OPTIMUM DECISION MAKING UNDER UNCERTAINTY: MODELLING, SOLVING AND APPLICATIONS

<u>Plenary Talk</u>

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Gautam Mitra

Department of Mathematical Sciences, Brunel University

and

OPTIRISK Systems

OPTIMUM DECISION MAKING UNDER UNCERTAINTY: MODELLING, SOLVING AND APPLICATIONS

Supported by researchers and colleagues including:

E F D Ellison, C A Lucas, N Jobst, P Valente,
N S Koutsoukis, C Poojari, B Dominguez-Ballesteros,
T Kyriakis, A Mirhassani

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Outline

1. Optimisation Models: Planning and Decision Making
   - background
   - our research interests

2. Uncertainty, Randomness and Optimum Decisions
   - why risk and return models
   - finance industry excitement and regrets
   - combining randomness and optimum resource allocation

3. Discrete Quadratic Programming
   - bond portfolio selection
   - portfolio rebalancing

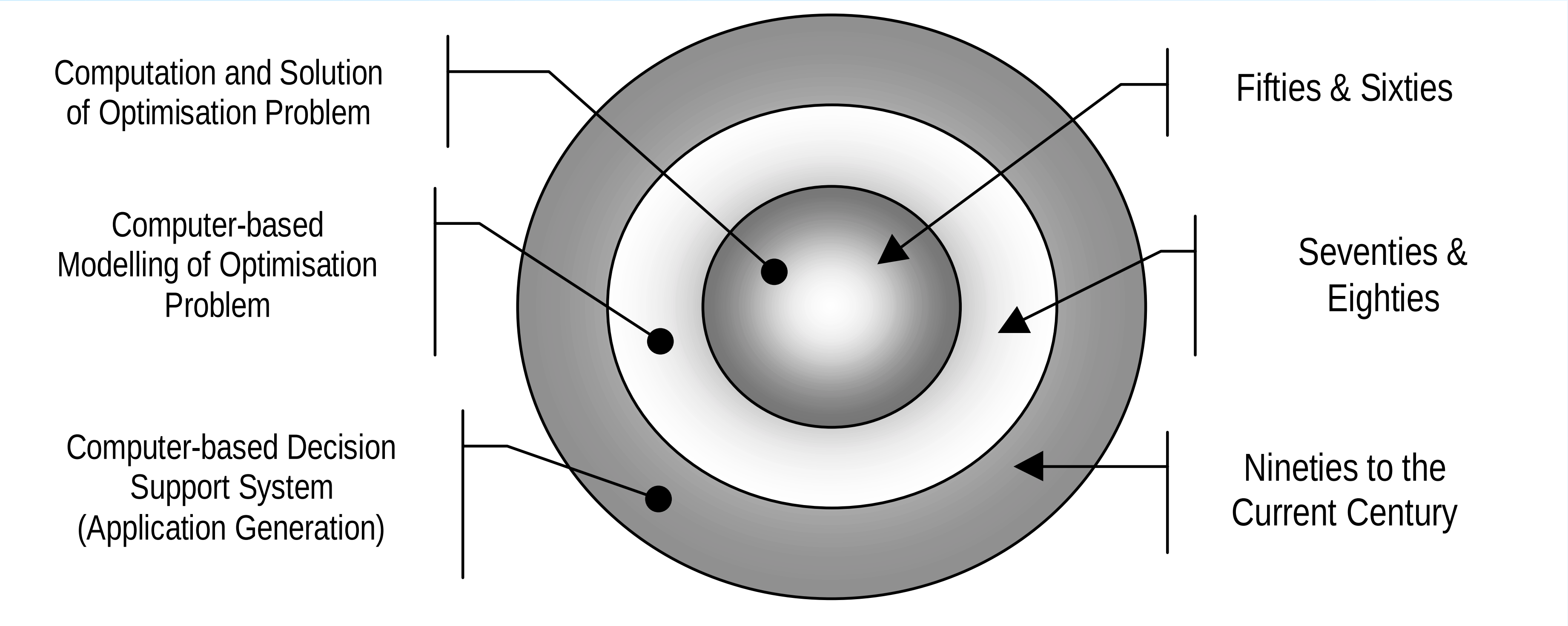
Outline

4. Stochastic Programming
   - background to stochastic programming
   - planning models for supply chains
   - solution of two-stage integer stochastic programs

5. New generation of models and software tools
   - stochastic programming integrated environment SPInE

6. Discussions

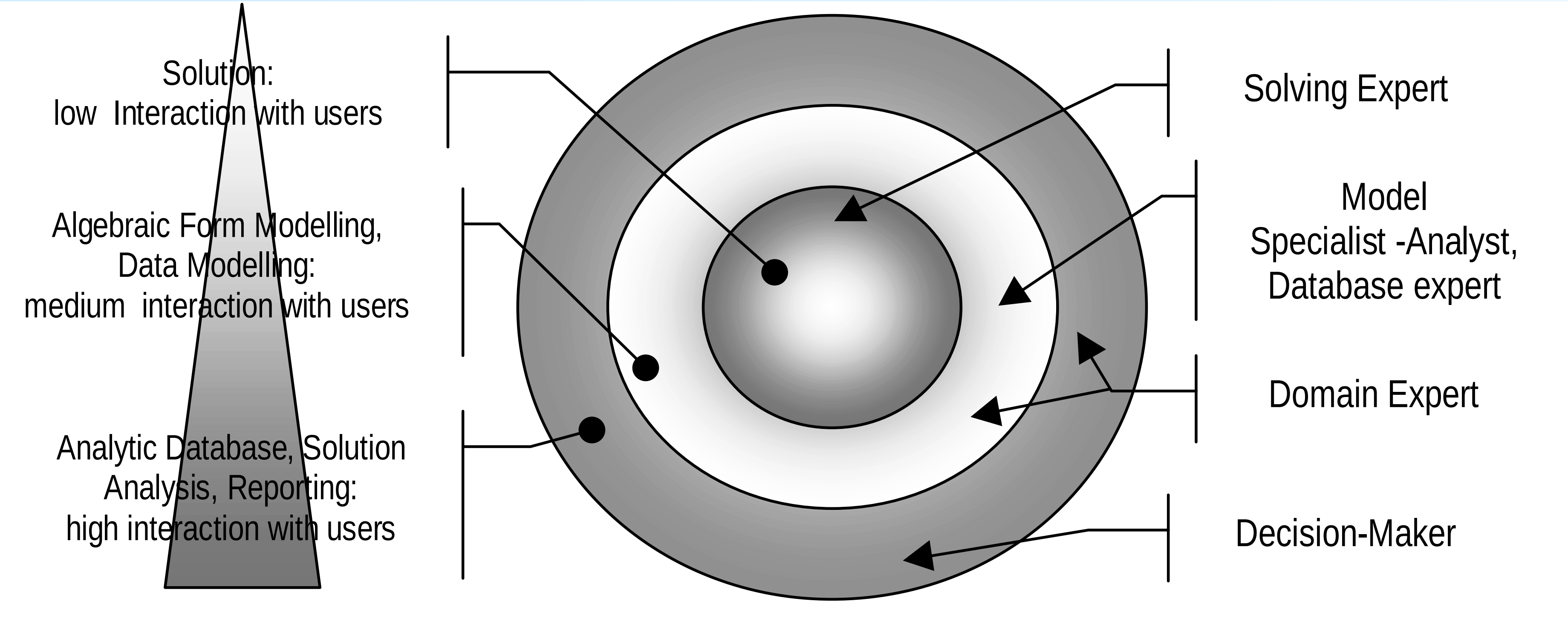
Background



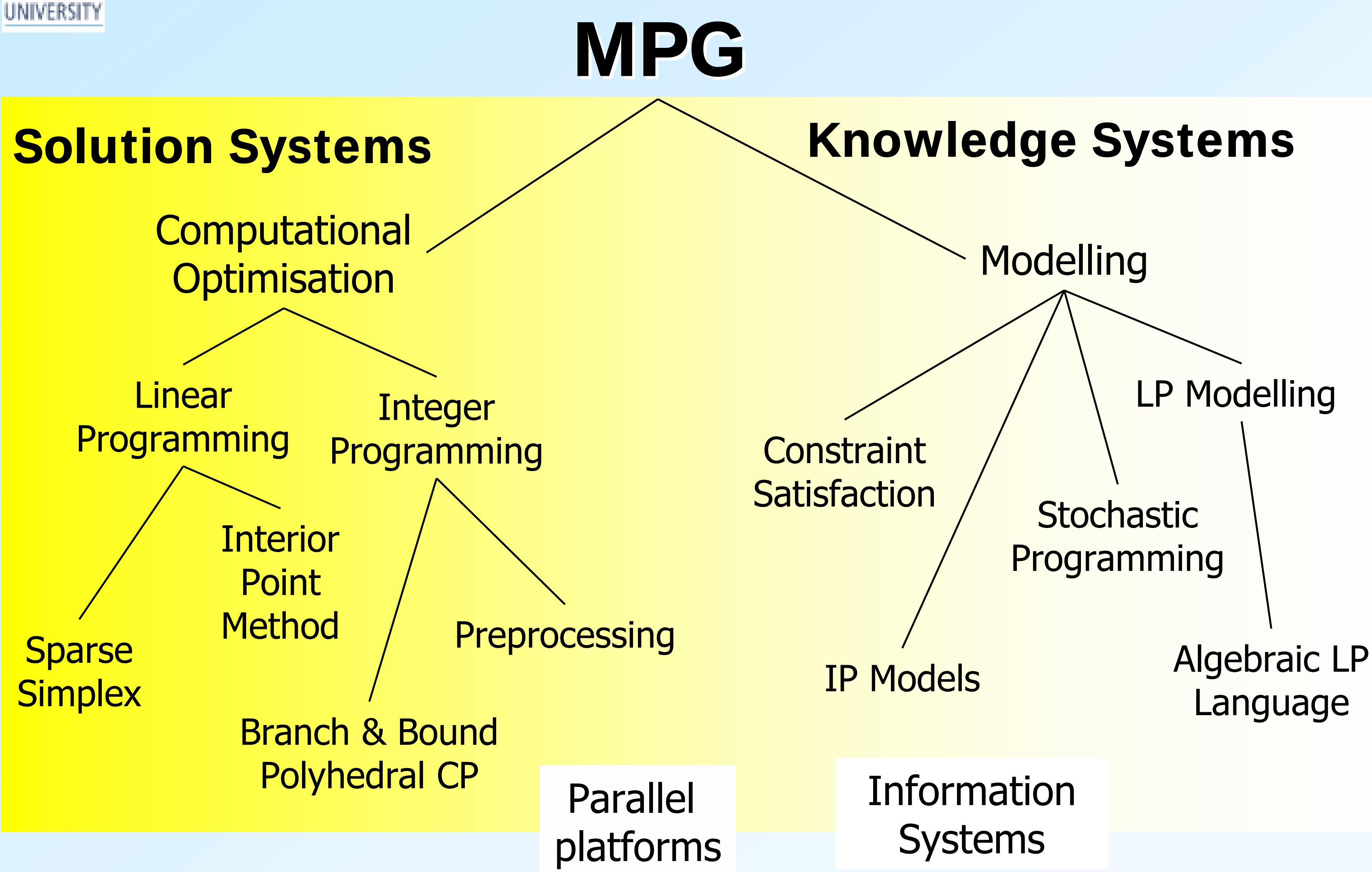
Computation and Solution of Optimisation Problem
Computer-based Modelling of Optimisation Problem
Computer-based Decision Support System (Application Generation)
Fifties & Sixties
Seventies & Eighties
Nineties to the Current Century

Background

Constituents and their interaction

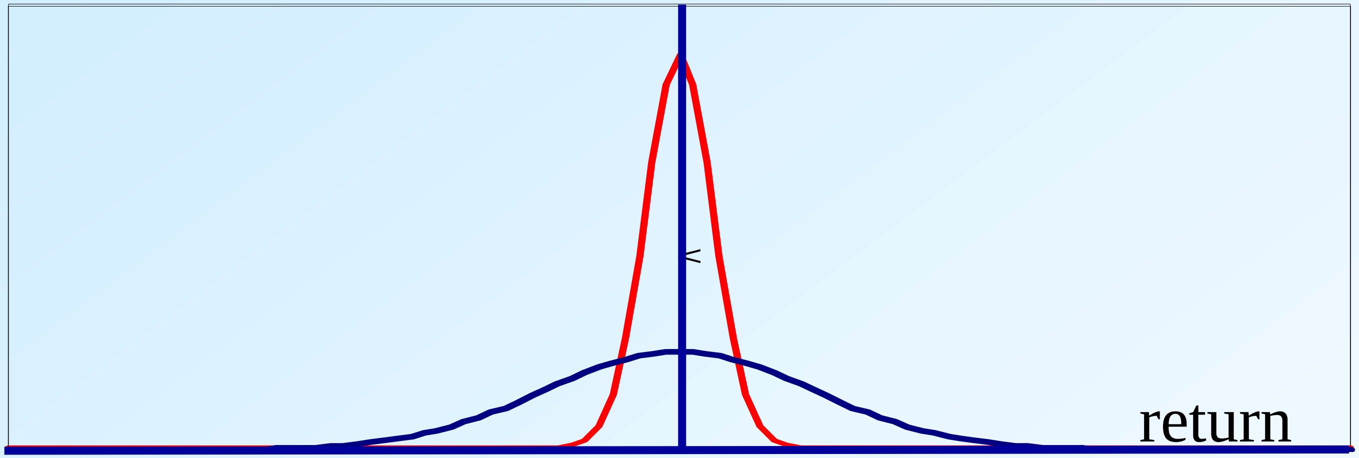


Our research interests



Uncertainty… optimum decisions

- Deterministic to an uncertain world
- Uncertainty …randomness…risk



•‵ Bell shaped world’
•Markowitz: Volatility of asset prices measured by Variance (standard deviation)

Uncertainty… optimum decisions

- Markowitz (Nobel Prize)
  - Mean variance (M-V Theory)
  - Diversification through ‘not strongly correlated assets’
- Sharpe (Nobel Prize)
  - Capital Asset Pricing Model (CAPM)
  - Behaviour of asset prices in relation to the market
- Ross
  - Arbitrage Pricing Theory (APT)…Multi factor models
  - Arbitrage…’no ? free lunch’ as the basis of economic behaviour
- Black,Scholes…Merton (Nobel Prize)
  - Pricing formula for derivative products
  - Brownian motion..continuous time finance.

