# An ALM model with Downside risk and cVaR constraints

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# Contents



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# Optimal Allocation Models

Fixed Mix Strategy → Dynamic Asset only Strategy → Asset and Liability Management → Asset and Liability Management & Achieving Goals

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# Asset and Liability Models under Uncertainty

- The basic concepts were developed by:
  - Kallberg, White, and Ziemba (1982)
  - Kusy and Ziemba (1986)
- Large Scale Applications include:
  - Mulvey, Gould, and Morgan (2000) "An Asset and Liability management system for the Towers Perrin-Tillinghast"
  - Carino and Ziemba, & Carino, Meyers, and Ziemba (1998)"Russel-Yasuda Kasai model"
  - Consigli and Dempster (1998) "The CALM stochastic programming model for dynamic ALM"
  - Dert (1995) "A dynamic model for ALM for defined benefit pension funds"
  - Zenios (1995) "Asset and Liability management under uncertainty"

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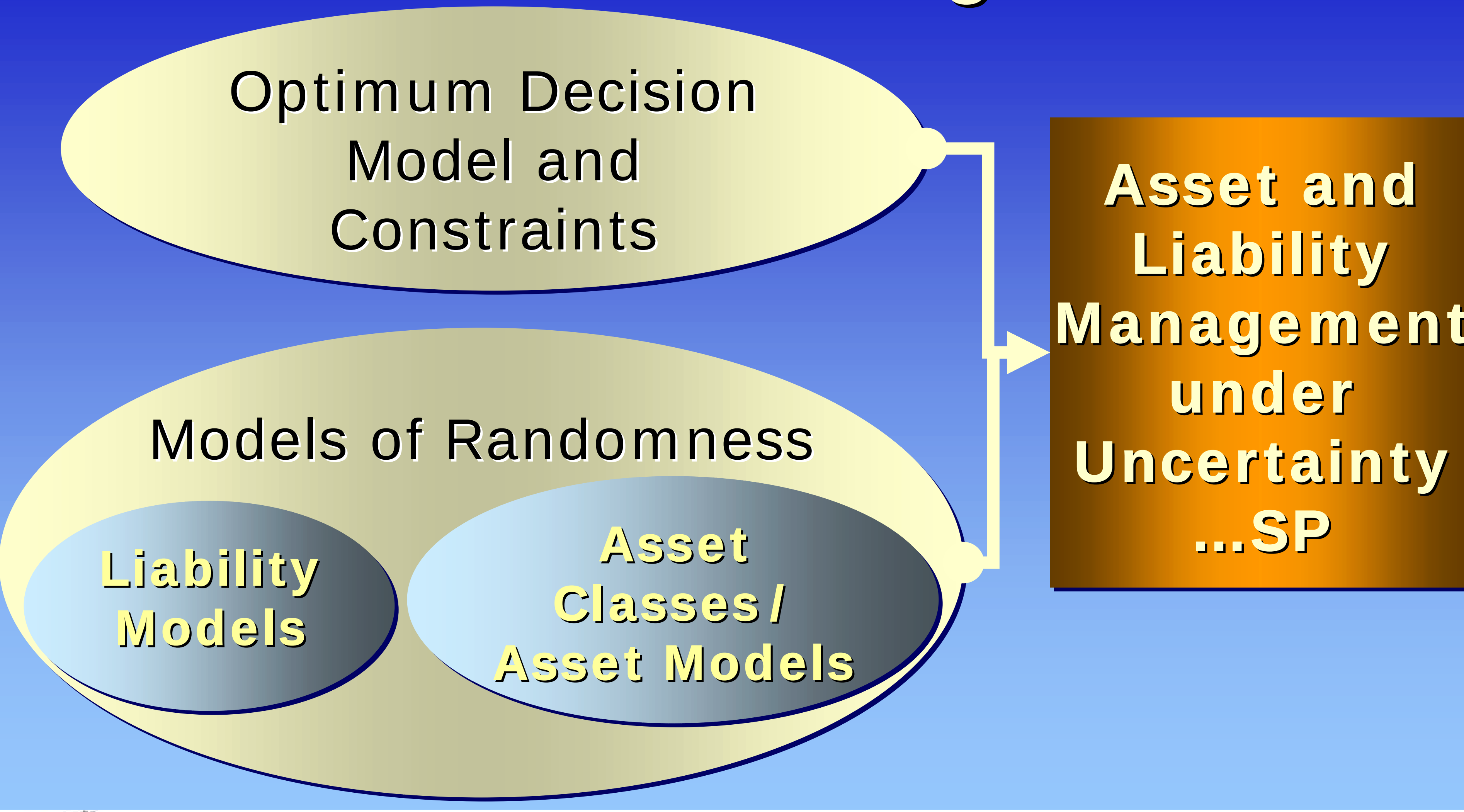
# ALM and SP Integration



# Constraint Classes

**Institution Specific Constraints**

- Asset Classes
- Planning Horizon
- Threshold Constraints
- Cardinality Constraints
- Transaction Costs
- ...

**Country Specific Constraints**

- Tax
- Regulatory Requirements
- Minimum Asset Reserve
- ...

**Risk Measures Constraints**

- Downside
- VaR
- cVaR
- Variance
- Semi-variance
- Mean Absolute Deviation
- ...

