A technique for solving stochastic programs with first stage integer variables

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Outline

1. Review of current solution approaches for Integer SP.
2. Discussion of our approach.
3. Analysis of Bounds.
4. Preliminary computational results.

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New Perspective of Investment Decisions

The issues

DCF is inadequate...

Three leading characteristics

- Investment Decisions (costs) irreversible
- Future returns are uncertain
- Another key aspect is timing
  - Invest
  - Disinvest
  - Not invest...postpone

Are all strategic decisions

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Decision making under uncertainty

Buy flexibility

≡

Hedge against uncertainty

≡

Make robust decisions

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General two-stage SP

$$Min\, c^{T}x + \int p(\omega) f(\omega) y(\omega)$$

Subject to

$$\begin{aligned} Ax &= b, \\ B(\omega)x + D(\omega)y(\omega) &= h(\omega) \\ x \geq 0,\ y &\geq 0. \end{aligned}$$

ω = random event,
$p(\omega)$ = probability,
$f(\omega)$ = Second-stage objective,
$B(\omega)$ = Technical matrix,
$D(\omega)$ = Recourse matrix,
$h(\omega)$ = Right-hand side,
x = *First-stage decision*,
$y(\omega)$ = *Second-stage decision*

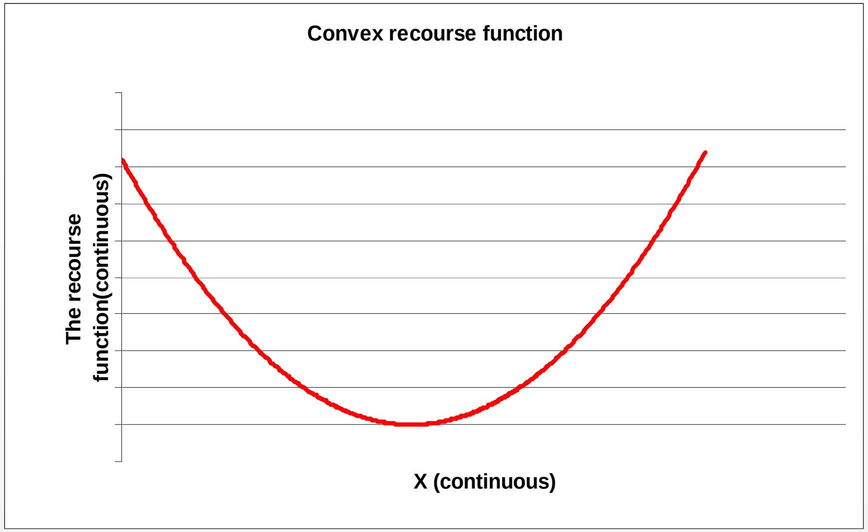
let $\xi_\omega = \{f, h, B, D\}_\omega$

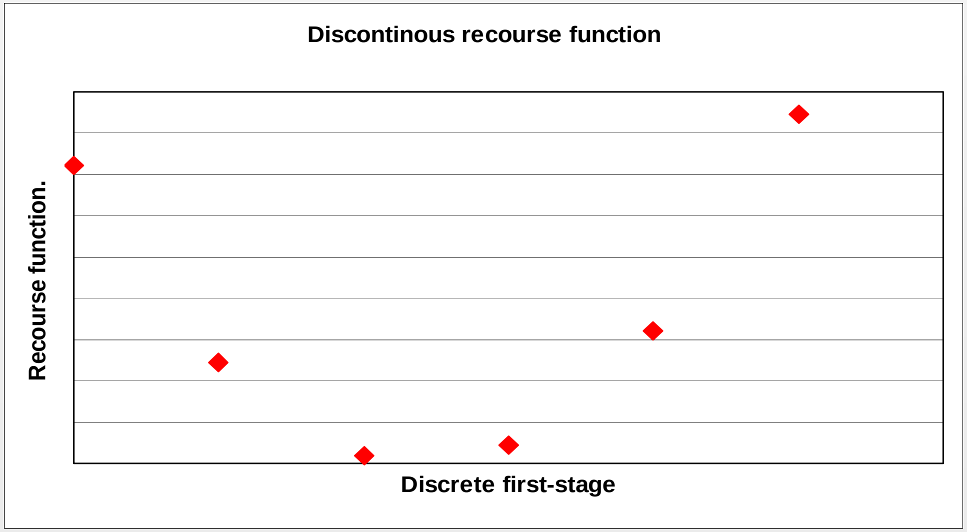
$$Min\, c^{T}x + E_\xi Q(x, \xi)$$

Subject to

$$Ax = b,$$
where
$$Q(x, \xi) = Min\{fy(\omega) \mid Dy(\omega) = h - Bx,\ y(\omega) \geq 0\}$$