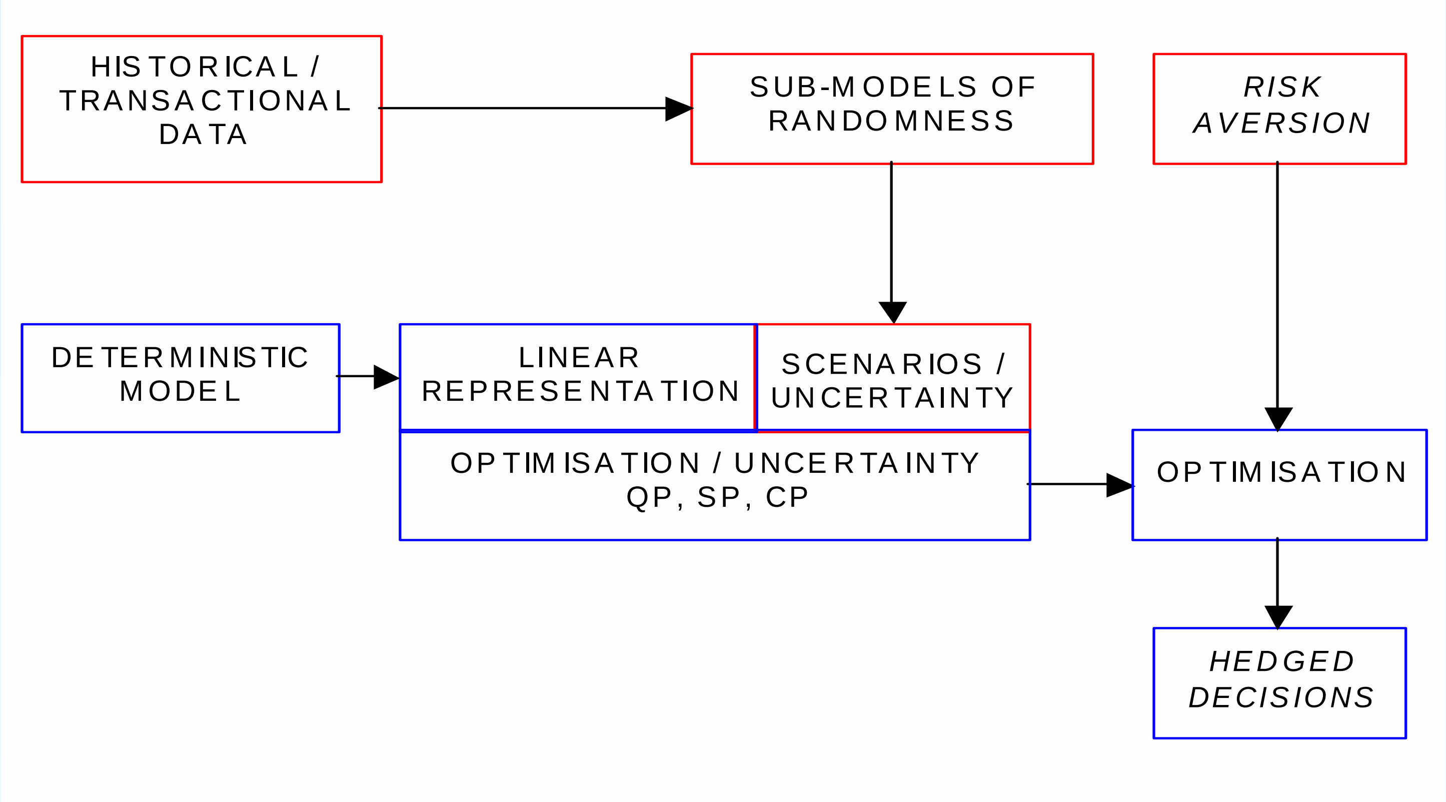
Uncertainty... optimum decisions

- Excitement and misadventures in the finance industry
- upside………→gain…..bet
- downside…..→risk……insurance
- Worldwide regulatory requirement to quantify and report risk in the finance industry
- Similar regulations emerging in the corporate sector

Uncertainty... optimum decisions

Modelling approach

- Construct decision models which capture return and risk (due to uncertainty)
- Combine models of optimum resource allocation and models of randomness



Uncertainty... optimum decisions

- Typically, models which have been developed to address these issues include:
  - Quadratic programs
  - Chance Constrained Programs
  - Stochastic LPs with recourse
    - ..two stage stochastic programs
    - ..multistage stochastic programs

Integer Quadratic Programming

Mean-Variance Model

- Markowitz (1952,1959)
- alternative formulations

QP1

$$Min \quad Z_{QP1} = \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \, x_j \sigma_{ij}$$

$$s.t. \quad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1, \ldots, N$$

QP2 (Arrow-Pratt absolute risk aversion index)

$$Max \quad Z_{QP2}^{R_A} = \frac{R_A}{2} \sum_{i=1}^{N} x_i \mu_i - \sum_{i=1}^{N} \sum_{j=1}^{N} x_i \, x_j \sigma_{ij}$$

$$s.t. \quad \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1, \ldots, N$$

$$R_A \geq 0$$

$i, j = 1, \ldots, N$: denotes the different risky assets

μ_i : expected return of asset i

σ_{ij} : covariance between asset i and asset j

ρ : desired level of return

x_i : the fraction of portfolio value invested in asset i

Mean-Variance Model

- Arrow Pratt absolute Risk Aversion Index

$$R_A = \frac{u''(w)}{u'(w)}$$

where

w is the portfolio wealth

u a Von Neumann-Morgenstern utility function with first and second derivatives

- Portfolios with similar Absolute Risk Aversion index lead to similar portfolios (weight-vector) regardless of functional form and parameters of the utility function (Kallberg and Ziemba 1983)

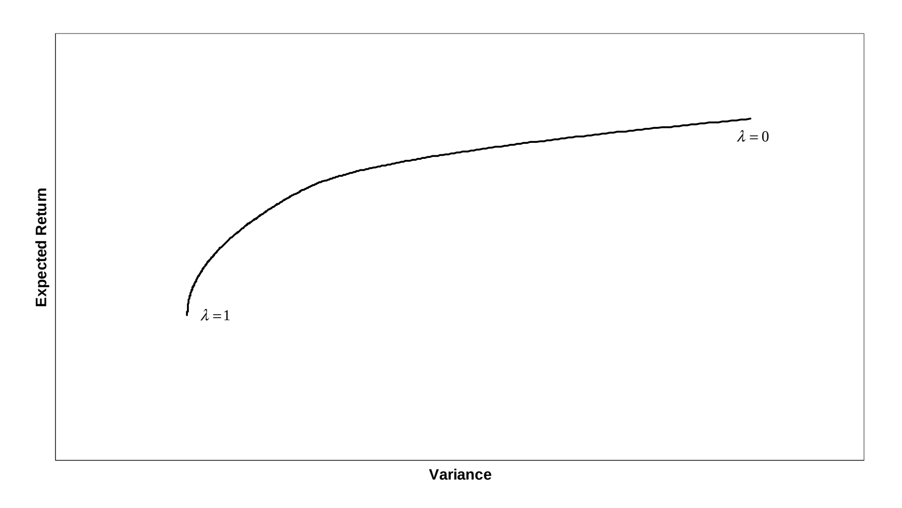
Integer Quadratic Programming

Mean-Variance Model

QP2 (Lambda-formulation)

$$\begin{aligned}
&Min \quad Z_{QP2} = \lambda \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij} - (1-\lambda)\sum_{i=1}^{N} x_i \mu_i \\
&s.t. \quad \sum_{i=1}^{N} x_i = 1 \\
&\qquad x_i \geq 0, \quad i = 1,\ldots,N \\
&\qquad 0 \leq \lambda \leq 1
\end{aligned}$$

Efficient Frontier



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Discrete Constraint Efficient Frontier (DCEF)

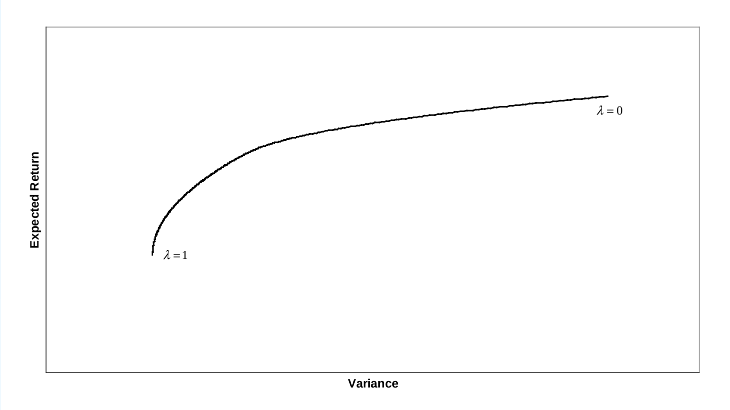
- Buy-in thresholds
  - min. level below an asset is not traded
  - eliminates unrealistically small trades
- Cardinality Constraints
  - controls the number of stocks in a portfolio
  - monitoring and control issues (management effort)
- Roundlots
  - trades only in multiples of ‘discrete’ numbers of assets possible

Integer Quadratic Programming

Discrete Constraint Efficient Frontier (DCEF)

Example:

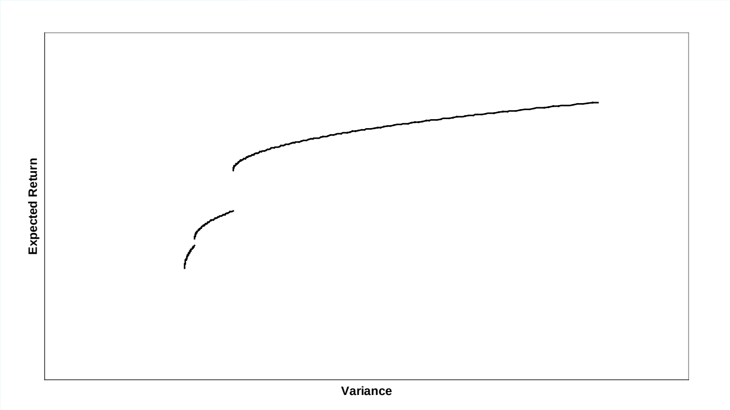
- 4 stock universe, EF



- Introduce cardinality constraint, i.e. build a portfolios containing 2 stocks only

  Effect:

  Discontinuities in the Efficient Frontier

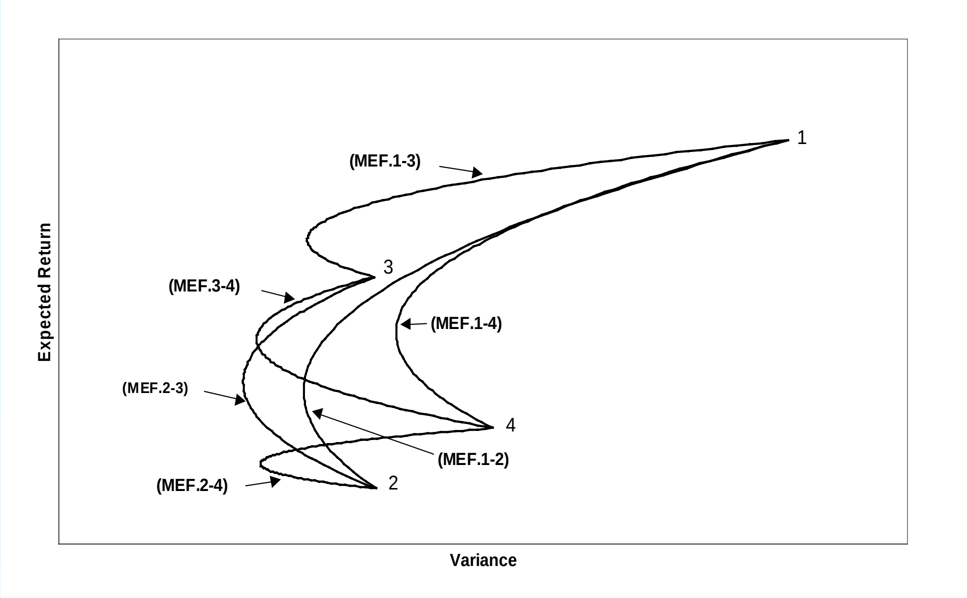


Integer Quadratic Programming

Discrete Constraint Efficient Frontier (DCEF)

Why discontinuities?

