Stochastic optimisation models in integrated market and credit risk ALM

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Outline

- Outline
- Credit Risk and Credit Risk Modelling
- Credit Risk Simulation
  - An extended CreditMetrics approach
- Credit Risk Optimisation
  - A stochastic programming index tracking model
- Numerical Results
- Extensions and future research

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Credit Risk and Credit Risk Modelling I

What is credit risk?

- one of the oldest forms of risks in financial markets

- the chance that a contractual counterparty does not meet its obligations stated in the contract, hence causing a financial loss to the creditor
    - e.g.: issuer of a bond fails to pay a promised coupon,
      default of a company (Enron,Worldcom..Russia,Argentina)

- more general: risk of drop in market value due to changes in the credit quality of a debtor (the issuer of the bond)

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Credit Risk and Credit Risk Modelling II

Risk factors

- credit spread changes,
- default events,
- recovery rates,
- migration (credit quality (rating) changes)
- market risk (interest rate changes)

- correlation between credit events (in portfolios)

Modelling approaches:

2 approaches developed in mathematical finance

- structural models (e.g. Merton 1974)
- reduced form models (e.g. Jarrow and Turnbull 1995)

Credit Risk and Credit Risk Modelling III

Structural models (e.g. Merton 1974)

- based on firm value evolution
- once firm value falls below a boundary process, default is triggered (e.g. boundary process: value of the firms liabilities)
- intuitively appealing, empirical work is less appealing

Reduced form models (e.g. Jarrow and Turnbull 1995, Duffie 1998)

- default process specified exogenously
- default hits the investor by surprise
- default process is usually defined by a point process (e.g. Poisson, Cox)
- good calibration to market data
- no direct link to the firms health

Hybrid models

- combine the features of both, reduced form and structural models

Credit Risk Simulation I

Wish list:

- Integrate disparate sources of risk
- Consistency with observed market prices (term structures)
- Capture correlations amongst risk factors
  - spreads and interest rates
  - correlated defaults and migrations ...
- Risk neutral vs. real probability measures
  - pricing vs. risk management and simulation
- Flexibility to include different securities s.t. market and credit risk
- Computational tractability

Credit Risk Simulation II

Reality:

- no single approach model that captures all elements
- Portfolio models such as CreditMetrics, CreditRisk+
  - focus on simulation of credit events for (large) portfolios
  - strong assumptions on individual price level
    - deterministic or constant credit spreads
    - deterministic or constant interest rates

Our approach:

- apply CreditMetrics approach to simulate correlated credit migrations and defaults

combined with

- reduced form pricing framework incorporating stochastic interest rates and credit spreads

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Credit Risk Simulation III

Reduced form models (intensity models):

- e.g. Recovery of Market value (Duffie and Singleton (1999))

$$v^{RMV}(t,T) = E_t^Q\left(e^{-\int_t^T (r(s)+q\lambda(s))ds} \right)$$

v^{RMV}	**Price of a risky zero coupon bond at time t with maturity T**
Q	**Risk-neutral measure**
$r(t)$	**Risk free short rate**
$\lambda(t)$	**Intensity or default intensity**
q	**Loss rate in default (fraction of pre-default value)**

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- spreads can be directly modelled by assuming: s(t) =q /Lambda

Credit Risk Simulation IV

Current portfolio valuation

$$W_0 = \sum_{n=1}^{N} w_n P_n^{\kappa_0}(0, T_n)$$

$$P_n^{\kappa_0}(0, T_n) = \sum_{t=1}^{T_n} F_n(t) v_n^{\kappa_0}(0, T_n)$$ current price of bond n

w_n holding of bond n in portfolio

κ_0 current rating of bond n (index n dropped)

$F_n(t)$ coupon

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Credit Risk Simulation V

Future bond prices and portfolio valuation (at a risk horizon $\bar{T}$)

$$P_n^{\kappa_{\bar{T}}}(\bar{T}, T_n) = \sum_{t=\bar{T}}^{T_n} F_n(t) v_n^{\kappa_{\bar{T}}}(\bar{T}, T_n)$$

price of risky coupon bond at $\bar{T}$

- $\kappa_{\bar{T}}$ rating of bond n at time
  simulated from CreditMetrics methodology (based on firm value and consistent with historical migration prob.)