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# Our Investigation

| Model | Objective | Risk | Methodology |
|---|---|---|---|
| Two-stage multi-time period SP | Maximise Terminal Wealth | Downside | Rolling & Backtesting |
| | | VaR | |
| Mean Variance | Maximise Single Period Wealth | Variance | Rolling & Backtesting |

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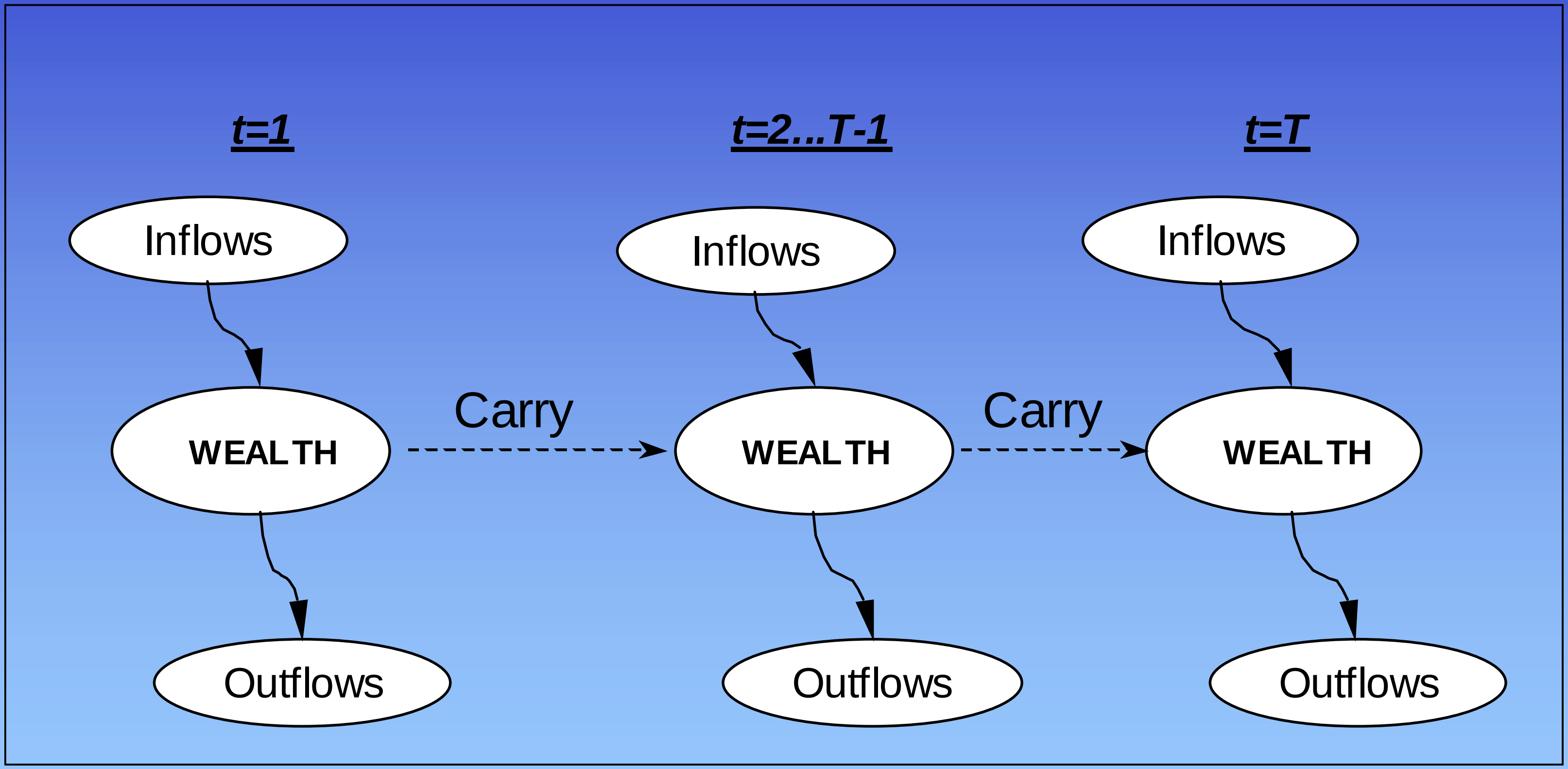
# Overview

- ✓Overview
- ✓Our investigation
- ✓The Models
- ➢Consideration of Risk
- ➢Experimental results and discussions

# An ALM Stochastic Programming Model

*Surplus Wealth* = assets – PV(liabilities) – PV(goals)



# Model Components

| | NOTATION | EXPLANATION | |
|---|---|---|---|
| INDEX | i | **Assets** | i=1..n |
| | s | **Scenarios** | s=1…S |
| | t | **Timeperiod** | t=1..T |
| | k | **RiskGroup** | k=1..K |
| DATA PARAMETERS | $P_{ist}$ | **Prices**[assets,scenarios,timeperiod] | |
| | $L_{ts}$ | **Liability**[timeperiod,secenario] | |
| | $\pi_s$ | **Probability**[scenario] | |
| | $H_{i0}$ | **Initial_Holdings**[assets, timeperiod=0] | |
| | $F_t$ | **Funding**[timeperiod] | |
| | 1+tr | **Transaction_Buy** | |
| | 1- tr | **Transaction_Sell** | |
| | $\rho_\kappa$ | **RiskGroup_Holdings** | |
| DECISION VARIABLES | $H_{ist}$ | **Amounthold**[assets, scenarios,timeperiod] | |
| | $B_{ist}$ | **Amountbuy**[assets, scenarios,timeperiod] | |
| | $S_{ist}$ | **Amountsell**[assets, scenarios,timeperiod] | |