- take investment opportunity set
- delete all inefficient portfolios (dominated points)



Integer Quadratic Programming

Mathematical Representation

Extending QP1 with discrete constraints

- Buy-in thresholds

$$Min \qquad Z_{BUY-IN} = \sum_{i=1}^{N}\sum_{j=1}^{N} x_i\, x_j \sigma_{ij}$$

$$s.t. \qquad \sum_{i=1}^{N} x_i \mu_i = \rho$$

$$\sum_{i=1}^{N} x_i = 1$$

$$l_i \delta_i \le x_i \le u_i \delta_i, \quad i = 1,\dots, N$$

$$\delta_i \in \{0,1\}, \quad i = 1,\dots, N$$

l_i, u_i : lower and upper bound on the stock weight

δ_i : binary variable

- Cardinality Constraints

k: number of assets

$$\sum_{i=1}^{N} \delta_i = k$$

Integer Quadratic Programming

Mathematical Representation

- Transaction Roundlots
  - integer number of blocks y_i
  - a lot can be illustratively expressed as fraction f_i of the portfolio wealth
  - re-express x_i as

$$x_i = y_i f_i, \qquad i = 1,\dots,N$$

$\varepsilon^-, \varepsilon^+$: undershoot, overshoot variable

γ : penalty

$$\begin{aligned}
Min \quad & Z_{LOT} = \sum_{i=1}^{N}\sum_{j=1}^{N} y_i f_i y_j f_j \sigma_{ij} + \gamma(\varepsilon^- + \varepsilon^+) \\
s.t. \quad & \sum_{i=1}^{N} y_i f_i \mu_i = \rho \\
& \sum_{i=1}^{N} y_i f_i + \varepsilon^- - \varepsilon^+ = 1 \\
& l_i \le y_i f_i \le u_i, \quad i = 1,\dots,N \\
& y_i \ge 0 \quad \text{integer}, \quad i = 1,\dots,N \\
& \varepsilon^-, \varepsilon^+ \ge 0
\end{aligned}$$

Modeling, Solution Paradigm

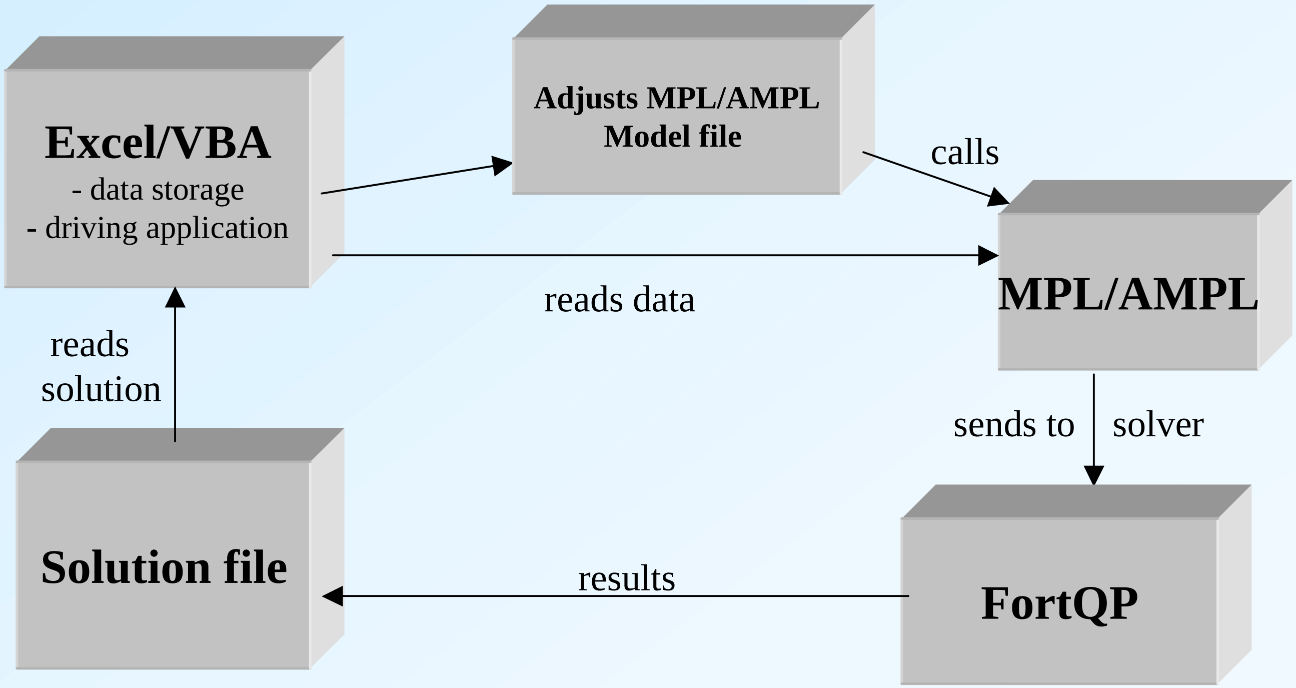
- **Integer Restart Heuristic**
  - solving say 500 QMIPS to optimality is unlikely, therefore it is likely to loose the “pareto efficient” property of the DCEF
  - Method starts from the highest return level and continues computing solutions for lower levels of return
  - uses the previous integer solution as the ‘first feasible and upper bounding QP’ value for computing the next point
  - restricted B&B search (stopped after fixed number of nodes)
  - within the band of sub-optimality, we retain the ‘efficient’ property of the frontier, i.e. risk and return increases or decreases jointly

Integer Quadratic Programming

Modeling, Solution Paradigm

- Software tools:
  - Solver System (FortQP)
  - Mathematical Programming Modeling language (MPL/ AMPL)
  - EXCEL/VBA - application

Data - Modelling - Solver Architecture



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Computational Study

- **Datasets**: taken from 5 markets
  - FTSE 100, Nikkei, Dax, S&P, Hang Seng
- **Model**: Cardinality Constraint
  - cardinality restriction: $k=10$
  - lower bound: $l_i=0.01$
- **Results** for both heuristics and comparison to modern heuristic approaches (Chang et al. (1999) are obtained
- **Metric** of comparison
  - integer optimal DCEF not available, we applied the metric proposed by Chang et al. (1999) and measure the distance to the Efficient Frontier (QP-optimal)

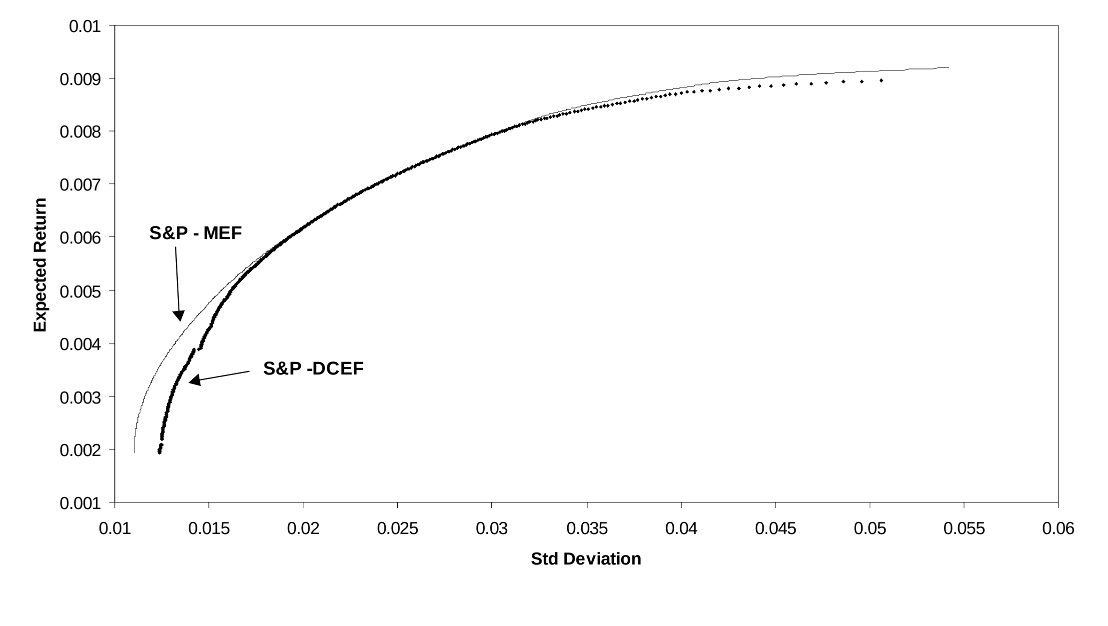
Integer Quadratic Programming

Computational Study

- Integer Restart Heuristic

* Pentium 500, 128 MB RAM

Index	No. of Stocks	Total no. of DCEF pts	No. of integer optimal pts	Solution time *	Mean Error	Median Error
Hang Seng	31	500	492	57.55	0.01415	0.00997
DAX	85	500	228	8405.33	0.01399	0.01159
FTSE	89	500	244	10978.12	0.01141	0.00860
S & P	98	500	192	15831.97	0.01586	0.01325
Nikkei	225	500	486	18345.56	0.00618	0.00252



Computational Study

- Portfolio Rebalancing Problem
  - cardinality constraints (limit on the number of trades)
  - additional variables and constraints:
  - balance constraint $x_i = n_i + b_i - s_i$

    x_i, b_i, s_i hold, buy, sell - variables for asset i

    n_i initial holding in asset i
  - binary variables δ_i^s, δ_i^b - indicator for either buying or selling
  - restriction that asset i can't be sold and bought at the same time

$$\delta_i^s + \delta_i^b \leq 1$$

  - buy-in threshold and upper bounds on buy/sell variables

$$\delta_i^s LB^s \leq s_i \leq \delta_i^s UB_i^s \qquad \delta_i^b LB_i^b \leq b_i \leq \delta_i^b UB_i^b$$

  - Cardinality constraint to restrict the number of trades

$$\sum_{i=1}^{N} \delta_i^s + \delta_i^b \leq k$$

Computational Study

- Factor model (C factors) used to describe asset returns r_i:

$$r_i = \alpha_i + \sum_{c=1}^{C} \beta_{ic} f_c + \varepsilon_i$$

f_c	level of factor c
β_{ic}	sensitivity of asset i to factor c
α_i	mean return of asset i
ε_i	specific return of asset i

- The variance of returns is given by:

$$Var(r_i) = \sigma_i^2 = \sum_{c=1}^{C} \beta_{ic}^2 \sigma_{f_c}^2 + \sigma_{\varepsilon_i}$$