- interest rates and spreads at time $\bar{T}$

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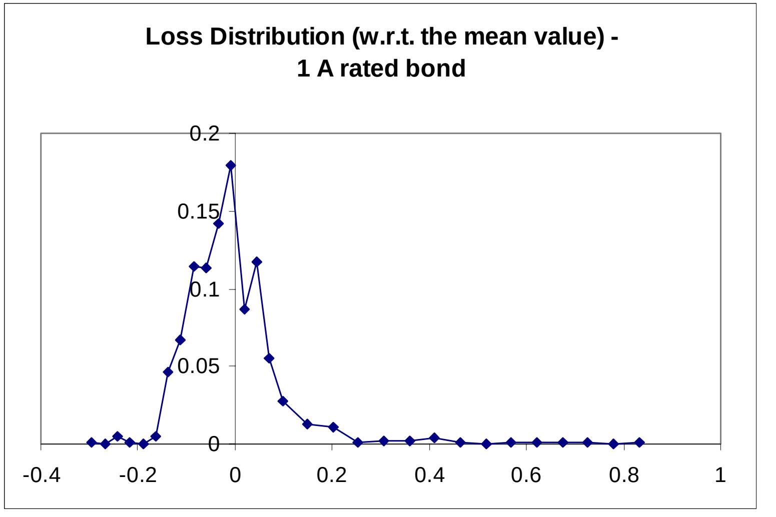
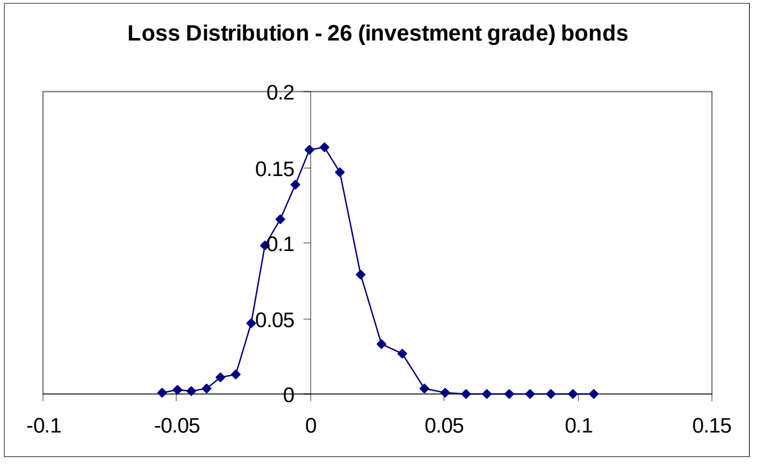
Credit Risk Simulation VI

Results:

a) Loss distribution
Single bond (A rated)



b) Loss distribution
Investment grade
portfolio



Non–normal distributions

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Credit Risk Simulation VII

Risk Measurement (in presence of credit risk):

VaR: *lowest possible value such that the probability of losses less then VaR exceeds α%.*
- industrial standard (Jorion 1996)

CVaR: *expected value of the losses, conditioned on the losses being in excess of VaR .*
- favourable measure in the insurance industry (e.g. Embrechts et al. 2000)
- satisfy axioms of “coherenence”

Further risk measures in the presence of asymmetric return distributions:

- semi-variance (e.g. Markowitz 1991)
- lower partial variance
- lower partial moments (in e.g. Zagst (2001) credit risk context)

Credit Risk Simulation VIII

Importance of Risk Factors (200 bond portfolios):

- interest rate and spread risk cannot be ignored (as frequently suggested) – especially for high quality debt