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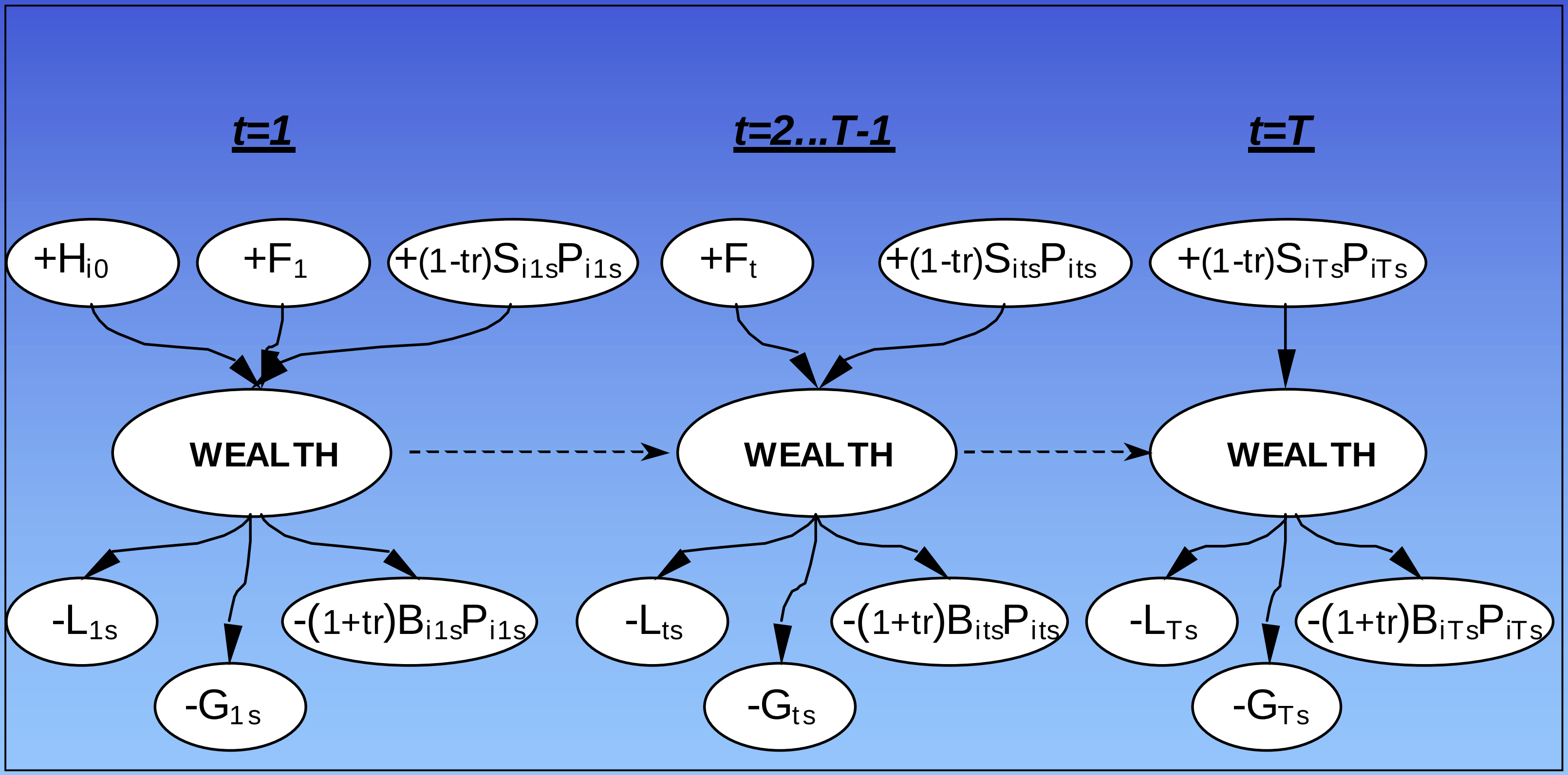
# An ALM Stochastic Programming Model

*Surplus Wealth* = assets – PV(liabilities) – PV(goals)

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# Model Constraints

- ***Asset holdings constraint:***
  - **C1.** $H_{ist} = H_{i0} + B_{ist} - S_{ist}$ , $t = 1, i=1...n, s=1...S$
  - **C2.** $H_{ist} = H_{ist-1} + B_{ist} - S_{ist}$ , $t = 2...T, i=1...n, s=1...S$
- ***Fund balance constraint***
  - **C3.** $\sum_{i=1}^{n} P_{ist}\,(1+tr_i)B_{ist} = \sum_{i=1}^{n} P_{ist}\,(1-tr_i)S_{ist} - L_{ts} + F_t,$