$\sigma_{f_k}^2$	factor variances
$\sigma_{\varepsilon_i}^2$	specific variances

Integer Quadratic Programming

Computational Study

- Tracking the target portfolio in terms of replicating the risk profile (vector of factor sensitivities)
- Sensitivity of the index to the factors given by I_c
- REBALANCE:

$$\mathit{Min} \quad \sum_{c=1}^{C} y_{P,c}^2 \sigma_c^2 + \sum_{i=1}^{N} x_i^2 \sigma_{\varepsilon_i}^2$$

$$s.t.$$

$$y_{P,c} = \left(\sum_{i=1}^{N} x_i \, \beta_{ic} \right) - I_c \qquad \forall c$$

$$\sum_{i=1}^{N} x_i \, \mu_i \ \geq \rho$$

$$\sum_{i=1}^{N} x_i \ = 1$$

$$x_i = n_i + b_i - s_i$$

$$\delta_i^b LB_i^b \leq b_i \leq \delta_i^b UB_i^b$$

$$\delta_i^s LB_i^s \leq s_i \leq \delta_i^s UB_i^s$$

$$\delta_i^b + \delta_i^s \leq 1$$

$$\sum_{i=1}^{N} \left(\delta_i^b + \delta_i^s \right) \leq k$$

$$x_i, b_i, s_i \geq 0$$

$$\delta_i^b, \delta_i^s = 0/1 \qquad \forall i$$

Computational Study

- Additional constraints are in the model (trans. costs, duration, cash infusion …)

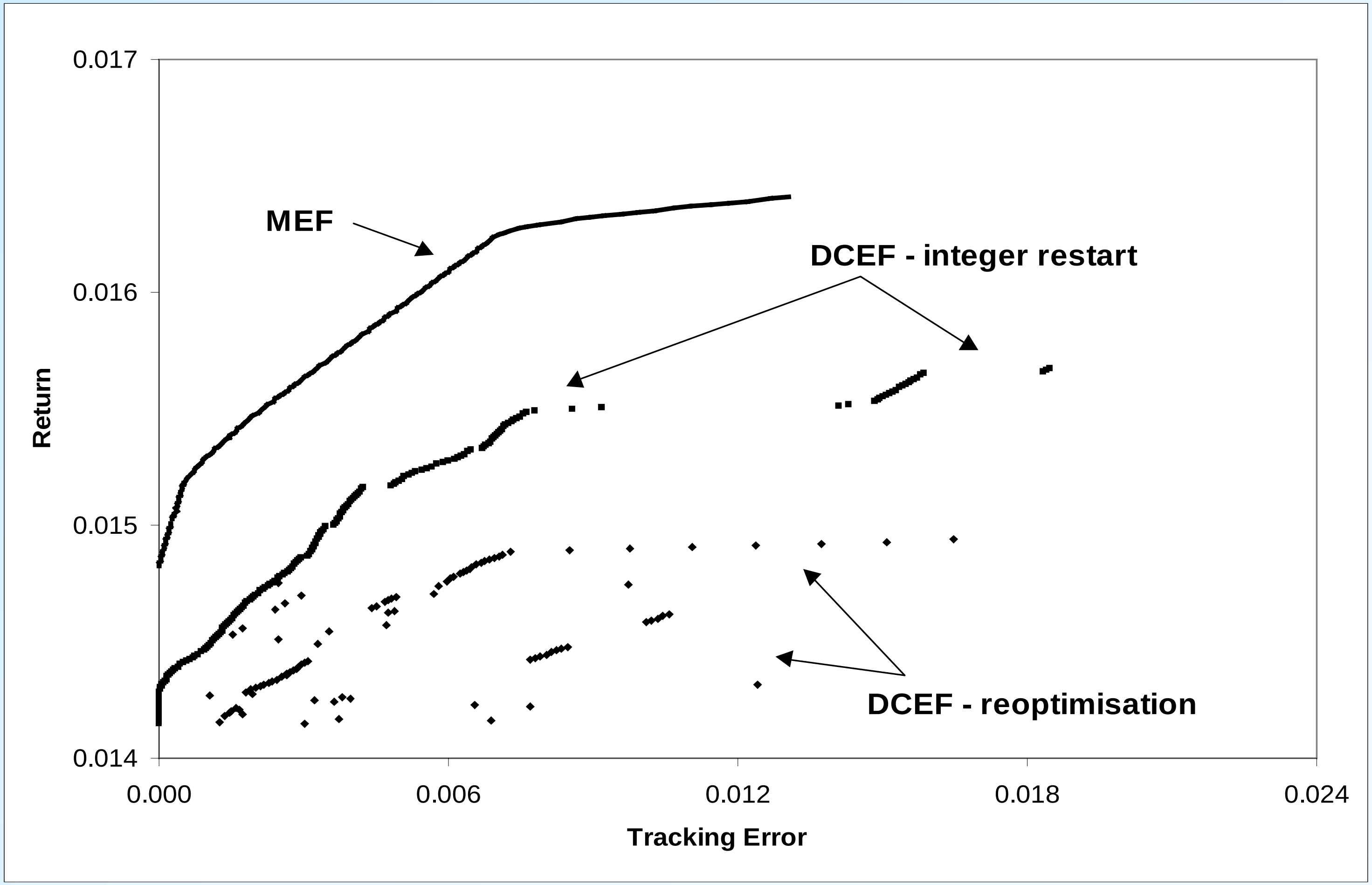
Empirical example:

- Models are implemented for 2 datasets from the fixed-income market
  - rebalancing a 20 bond portfolio, at most 6 trades, tracking an index of 330 bonds
  - rebalancing a 49 bond portfolio, at most 10 trades, tracking an index of 391 bonds
- 3 factors were used to describe return dynamics (explaining approx. 95% of total variance)
- residual risk term is ignored (focus is on factor risk)
- 200 points on the frontier are plotted for each problem

Integer Quadratic Programming

Computational Study

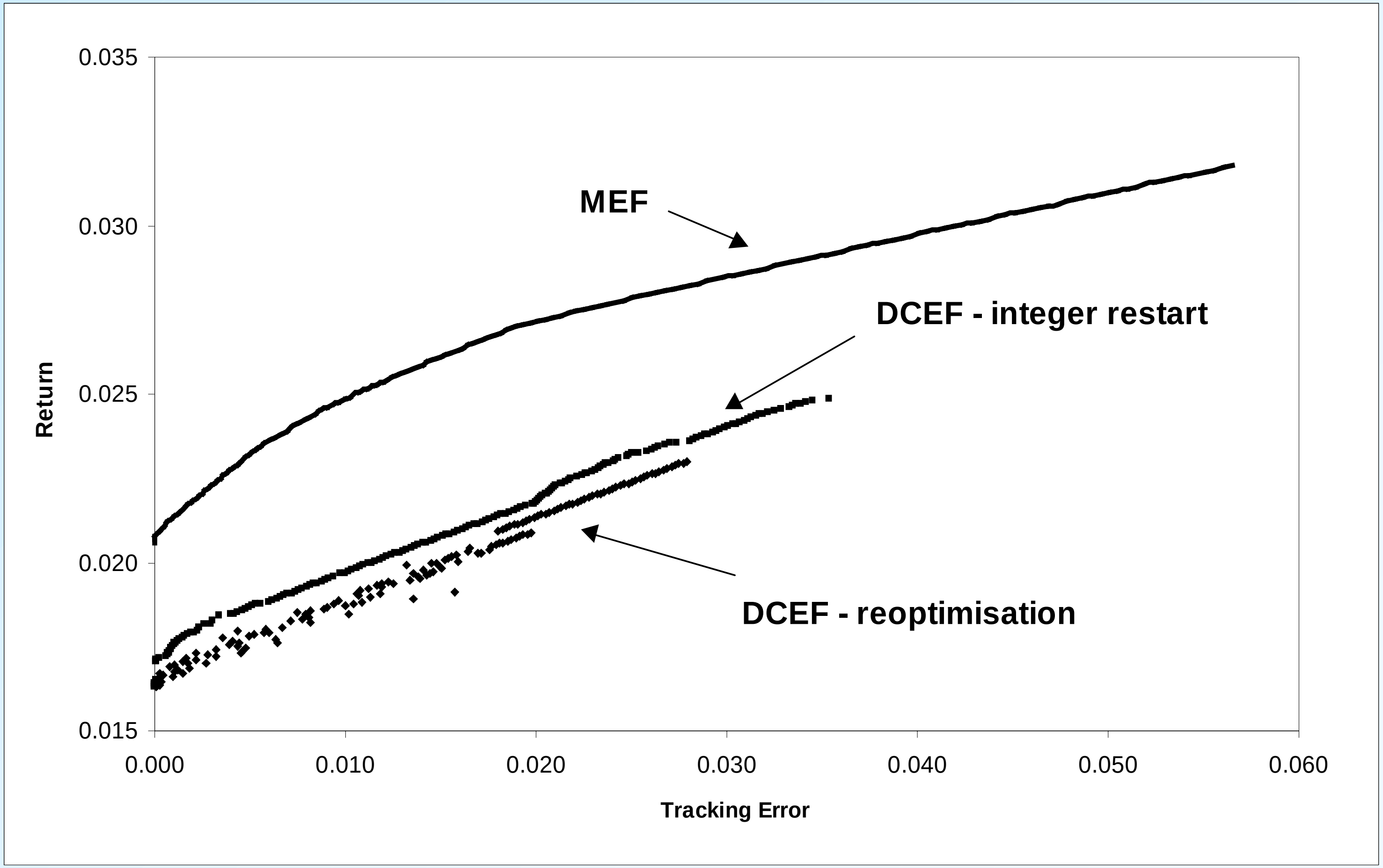
- *Dataset 1 - 6/20 bond rebalancing*



Integer Quadratic Programming

Computational Study

- *Dataset 2 - 10/50 bond rebalancing*



Stochastic Programming

Linear Program

We consider first a linear programming problem:

$$
\begin{aligned}
Z &= \min cx \\
\text{subject to } \quad Ax &= b \qquad\qquad (1) \\
x &\geq 0
\end{aligned}
$$

$$\text{where} \quad A \in R^{m \times n};\ c, x \in R^{n};\ b \in R^{m}$$

Deterministic optimisation model inapplicable to optimum decision making under uncertainty

Stochastic Programming

Optimum Decision Making under Uncertainty

- Quadratic programs
- Two stage stochastic programs
- Multistage stochastic programs
- Chance constrained programs

Random Parameters

Let $(\Omega,\ P)$ denote a probability space, $\omega \in \Omega$ the realizations of the uncertain parameters and $p(\omega)$ the corresponding probability.

Let us denote the realizations of A, b, c for a given ω as

$$(A,b,c)_\omega = \xi_\omega \quad \text{or} \quad \xi(\omega).$$

Let

$$C^\omega = \{x \mid Ax = b, x \geq 0\} \quad \text{for} \quad (A,b,c)_\omega \ \text{or} \ \xi(\omega)$$

We can reconsider (1) as an expected value or an average value problem where

$$\bar{\xi}(\omega) = E[\xi(\omega)] = \sum_{\omega \in \Omega} p(\omega)\xi(\omega)$$