Shape of the recourse function





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Stochastic integer programs

$$x \in Z^{n_1}\ , \text{OR}\ y(\omega) \in Z^{n_2}\ , \text{OR}\quad x \in Z^{n_1}\ \text{AND}\ y(\omega) \in Z^{n_2}$$

Applications

First Stage integer

- Supply chain planning, Mitra et al.
- Asset allocation, Mulvey and Rusczynski
- Electric utility planning, Bienstock and Shapiro
- Goods distribution, Cheung and Powell

Second Stage integer

- Scheduling decisions, Dempster et al.
- Routing decisions Spaccamela et al.
- Fixed-charge, Bitran et al.
- Change over costs, Caroe et al.

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Current approaches

- Cutting plane - *Wollmer*
- Scenario decomposition- *Rockafellar and Wets*
- Integer L-shaped – *Laporte and Louveaux*
- Convex hull for Simple integer recourse – *Haneveld, Stougie, Van der Vlerk*

Current approaches

- Progressive hedging - *Takriti et al.*
- Groebner bases – *Schultz et al.*
- Integer L-shaped – *Caroe and Tind*
- Dual decomposition – *Caroe and Schultz*
- Branch and bound based approach – *Ahmed et al.*

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Problem Statement

$$\text{Min} \quad c^{\mathbf{T}} x + E_{\xi} Q(x, \xi)$$

Subject to

$$Ax = b,$$
$$where$$
$$Q(x,\xi) = Min\{ fy(\omega) \mid Dy(\omega) = h - Bx \}$$
$$x = (x^{'}, x^{''}), x \in \{0,1\}^{n_1^{'}}, x^{n_1^{''}} \in \Re^{n_1^{''}}, y(\omega) \in \Re^{n_2}$$

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Motivation for the algorithm

A Strategic Supply Chain model

Rows	6768
Discrete	2096
Continuous	54400
Non-Zeroes	1154034
Scenarios	100

Assumptions

- Relatively complete recourse
- Discrete distribution

The Approach

1. Lagrangean relaxation used to generate a candidate set of integer feasible solutions for each WS model.
2. Construct a convex hull of this set of integer feasible solutions.
3. Not necessarily a convex hull to the original 2-stage ISP.
4. We compute bounds to the 2-stage SP model and qualify the approximated model.

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Algorithm

$$P_{2SP}$$

$$\min Z = cx + \sum_s p_s f_s y_s$$

subject to

$$Ax = b$$

$$B_s x + D_s y_s \le d_s \quad \forall s \in \{1,\ldots,S\}$$

$$x \in \{0,1\}^{n_1}$$

$$y_s \ge 0$$

$$P_{WS}(s)$$

$$\min Z = cx + f_s y_s$$

subject to

$$Ax = b$$

$$B_s x + D_s y_s \le d_s$$

$$x \in \{0,1\}^{n_1} \qquad \text{Discrete Variables}$$

$$y_s \ge 0 \qquad \text{Continuous Variables}$$

Algorithm

$P_{WS}(s)$

$$\begin{aligned}
&\min \; Z_{WS} = cx + f_s y_s \\
&\textit{subject to} \\
&Ax = b \\
&B_s^R x + D_s^R y_s^R \le d_s^R \\
&B_s^{NR} x + D_s^{NR} y_s^{NR} \le d_s^{NR} \\
&x \in \{0,1\}^{n_1}, \quad y \ge 0
\end{aligned}$$