    $t = 1...T, s=1...S$

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# Model Constraints

- ***RiskGrades™ (JPMorgan - RiskMetrics) based classification of risk groups…partitions asset set N={1…n}***
  - $g_k \subseteq N$ , $k = 1 \ldots K$
- ***Risk Groups constraint:***
  - $\bigcup_k g_k = N$ , $g_j \bigcap g_k = 0 \quad \forall (j,k)$
  - **C4.** $\sum_{i \in g_k} H_{ist} \leq \rho_k * \sum_{i=1}^{n} H_{ist}$ , $t = 1 \ldots T$, $s = 1 \ldots S$
  - $\sum_{k=1}^{K} \rho_k = 1$
- ***S&P100 is the benchmark***

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# Model Constraints

## ➢ *Non-anticipativity Constraints*

- **C5.1** Amount of asset classes/stocks the investor holds

$H_{ist} - H_{is't} = 0, \qquad t=1,\ s=1,\ \forall\ i=1...n,\ s^{/}=2...S$

- **C5.2** Amount of asset classes/stocks the investor buys

$B_{ist} - B_{is't} = 0, \qquad s=1,\ t=1\ \forall\ i=1...n,\ s^{/}=2...S$

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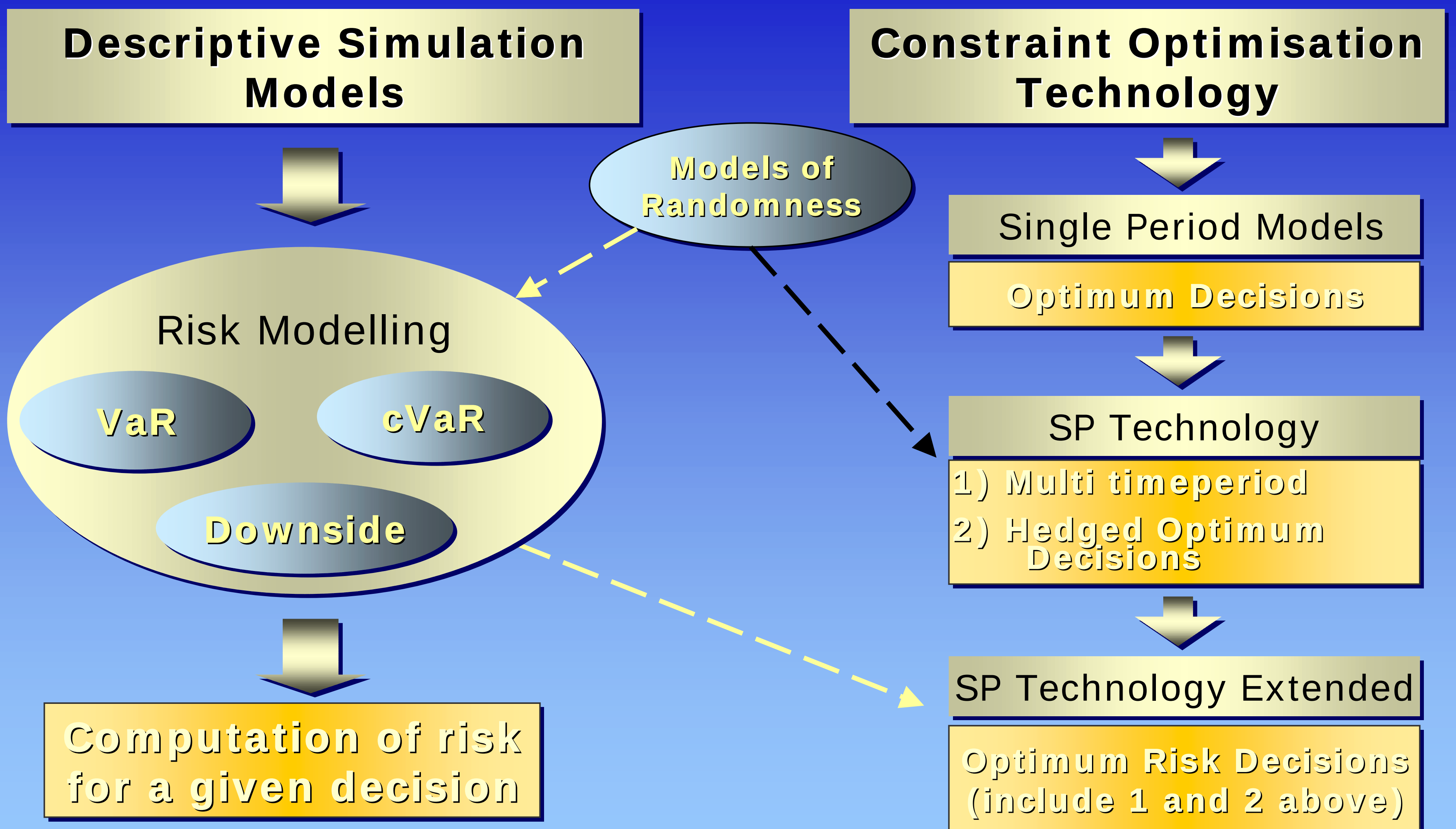
# Risk Decisions