$$Z_{ev} = \min f(x, \bar{\xi}) = \min \bar{c}x \tag{2}$$

Classical Stochastic Linear Program with Recourse

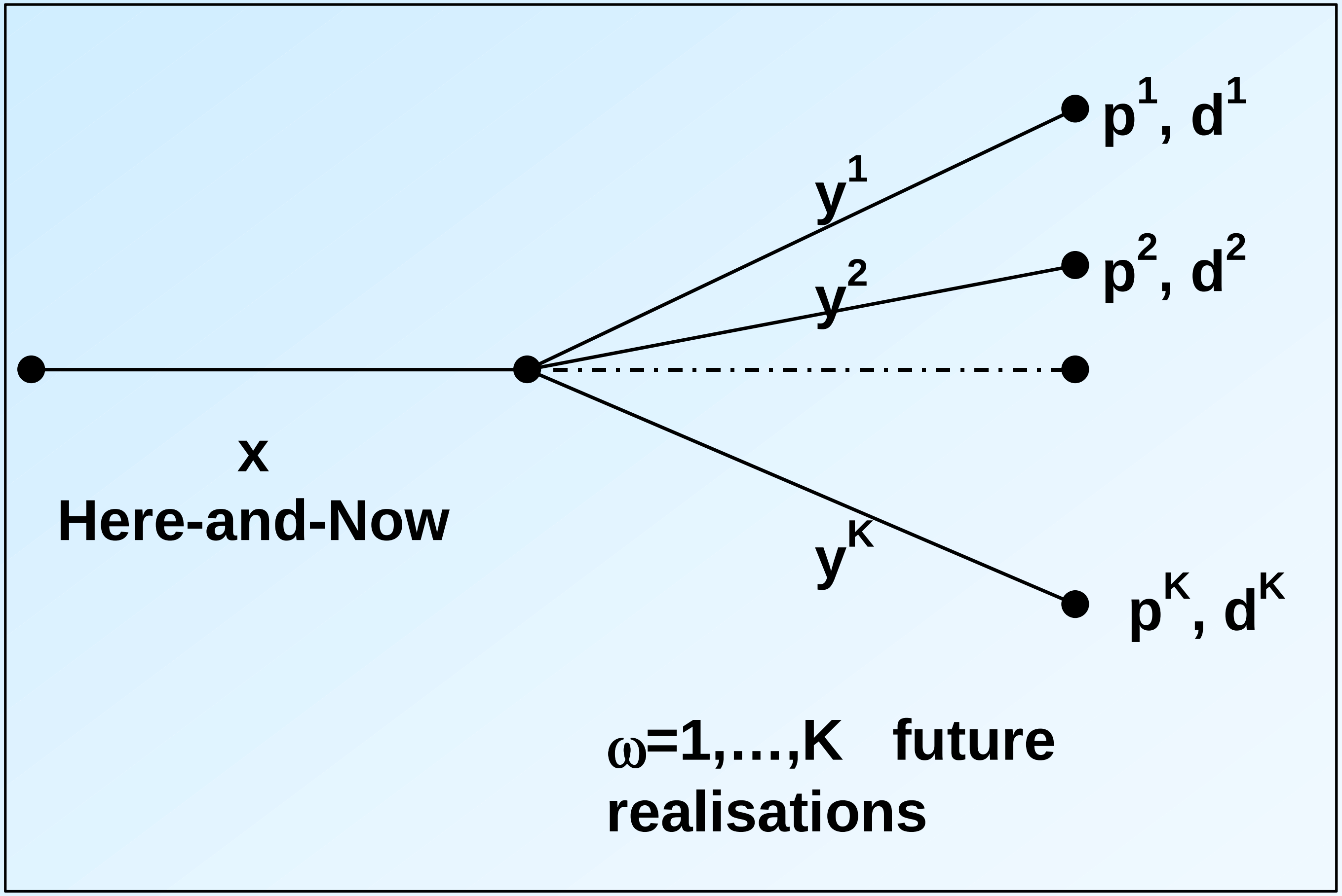
The SLPR is stated as

$$\begin{aligned} Z = \ \min \quad & cx + E_{\omega} Q(x,\omega) \\ \text{subject to} \quad & Ax = b \\ & x \geq 0, \end{aligned} \tag{10}$$

where

$$\begin{aligned} Q(x,\omega) = \quad & \min f(\omega) y \\ \text{subject to} \quad & D(\omega) y = d(\omega) + B(\omega) x \\ & y \geq 0. \end{aligned} \tag{11}$$

Two Stage Stochastic Program



Two Stage Stochastic Linear Programs

A two-stage recourse problem is transformed into a two-stage stochastic linear program:

$$
\begin{aligned}
\min Z \quad & = cx + E^{\omega}\left[fy^{\omega}\right] \\
\text{subject to} \quad & Ax = b \\
& -B^{\omega}x + D^{\omega}y^{\omega} = d^{\omega} \\
& x, y^{\omega} \geq 0; \quad \omega \in \Omega
\end{aligned}
\tag{12}
$$

(Deterministic) Equivalent LP

Let us consider the case of Ω being discrete and finite, and Ω as an index set Ω={1,...,K}, meaning that the parameter ω may take on K different values.

$$
\begin{array}{llllllll}
\min Z & = & cx & + p^1 f y^1 & + p^2 f y^2 & + \ldots & + p^K f y^K & \\
\text{subject to} & & Ax & & & & & = b \\
& & -B^1 x & + D^1 y^1 & & & & = d^1 \\
& & -B^2 x & + & D^2 y^2 & & & = d^2 \\
& & \vdots & & & \ddots & & \vdots \\
& & -B^K x & + & & & D^K y^K & = d^K \\
& & x, & y^1, & y^2, & \ldots & y^K & \geq 0 \\
& & 0 \leq p^\omega \leq 1 & \text{and} & \sum p^\omega = 1.0 & & &
\end{array}
\qquad (13)
$$

Expectation of the Expected Value Problem

Let x^*_{ev} be the optimum solution of the above expected value problem. This solution can be evaluated for all possible scenarios $\omega \in \Omega$ and we can determine the corresponding objective function values and compute the expected value of the objective function as Z_{eev}.

$$Z_{eev} = E\left[c x^*_{ev}\right]$$

Note c is a component of ξ. In particular we note that x^*_{ev} may not be feasible for all c^{ω}, that is for some ω, $x^*_{ev} \notin C^{\omega}$; in this case we set $Z_{eev} \rightarrow +\infty$.

Wait and See Models

The corresponding problem is stated as:

$$Z^{\omega} = \min cx \qquad (3)$$

$$\text{subject to} \quad x \in C^{\omega}$$

Let Z_{ws} denote the expected value of Z^{ω}, then

$$Z_{ws} = E\left[Z^{\omega}\right] = \sum_{\omega \in \Omega} Z^{\omega} p(\omega) \qquad (4)$$

Here and Now Decision Problem

The value *x* is chosen such that the expected costs E(cx) assume a minimum:

$$Z_{hn} = \min E[cx] \qquad (5)$$

where $x \in C$

and $$C = \bigcap_{\omega \in \Omega} C^{\omega} \qquad (6)$$

The optimal objective function value Z_{hn} denotes the minimum expected costs of the stochastic optimization problem.

Inter Relationship and Bounds

Z_{ws}, Z_{hn}, Z_{eev} are connected by the following relationships:

$$Z_{ws} \le Z_{hn} \le Z_{eev} \qquad (7)$$

The difference ($Z_{hn} - Z_{ws}$) is known as the expected value of perfect information (EVPI). Thus:

$$EVPI = Z_{hn} - Z_{ws} \qquad (8)$$

There is another measure known as the value of the stochastic solution (VSS):

$$VSS = Z_{eev} - Z_{hn} \qquad (9)$$

Birge shows that EVPI and VSS are bounded:

$$0 \le EVPI \le Z_{hn} - Z_{ev} \le Z_{eev} - Z_{ev}$$

$$0 \le VSS \le Z_{eev} - Z_{ev}$$

Multistage Stochastic Programs (MSP)

$$
\begin{array}{ll}
\min\limits_{x_1} & \left\{ c_1 x_1 + E_{\xi_2}\left[\min\limits_{x_2} c_2 x_2 + E_{\xi_3|\xi_2}\left[\min\limits_{x_3} c_3 x_3 + \ldots + E_{\xi_T|\xi_{T-1}|\ldots|\xi_2} \min\limits_{x_T} c_T x_T \right]\right]\right\} \\
\text{st} & \begin{array}{lllll}
A_{11}x_1 & & & & = b_1 \\
A_{21}x_1 + & A_{22}x_2 & & & = b_2 \\
A_{31}x_1 + & A_{32}x_1 + & A_{33}x_3 & & = b_3 \\
\vdots & & \ddots & & \vdots \\
A_{T1}x_1 + & A_{T2}x_1 + & A_{T3}x_3 + & \ldots & + A_{TT}x_T = b_T
\end{array} \\
& \ell_t \le x_t \le u_t;\ t = 1,\ldots,T
\end{array}
\qquad (19)
$$

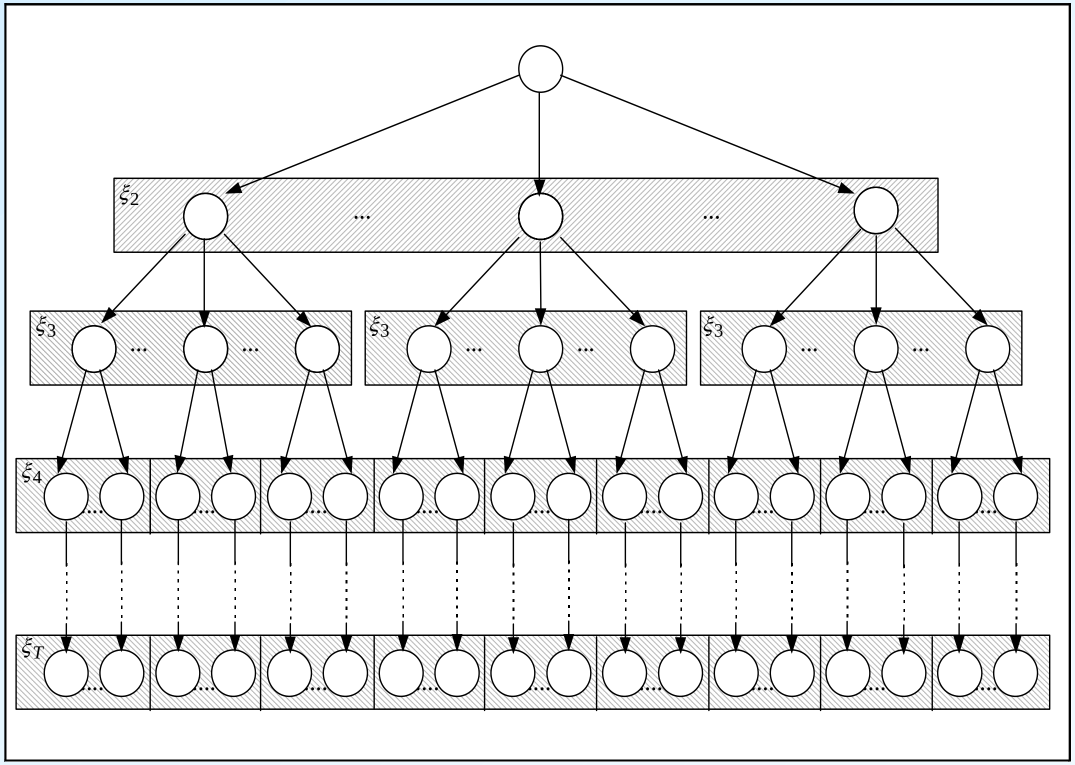
Stochastic Programming

Scenario Tree for MSP

A probability π_n can be associated with each node n at level t such that

$$\pi_n = p\{\xi_t \mid \xi_{t-1} \mid \dots \mid \xi_2\}$$

Hence, arcs in the tree represent the probability distribution of ξ_t



Stochastic Programming

Planning models for supply chains

- Planning and utilisation of production & distribution capacities ….Optimal decisions
- Leading problem in manufacturing and supply chain logistics…
- Need for regular evaluation of strategic asset allocation decisions (rolling plan)…uncertain business environment

New Perspective on Investment Decisions

- The issues:
- DCF is inadequate…
- Three leading characteristics
  - Investment Decisions (costs) irreversible
  - Future returns are uncertain
  - Another key aspect is timing
    - Invest
    - Disinvest
    - Not invest…postpone
- Are all strategic decisions

Stochastic Programming

- Deterministic Optimisation Model LP cannot be applied !!
- We do not know demands and production rates with certainty !
- With hind sight we can make optimum decisions.
  -> 'Wait-and-See' for actual realisations ?
- But the decisions must be made 'Here-and-Now'

Stochastic Programming

Answer:

- BUY FLEXIBILITY

=

- HEDGE AGAINST UNCERTAINTY

=

- MAKE ROBUST DECISIONS

Three worlds

- ***Deterministic***:

in a deterministic world all parameters known with certainty

- ***Probabilistic*** *(Uncertain)*:

in an uncertain world we have random parameters … probability distributions

- ***Pragmatic***:

in a pragmatic world we have alternative scenarios and discrete weights … probability

"crude but possibly acceptable approximation to the actual randomness"

Stochastic Programming

Role of time

"Wait and See"