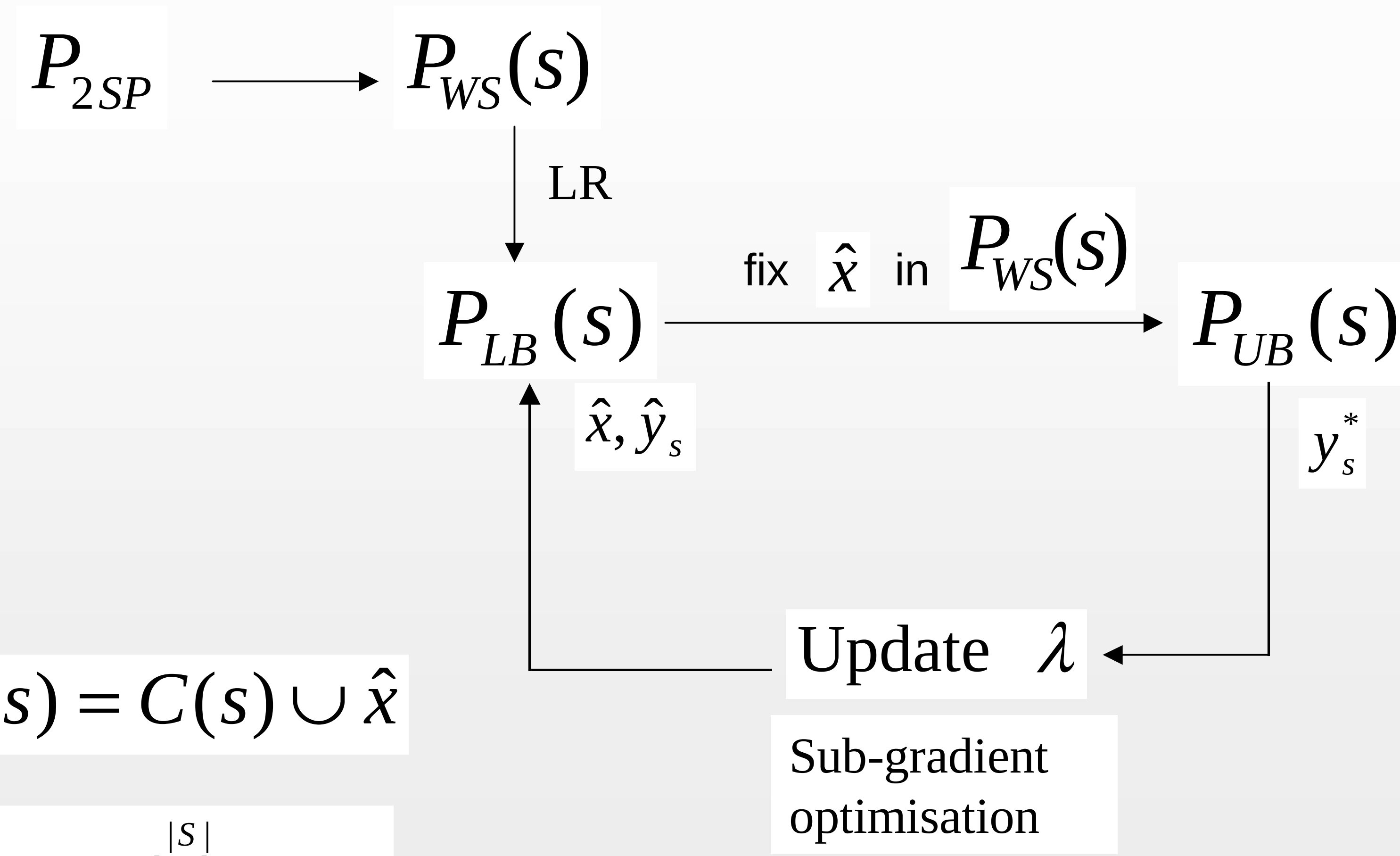
$P_{LB}(s)$

$$\begin{aligned}
&\min \; Z_{LB} = (c + \lambda B_s^R)x + (f_s + D_s^R) y_s^R - \lambda_s^R d_s^R \\
&\textit{subject to} \\
&Ax = b \\
&B_s^{NR} x + D_s^{NR} y_s^{NR} \le d_s^{NR} \\
&x \in \{0,1\}^{n_1}, \; y \ge 0
\end{aligned}$$

$P_{UB}(s)$

$$\begin{aligned}
&\min \; Z_{UB} = c\hat{x} + f_s y_s \\
&\textit{subject to} \\
&A\hat{x} = b \\
&B_s \hat{x} + D_s y_s \le d_s \\
&\hat{x} \in \{0,1\}^{n_1}, \quad y \ge 0
\end{aligned}$$

Algorithm



$$C(s) = C(s) \cup \hat{x}$$

$$ACS = \bigcup_{s=1}^{|S|} C(s)$$

Algorithm

Stopping Criteria:

1. $$\frac{\text{BUB} - \text{BLB}}{|\text{BLB}| + 1} < \text{Tolerance}$$

2. Pass $\geq$ Maximum Number of Iterations

3. Satisfaction of the relaxed constraints, $B_s^R \hat{x} + D_s^R \hat{y}_s \leq d_s^R$

The Approximated model

$APXP_{2SP}$

$$\min\ Z = cx + \sum_s p_s f y_s$$

subject to

$$Dy_s \le h - Bx,$$

$$Ey_s = d_s, \quad \forall\, s \in \{1,..,\ S\}$$

$$x = \sum_{n=1}^{|ACS|} \beta_n \hat{x}_n$$

$$\sum_{n=1}^{|ACS|} \beta_n = 1,$$

$$x \ge 0,\ y_s \ge 0,\ \forall s$$

$$\beta_n \in \{0,1\} \quad \forall n$$

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Bounds

Upper Bound

$Z^*_{HN}(I) \equiv$ The optimum objective function value to the ISP.