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# Downside Risk

- The definition was introduced by Bawa (1975) and Fishburn (1977)
- Penalise only negative returns relative to a given benchmark – target $\theta$
- $R_\gamma = E\left([\max\{\theta - x, 0\}]^\alpha\right) = \int_{-\infty}^{\theta} (\theta - x)^\alpha dF(x)\,,\ \alpha \geq 0$
  - *F(x)* : the probability distribution function over portfolio returns *x*
  - $\alpha = 0$ : Shortfall Probability
  - $\alpha = 1$ : Expected Shortfall
  - $\alpha = 2$ : Downside Variance

# Downside Risk

## ➢ *Downside Risk Constraint*

- **C6.1** $A_t - (\sum_{i=1}^{n} P_{ist} H_{ist} - L_{ist} + F_{ist}) \le R_{ts}$ , t=2..T, s=1...S.

- **C6.2** $\sum_{s=1}^{S} \pi_s R_{ts} \le (1-\gamma) * R_t^{max}$

  $0 \le \gamma \le 1$, i = 1...n, t = 2...T and s= 1...S

- $R_t^{max}$: maximum amount of downside risk the portfolio can have each time period
- $R_{ts}$: the downside risk for each scenario
- $A_t$ : target level
- $\gamma$ : parameter representing the risk aversion of the investor.

# Value at Risk

- JP Morgan (1994), Uryasev et al. (1999), Bucay et al. (1999), Rockafellar & Uryasev (2000)
- VaR:The level of underperformance for a given portfolio with probability β
  - $\Psi(x, \alpha) = \int_{f(x,y) \leq a} p(x,y)dy$
    - The probability that the loss function *f(x,y)* does not exceed some threshold value α
  - $VaR(x, \beta) = \min \{\alpha \in R \mid \Psi(x, \alpha) \geq \beta\}$

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# Value at Risk

- cVaR:The mean of the level of under performance for a given portfolio with probability β
  - $\text{cVaR}(x,\beta) = (1-\beta)^{-1} \int_{f(x,y)\geq VaR(x,\beta)} f(x,y)p(y)d(y)$
- cVaR $\geq$ VaR
- cVaR can be minimised for discrete scenarios within the linear programming framework and we extend it to the SP setting

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# cVaR

## cVaR Risk Constraint (C7)

- $\sum_{i=1}^{n} \{(F_0 + H_{i,0} * P_{i,0}) - (F_t - L_{t,s} + H_{i,t-1,s} * P_{i,t,s})\} - \alpha_t \le z_{s,t}$ ,

  $t = 1, \quad s = 1 \ldots S$

- $\sum_{i=1}^{n} \{(F_{t-1} - L_{t-1,s} + H_{i,t-1,s} * P_{i,t-1,s}) - (F_t - L_{t,s} + H_{i,t-1,s} * P_{i,t,s})\} - \alpha_t \le z_{s,t}$ ,

  $t = 2 \ldots T, \quad s = 1 \ldots S$

- $\alpha_t + S^{-1} * (1-\beta)^{-1} * \sum_{s=1}^{S} z_{st} \le \varpi_t$

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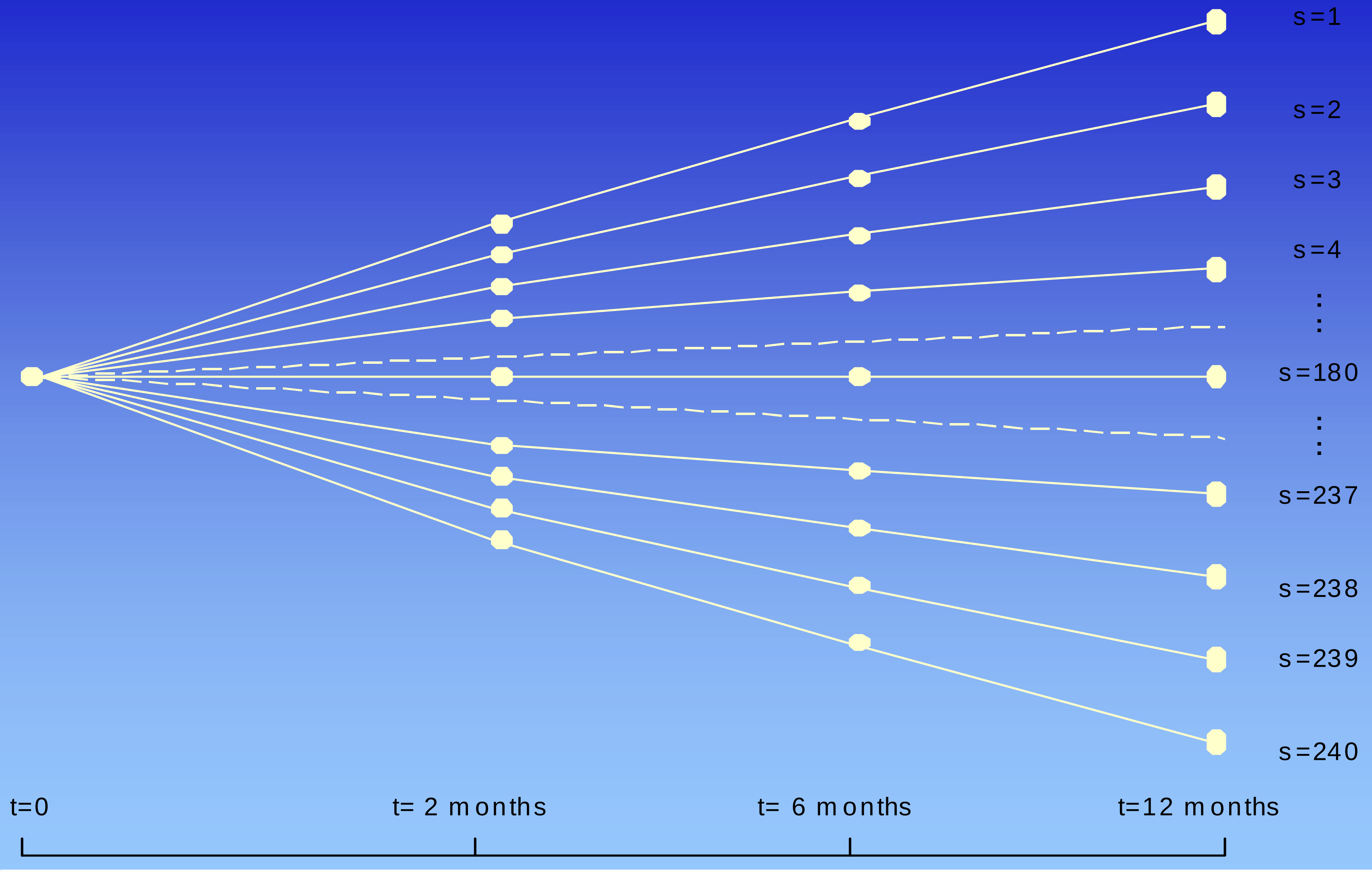
# Model specific characteristics and data