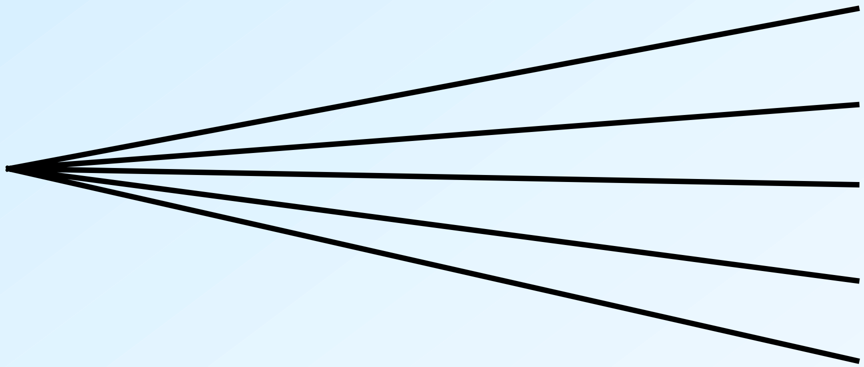
optimum decisions

"Here and Now"

decisions to be made

Here and Now **Future**

Scenario 1 **P_1**



Scenario 100 **P_{100}**

Strategic Decision

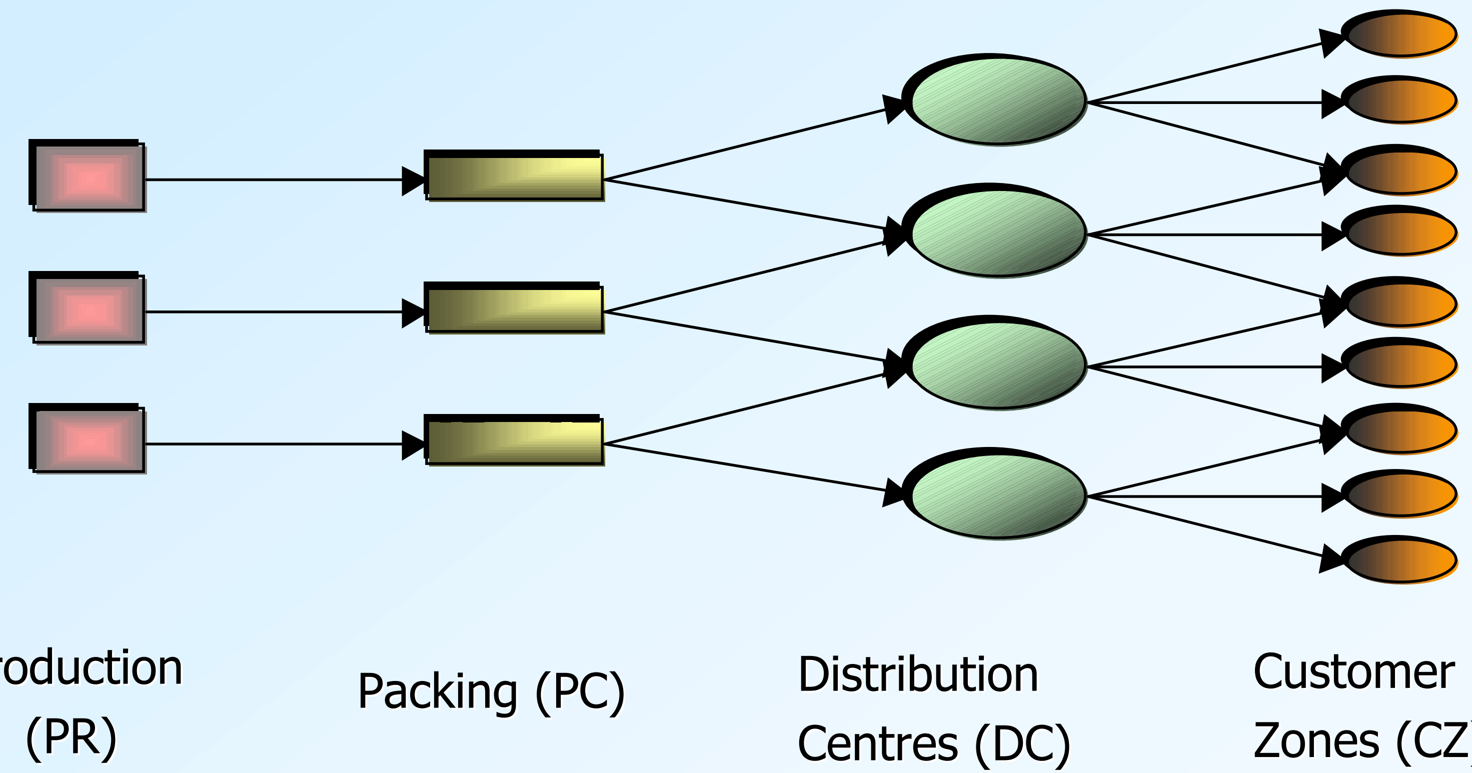
Recourse

(best corrective action)

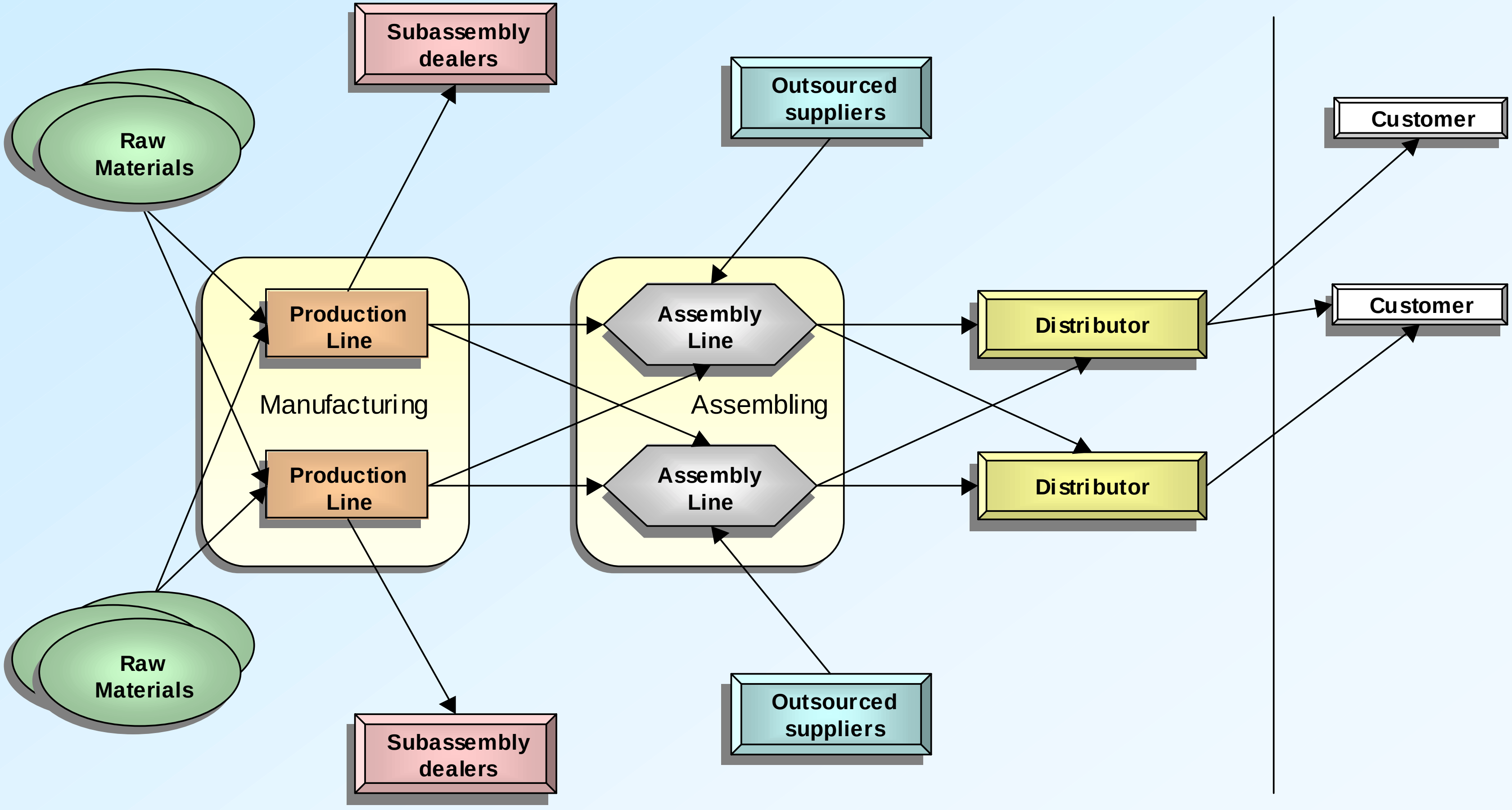
Stochastic Programming

- Stochastic Programming with recourse models are ideally suited .. two perspectives
  - (near) optimum resource allocation
  - hedge against uncertain future outcomes
  - Decisions not optimum for any one outcome, good for many outcomes !
- Two stage models
  - First Stage: ‘ Here-and-Now’ asset allocation decisions … takes into consideration scenarios(outcomes)
  - Second Stage: Recourse decisions optimal corrective actions as future unfolds…

Supply Chain Model 1



Supply Chain Model 2



Model and Data instances

P0:

$$\min Z = cx + \sum_s p_s f y_s$$

subject to

$$\begin{aligned} &Ax && = b && \\ &Bx + Dy_s && \leq h && \forall s \in \{1,\ldots,S\} \\ &\quad\quad Ey_s && = d_s && \forall s \in \{1,\ldots,S\} \\ &x \in \{0,1\}^{n_1} && && \\ &y_s \geq 0 && && \end{aligned}$$

P1:

$$\min Z = cx + fy$$

subject to

$$\begin{aligned} &Ax && = b && \text{Logical Constraints} \\ &Bx + Dy && \leq h && \text{Capacity Constraints} \\ &\quad\quad Ey && = d_s && \text{Demand Constraints} \\ &x \in \{0,1\}^{n_1} && && \text{Discrete Variables} \\ &y \geq 0 && && \text{Continuous Variables} \end{aligned}$$

Stochastic Programming

Model and data instances

Network Components	Dimensions
The number of Sites, I :	**8**
The types of packing line technology, Y_C:	**4**
The types of production line technology, Y_R:	**2**
The number of distribution centres , J :	**15**
The types of DC line technology , Y_D:	**2**
The number of Customer Zones , H :	**30**
The number of Products , P :	**13**
The number of time periods, T :	**6**

Scenarios: 100

<table>
<tr><th colspan="4">Model Statistics</th></tr>
<tr><td colspan="2">Logical Constraints:
Sites, DCs opening and closing, Limit on number of Sites, DCs, and Lines</td><td>968</td><td rowspan="3">6768</td></tr>
<tr><td rowspan="2">Other Constraints: Production, Packing, Ordering, Transportation, Balance, Demand, and also Production and Packing Capacities.</td><td>Mixed</td><td>850</td></tr>
<tr><td>Continuous</td><td>4950</td></tr>
<tr><td colspan="2">Discrete Decision Variables:
Sites, DCs, Production lines, Packing lines, DC lines.</td><td>2096</td><td rowspan="2">56496</td></tr>
<tr><td colspan="2">Continuous Variables:
Production, Packing, Ordering, Transportation, and Shortage quantities.</td><td>54400</td></tr>
<tr><td colspan="2">Non zeroes</td><td colspan="2">1154034</td></tr>
</table>

Stochastic Programming

Stochastic Programming

Two-stage integer stochastic program in serial

P2:

$$
\begin{aligned}
&P_{MIP}(s): \quad \min Z = cx + fy \\
&\textit{subject to} \\
&Ax = b \\
&Bx + Dy \le h \\
&Ey = d_s \\
&x \in \{0,1\}^{n_1},\ y \ge 0
\end{aligned}
$$

P3:

$$
\begin{aligned}
&P_{LP}(s,n): \quad \min Z = c\bar{x}_n + fy \\
&\textit{subject to} \\
&Dy \le h - B\bar{x}_n \\
&Ey = d_s \\
&y \ge 0
\end{aligned}
$$

Stochastic Programming

Two-stage integer stochastic program in serial

P4:

$$\min Z = cx + fy + \lambda(Bx + Dy - h)$$

OR

$$\min Z = (c + \lambda B)x + (f + \lambda D)y - \lambda h$$

subject to

$$Ax = b$$

$$Ey = d_s$$

$$x \in \{0,1\},\ y \geq 0,\ \lambda \geq 0,\ \lambda \in \mathrm{R}^{m_2}$$

Lagrangean Decomposition

P5:

$$\min Z = (c + \lambda B)x$$

subject to

$$Ax = b$$

$$x \in \{0,1\},\ \lambda \geq 0$$

P6:

