\left|R(x,\tilde{r}) - R(x,\bar{r})\right|\right]$$

- Using scenarios we estimate the mean absolute deviation:

$$\hat{\delta}_p(x) = \sum_{s \in S} p^s \left| \sum_{i=1}^{m} (r_i^s - \bar{r_i}) x_i \right|$$

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Downside Risk

- The definition was introduced by Bawa (1975) and Fishburn (1977)
- Penalise only negative returns relative to a given benchmark – target θ
- $R_\gamma = E([\max\{\theta - x, 0\}]^\alpha) = \int_{-\infty}^{\theta} (\theta - x)^\alpha dF(x) \quad , \alpha \geq 0$
  - $F(x)$: the probability distribution function over portfolio returns x
  - $\alpha = 0$: Shortfall Probability
  - $\alpha = 1$: Expected Shortfall
  - $\alpha = 2$: Downside Variance

Value at Risk

- JP Morgan (1994) and RiskMetrics introduced the concept
- Uryasev et al. (1999), Bucay et al. (1999), Rockafellar & Uryasev (2000)
- VaR:The level of underperformance for a given portfolio with probability β
  - $\Psi(x, \alpha) = \int_{f(x,y) \leq a} p(x,y)dy$
    - The probability that the loss function *f(x,y)* does not exceed some threshold value α
  - $VaR(x, \beta) = \min \{\alpha \in R \mid \Psi(x, \alpha) \geq \beta\}$

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