$Z^*_{HN}(C) \equiv$ The optimum function value of the LP relaxation.

$Z_{HN}(F_n) \equiv$ The objective function value of the n^{th} integer feasible solution

$$\text{where } Z_{HN}(F_n) = c\hat{x}_n + \sum_{s=1}^{|S|} p_s f_s y_s \quad \text{and} \quad n = 1 \ldots |ACS|$$

Let $Z_{HN}(F^*) = \text{Min}\{Z_{HN}(F_1), \ldots . Z_{HN}(F_{|ACS|})\}$

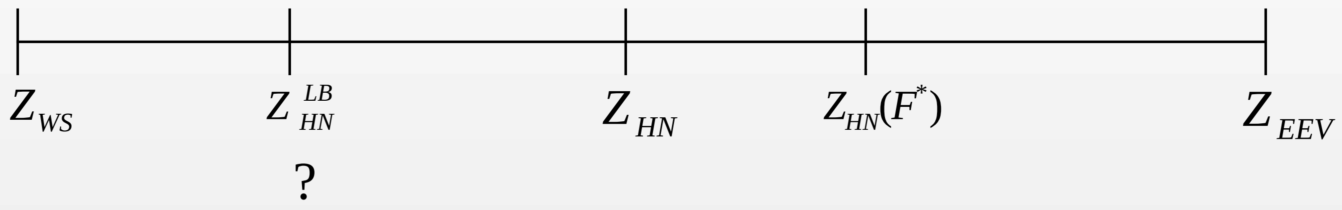
Lower Bound

$$Z^{LB}_{HN} = \text{Max}\{Z_{WS}, Z^*_{HN}(C)\}$$

Then we have $Z^{LB}_{HN} \leq Z^*_{HN}(I) \leq Z_{HN}(F^*)$

Qualifying the bound

For any 2 - stage SP minimisation model $Z_{WS} \leq Z_{HN} \leq Z_{EEV}$



The ratio $$\frac{Z_{EEV} - Z_{HN}(F^*)}{Z_{HN}(F^*) - Z_{HN}^{LB}}$$ measures the compactness of the bound

Computational Results

Model description

SCP-1 – Strategic supply chain, opening and closing of sites, DCs,lines.

SCP-2 – Trade-off between capacity acquisition and maintaining inventory.

Model Statistics

Model	Constraints	Variables (discrete/continuous)	Non-Zeroes
SCP-1	6768	2096/54400	1154034
SCP-2	574	216/240	1342

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Network Dimensions		Planning Problem	
The number of Sites, I :		8	
The types of packing line technology, Y_C:		4	
The types of production line technology, Y_R:		2	
The number of distribution centres , J :		15	
The types of DC line technology , Y_D:		2	
The number of Customer Zones , H :		30	
The number of Products , P :		13	
The number of time periods, T :		6	
Model Statistics			
Logical Constraints: Sites, DCs opening and closing, Limit on number of Sites, DCs, and Lines		m_1 = 968	6768
Other Constraints: Production, Packing, Ordering, Transportation, Balance, Demand, and also Production and Packing Capacities.	Mixed	m_2 = 850	
	Continuous	m_3 = 4950	
Discrete Decision Variables: Sites, DCs, Production lines, Packing lines, DC lines.		n_1 = 2096	56496
Continuous Variables: Production, Packing, Ordering, Transportation, and Shortage quantities.		n_2 = 54400	
Non-zeros		1154034	
Scenarios		100	

Computational Results

Generation of integer solutions

Model	Number of scenarios	Number of Lagrangean iterations	Number of unique solutions
SCP-1	100	25	2050
SCP-2	43	25	143
SCP-2	43	15	85

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Computational Results

Generation of integer solutions

Model	Z_{WS}	Z_{HN}^{LB}	$Z_{HN}(I)$	$Z_{HN}(F^*)$	Z_{EEV}	Quality
SCP-1	*	$2.52*10^6$	?	$2.64*10^6$	$3.59*10^6$	7.91
SCP-2(25)	$-1.23*10^6$	$-1.23*10^6$	$-1.21*10^6$	$-1.18*10^6$	∞	∞
SCP-2(15)	$-1.23*10^6$	$-1.23*10^6$	$-1.21*10^6$	$-1.14*10^6$	∞	∞

Comparison of computational time

Model	DEQ	Heuristic
SCP-1	158 hrs 35 mins	26 hrs 23 mins
SCP-2(25)	666s	980s
SCP-2(15)	666s	400s

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Outstanding Issues

- Step-size in subgradient optimisation.
- The constraints to be relaxed.
- Parallelisation of the algorithm.
- Study the performance of the algorithm to SP test set.

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Thank you !