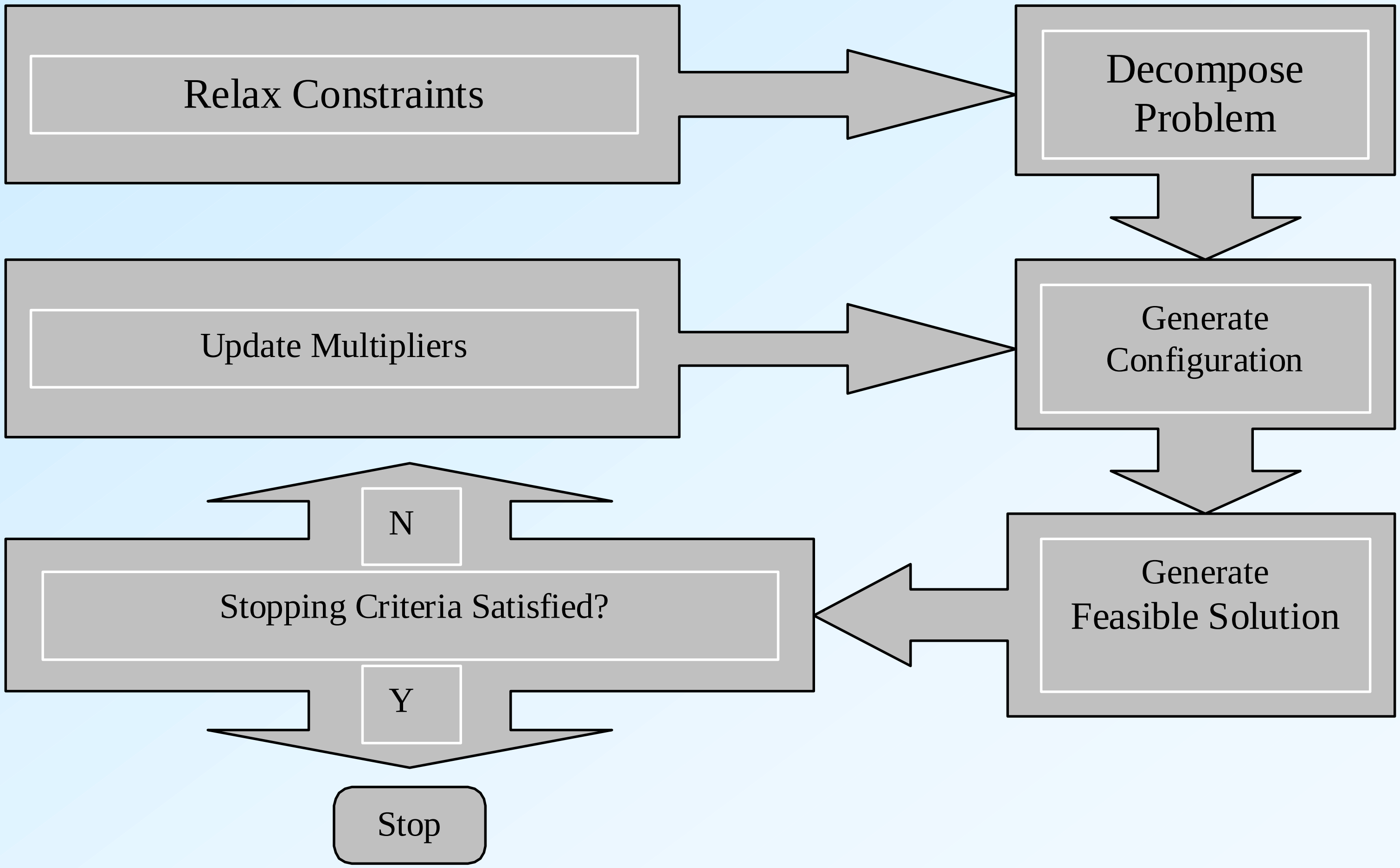
$$\min Z = (f + \lambda D)y$$

subject to

$$Ey = d_s$$

$$y \geq 0,\ \lambda \geq 0$$

Two-stage integer stochastic program in serial



The approximate models

APXP0

$$\min Z = cx + \sum_s p_s f y_s$$

subject to

$$Dy_s \le h - Bx,$$
$$Ey_s = d_s, \quad \forall s \in \{1,..,S\}$$
$$x = \sum_n \beta_n \bar{x}_n$$
$$\sum_n \beta_n = 1,$$
$$x \ge 0, y_s \ge 0, \forall s$$
$$0 \le \beta_n \le 1, \ \beta_n \in Z^+ \quad \forall n$$

APXP1

$$\min Z = cx + fy$$

subject to

$$Dy \le h - Bx,$$
$$Ey = d_s,$$
$$x = \sum_n \beta_n \bar{x}_n$$
$$\sum_n \beta_n = 1,$$
$$x \ge 0, y \ge 0$$
$$0 \le \beta_n \le 1, \beta_n \in Z^+ \quad \forall n$$

Stochastic Programming

Computational results

- 500 MHz, 128 MB RAM , 100 MB hub, PVM

•. Time taken for Lagrangean as a column generator on a cluster of 3 machines

	Phase 1	Phase 2
Time (hrs)	9	150

•In Phase 1

We perform 25 iterations per scenario(100)

At each iteration we solve

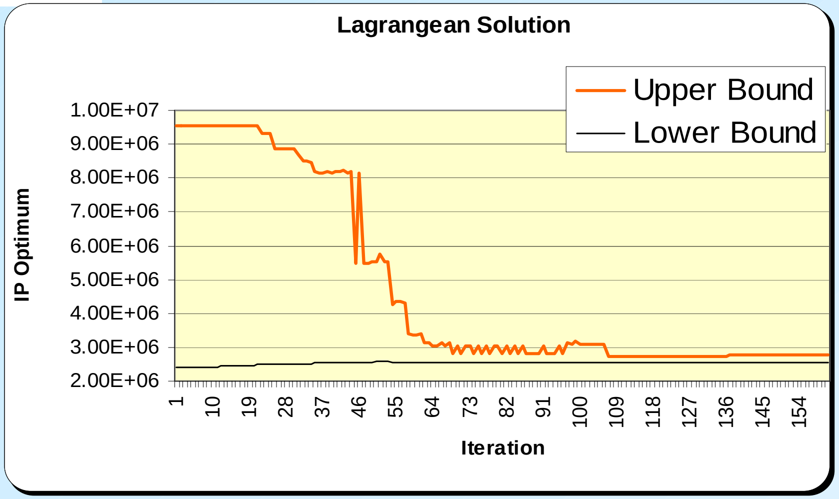
- A 968 * 2096 model to first integer
- A 5800 * 54400 model to LP optimum
- A 4969 * 54400 model to LP optimum

We generate 2050 configurations

•In Phase 2

We solve 2050 * 100 LPs of dimension 5800 * 54400 to optimality

Stochastic Programming



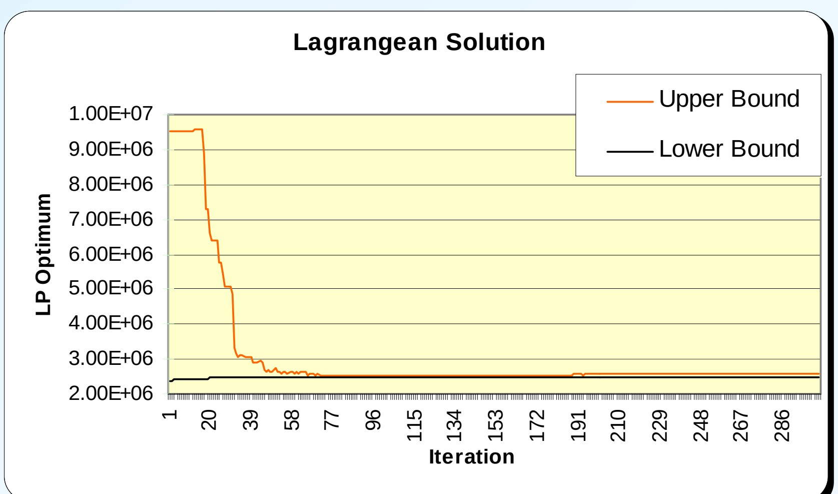
Lagrangean on the DEQ

Constraints = 580,968

Variables = 5,442,096

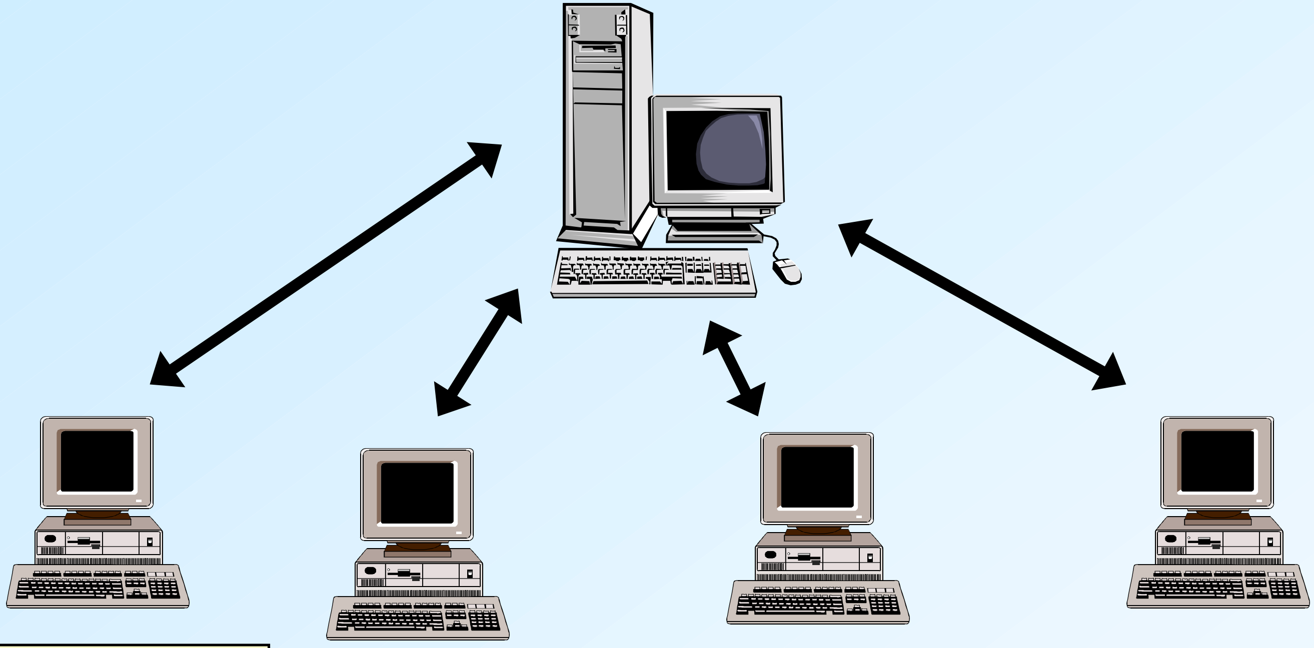
LB	Best obj	UB	VSS	EVPI
2.7428e6	2.7563e6	2.7689e6	1.26e5	1.34e5

LUB	GLB
2.53E+06	2.47E+06



Stochastic Programming

Two-stage integer stochastic program in parallel



N = Number of Processors, n∈N
S = Set of Scenarios to be solved for generating configurations.
S_n= Set of Scenarios to be solved on Processor n.

$$\bigcup S_n = S,$$
$$\bigcap S_n = \varphi$$

On each Processor n, the following algorithm is run.

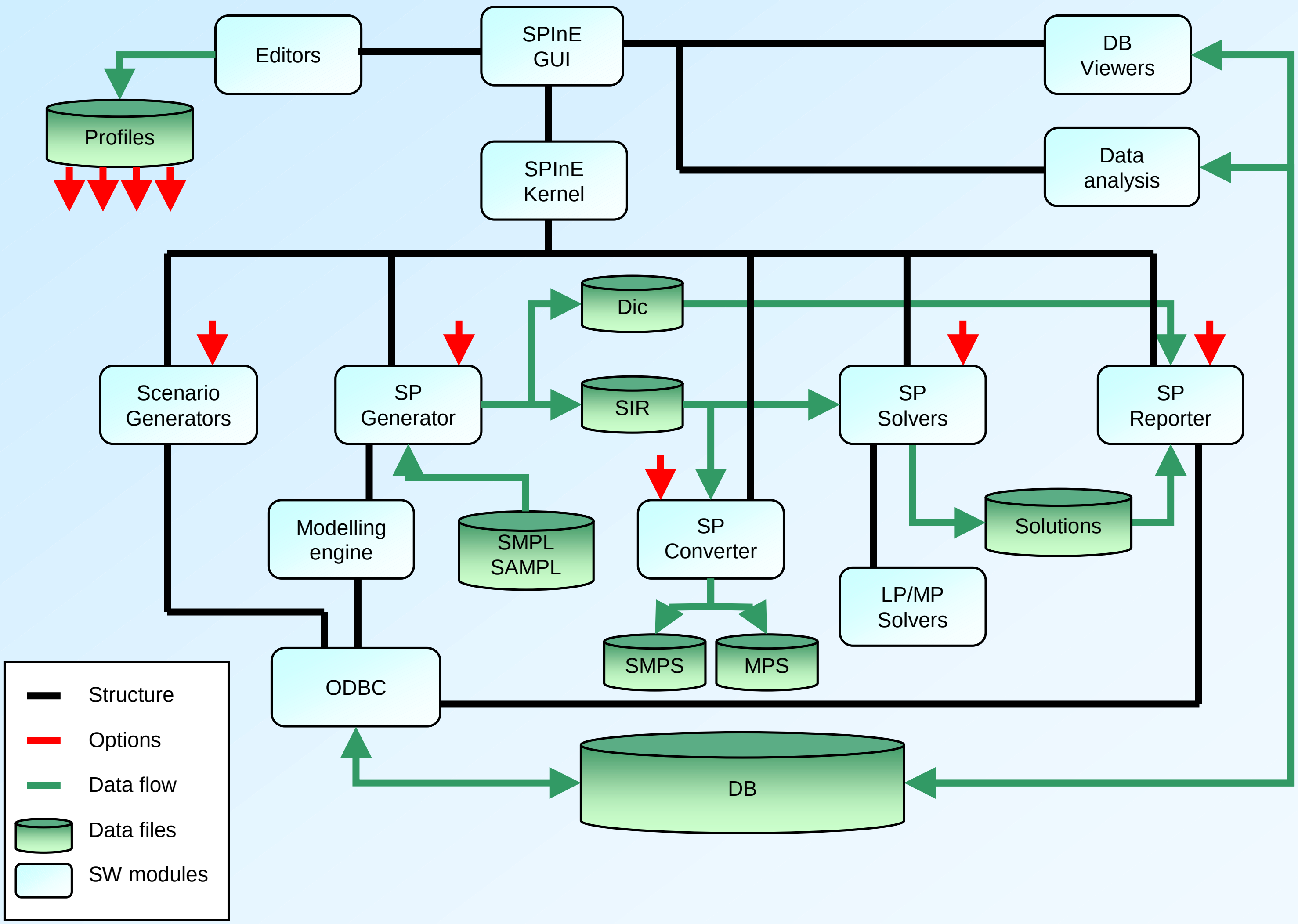
New generation of models and software tools