- 59 stocks constituting the S&P100
- 12 months planning horizon
- Rebalancing in months 2 and 6 The level of underperformance for a given portfolio with probability β
- 240 scenarios
- Classification of stocks into 5 Risk Groups
- 5 risk profiles:
  - Minimum, Medium Low, Medium, Medium High, and High risk
- 10 years backtesting

# Two Stage Scenario Tree



# Rolling Decisions

- Why rolling ?
  - Data and information in the financial markets is continuously updated
  - In real life applications only the first stage decisions are followed
- What do we achieve ?
  - We simulate the actions taken by the financial institution or the investor
  - We simulate the impact that these actions have on the portfolio wealth
  - We gain some insight into the behaviour of the models in respect of historical data

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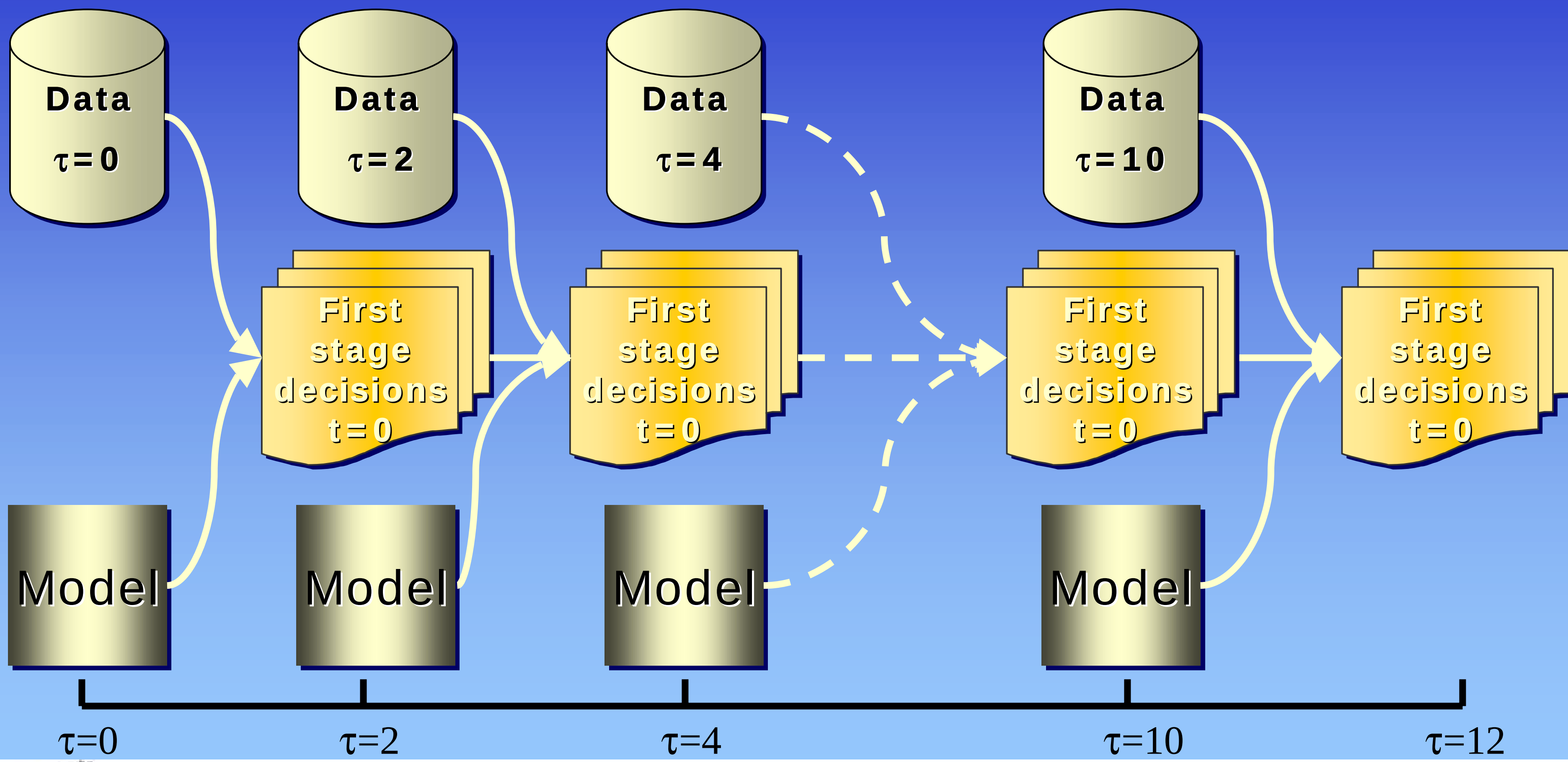
# Rolling Decisions