Current Developments

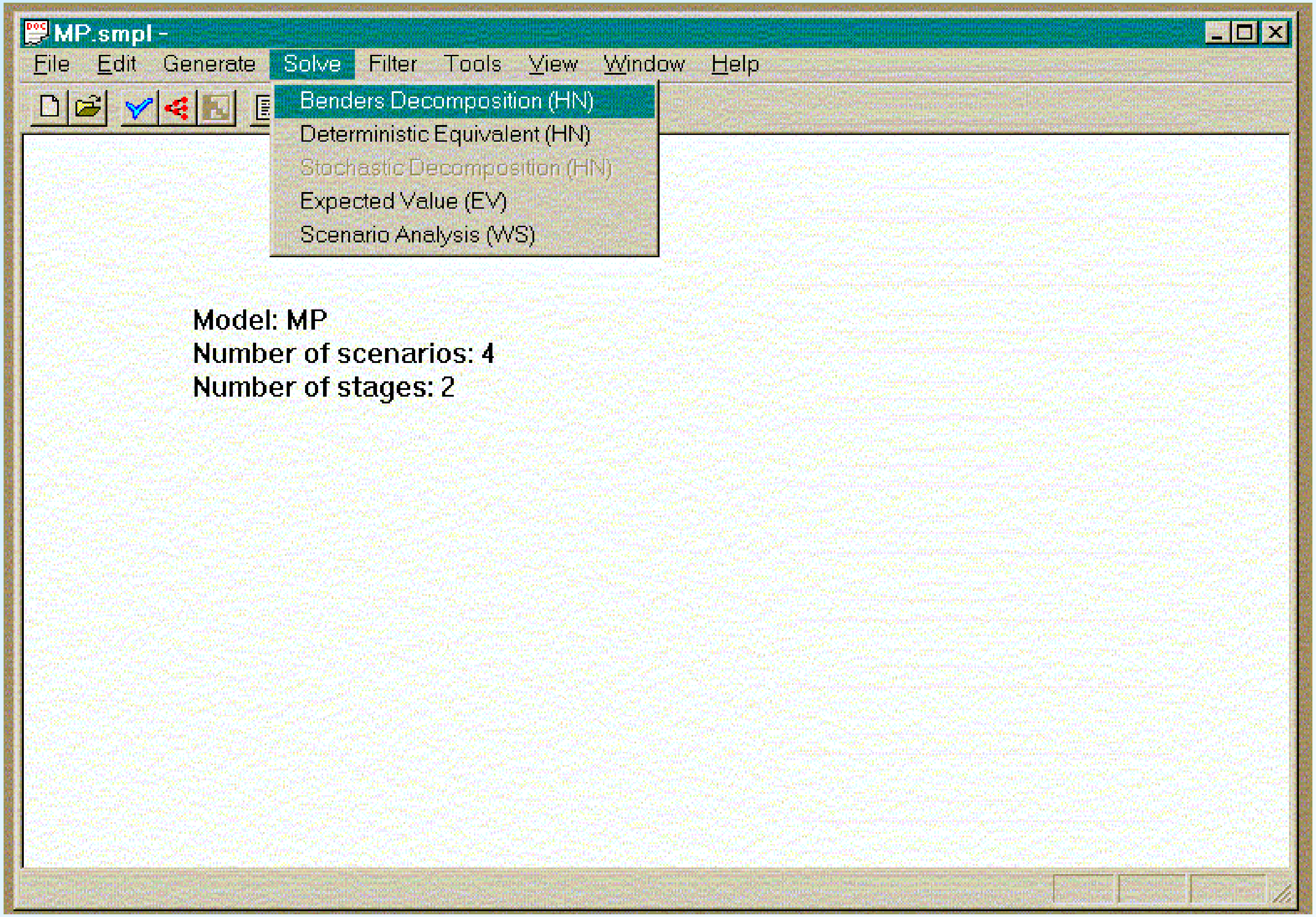
Name	Affiliation	System Name
A. King ,et al.	IBM	SPOSL
G. Mitra ,et al.	Brunel Univ./UNICOM	SPInE
J. Bisschop ,et al.	Paragon D.T.	AIMMS
P. Kall ,et al.	Univ. of Zurich	SLP-IOR
M. Dempster ,et al.	Cambridge Univ.	STOCHGEN
R. Fourer ,et al.	North Western Univ.	SP/AMPL
H.I.Gassman ,et al.	Dalhousie Univ.	MSLiP
E. Fragniere,et al.	Univ.Geneva/Edinburgh	SETSTOCH
G. Infanger ,et al.	Stanford Univ.	DECIS

New generation of models and software tools

SPInE system architecture

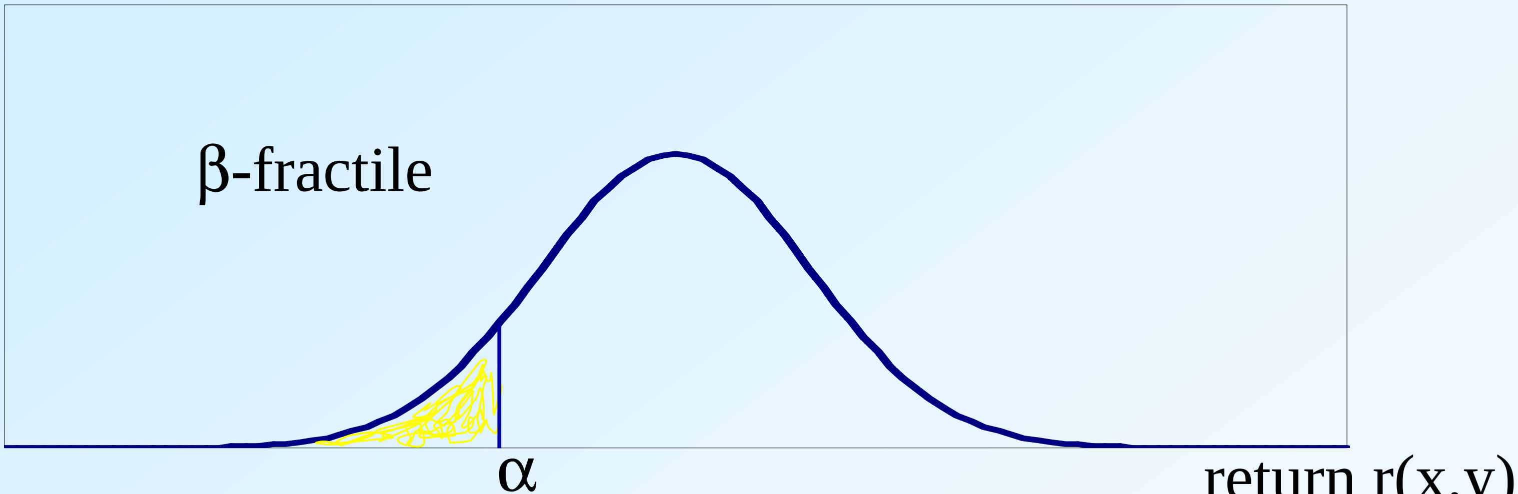


SPInE interface



Discussions

- Finance industry has introduced Value at Risk (VAR) also known as the β-var.



$$\text{probability function}: \ \Psi(x,\alpha) = \int_{r(x,y)\le\alpha} p(x,y)\,dy$$

$$\text{quantile function}: \quad \alpha(x,\beta) = \min\{\alpha : \Psi(x,\alpha) \ge \beta\}$$

Discussions

- Using simulation we can always compute the β-var of a given optimum! hedged decision
- Should we? And how can we? Make a decision that instead of best hedging makes least β-var or conditional VAR loss
- This will keep the regulators happy and more … fun for the “risk quant boys” !!

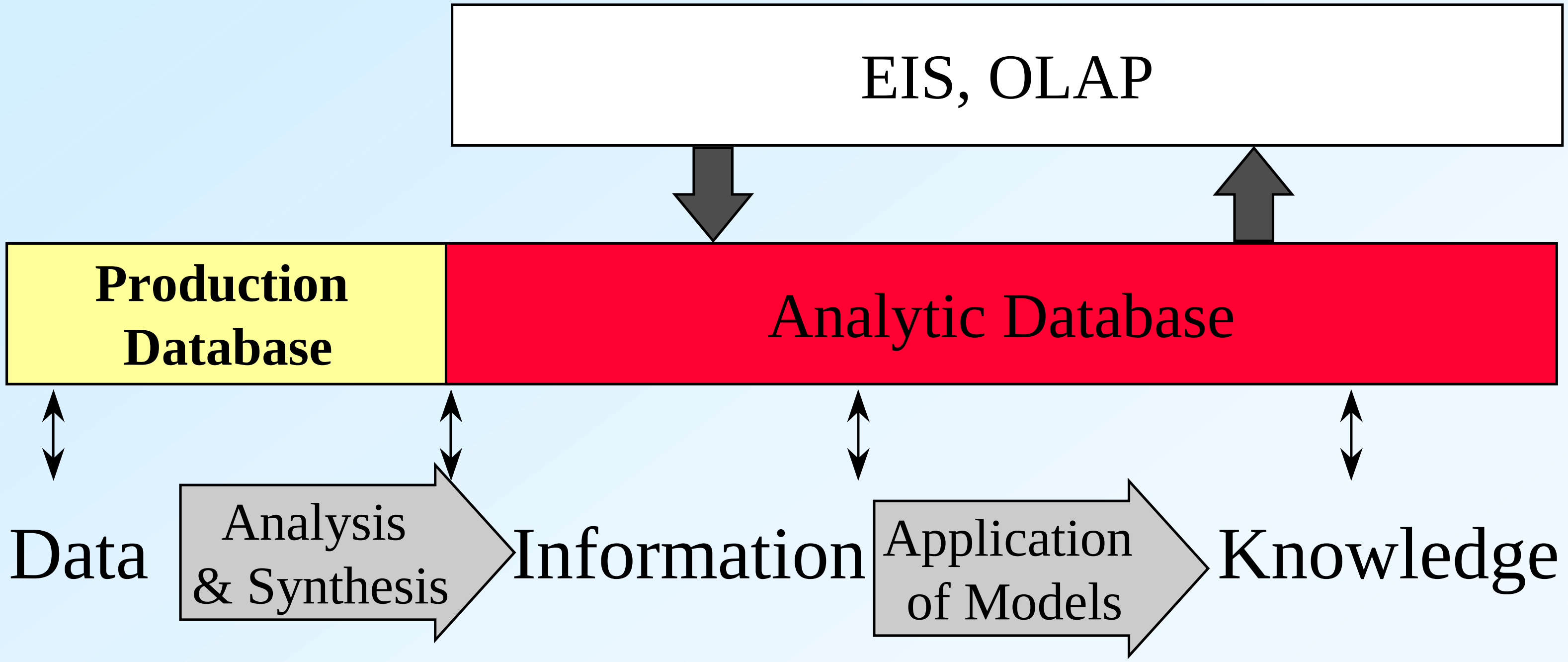
Discussions

- Because of many uncertainties
  - Economic
  - Physical(weather)
  - Political

  decision models for real world problems must go beyond the paradigm of optimisation

- The new generation of decision models need to capture risk as well as return.
- Temporal aspects of these problems (not knowing the future) lead to extremely challenging dynamical probelms

Information & Knowledge: The Value Chain



Alternative View