- Explanation of the framework

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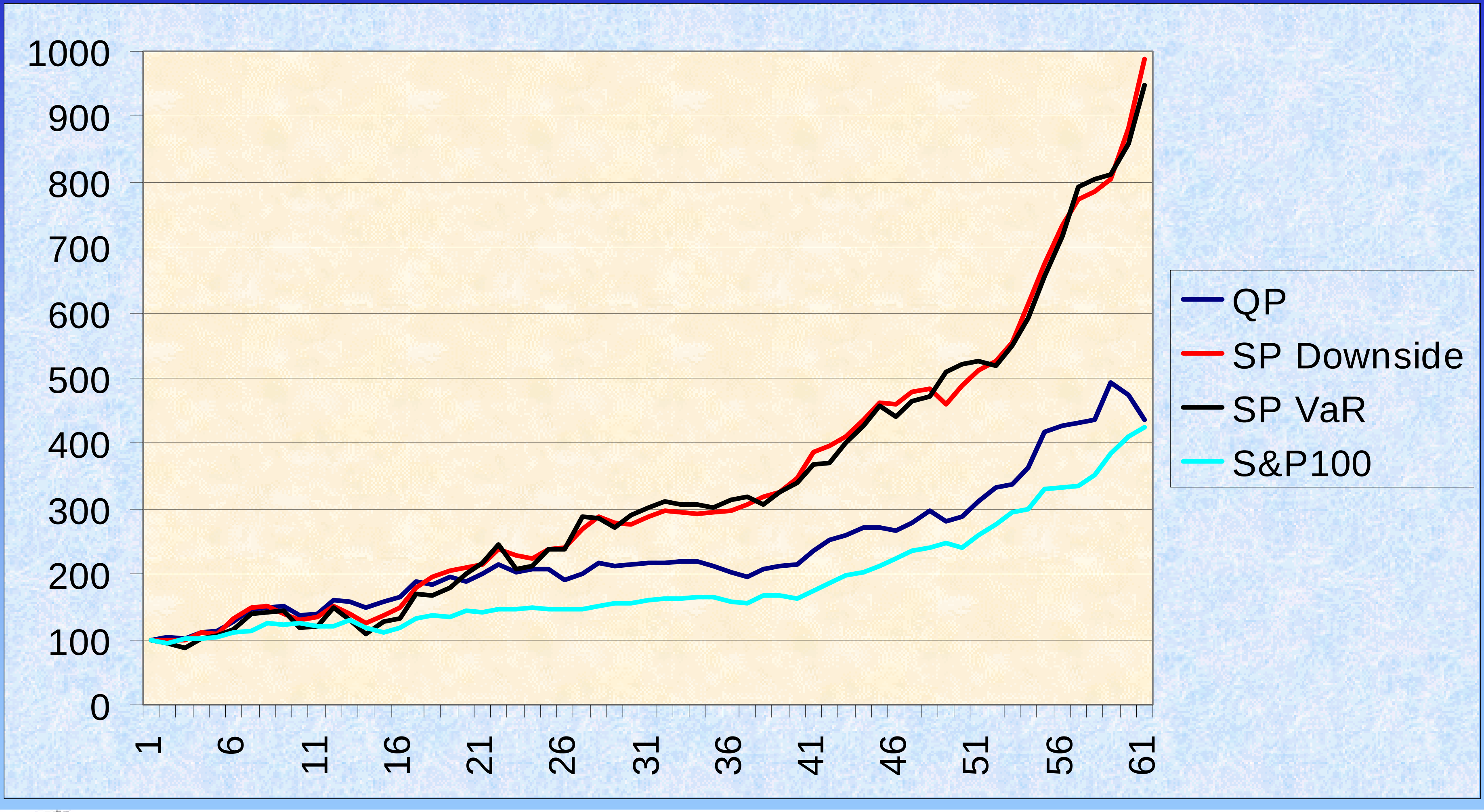
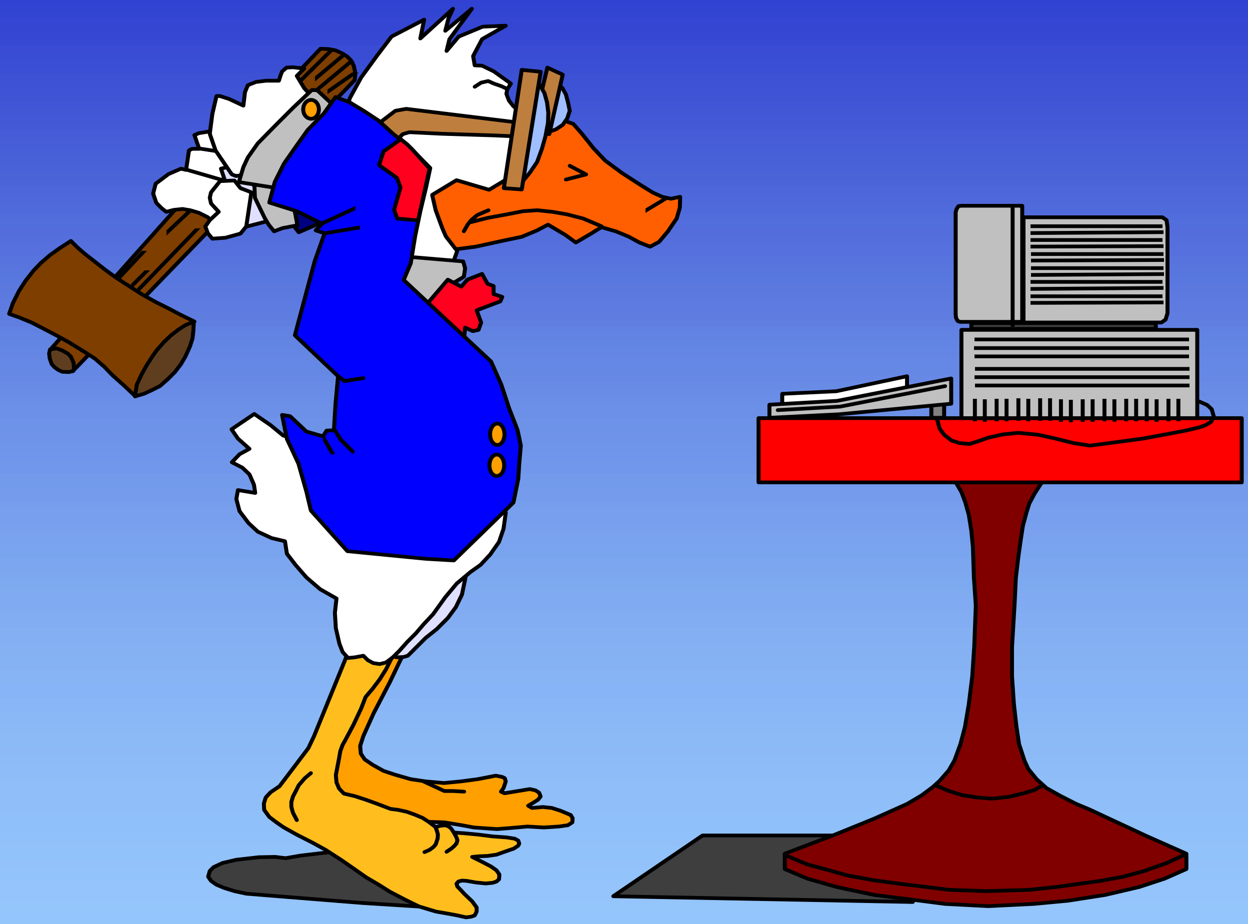
# Visual Demonstration of Results



# Computational Results

| Year | Risk Profile | SP Downside | SP VaR | QP |
|---|---|---|---|---|
| 1989 | Min | 40.63% | 39.74% | 34.58% |
| | Medium Low | 45.90% | 41.58% | 42.67% |
| | Medium | 49.74% | 52.52% | 45.79% |
| | Medium High | 49.11% | 41.68% | 41.53% |
| | High | 40.55% | 43.75% | 33.46% |
| 1990 | Min | -8.27% | -7.14% | 20.04% |
| | Medium Low | -6.50% | -4.09% | 10.46% |
| | Medium | -6.36% | -9.54% | 8.47% |
| | Medium High | -5.08% | -14.67% | 8.91% |
| | High | 1.06% | 0.80% | 10.59% |
| 1993 | Min | 15.15% | 20.20% | 5.92% |
| | Medium Low | 20.06% | 20.16% | 5.79% |
| | Medium | 20.84% | 21.90% | 4.39% |
| | Medium High | 21.61% | 22.21% | 4.68% |
| | High | 26.30% | 28.79% | 6.39% |
| 1998 | Min | 50.85% | 50.41% | 4.04% |
| | Medium Low | 45.61% | 50.32% | 4.53% |
| | Medium | 46.50% | 39.65% | 4.60% |
| | Medium High | 46.92% | 41.96% | 3.32% |
| | High | 46.50% | 42.75% | 10.27% |

# And in the end when it all gets too much...



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