VAR statistics		Asset Correlation		0.1	Asset Correlation		0.2	Asset Correlation		0.3
RATING	**VaR-Level**	**CM**	**CM + S**	**CM + S + I**	**CM**	**CM + S**	**CM + S +**	**CM**	**CM + S**	**CM + S + I**
AAA	**95**	0.000602	0.011436	0.025329104	0.000929054	0.011396	0.025451	0.001238	0.011364	0.025459419
	99	0.001249	0.012336	0.028386582	0.00212476	0.012664	0.028237	0.002881	0.01297	0.028161252
AA	**95**	0.002172	0.015403	0.019687956	0.002879703	0.015907	0.020112	0.003538	0.016501	0.021227393
	99	0.005792	0.02616	0.0357081	0.008888288	0.026225	0.035813	0.012639	0.026651	0.036296723
A	**95**	0.002737	0.035447	0.047238189	0.00395244	0.035738	0.047401	0.005193	0.036083	0.047786642
	99	0.006365	0.039019	0.064644969	0.010827233	0.040184	0.065015	0.014882	0.042578	0.065260409
BBB	**95**	0.00726	0.110828	0.123547248	0.010362826	0.111221	0.123032	0.012825	0.111498	0.121680668
	99	0.015041	0.146509	0.140964939	0.025852533	0.147853	0.142105	0.036249	0.149461	0.143570029
BB	**95**	0.049338	0.065658	0.068586352	0.032518398	0.035707	0.037549	0.092805	0.100984	0.102828463
	99	0.084871	0.099165	0.100996689	0.073793677	0.074702	0.074973	0.185305	0.190039	0.190300531
B	**95**	0.049338	0.065658	0.068586352	0.071922717	0.083216	0.085444	0.092805	0.100984	0.102828463
	99	0.084871	0.099165	0.100996689	0.142173294	0.145926	0.147108	0.185305	0.190039	0.190300531

Asset Correlation is driving migration and default events (CreditMetrics latent variable approach)

CM	CreditMetrics
CM + S	CreditMetrics extended to incorporate stochastic spreads
CM + S + I	CreditMetrics extended to incorporate stochastic spreads and stochastic interest rates

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Credit Risk Simulation VIIII

Diversification of Credit Risk – VaR stats:

- Simulation model: CM+S+I

Rating	NrBonds	100	50	10	5
	95	0.025353	0.025274	0.025189	0.02519
AAA	***99***	0.028242	0.0282	0.028188	0.028766
	95	0.020361	0.020898	0.023384	0.026518
AA	***99***	0.036083	0.036214	0.036973	0.03943
	95	0.047532	0.047683	0.047775	0.048874
A	***99***	0.065066	0.065133	0.066196	0.067699
	95	0.121429	0.121311	0.120077	0.119883
BBB	***99***	0.141577	0.14217	0.147012	0.158776
	95	0.089556	0.095316	0.127541	0.179379
B	***99***	0.142067	0.145391	0.213879	0.322229
	95	0.112334	0.128086	0.201343	0.294721
C	***99***	0.154215	0.175796	0.302355	0.453071

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Credit Risk Optimisation

Alternative approaches:

- based on MV-analysis (variance of portfolio losses)

  (not advisable as reducing the variance does not prevent from low probability extreme events)

- Lower Partial Moments (e.g. Zagst 2001)
  - measuring the risk of falling below a benchmark
  - suitable for asymmetric distributions
  - leads to mixed-integer programs

- CVaR (e.g Rockafellar and Uryasev 2000, Andersson et al. 2001)
  - linear programming formulation
  - addresses tail events

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Credit Risk Optimisation II

Empirical studies:

- not many
- hardly real world examples
- nothing on stochastic programming (with recourse)

Areas of Applications:

- Enterprise wide risk management
- Minimising the risk of a given position (e.g. loan portfolio)
- Active management of corporate bond portfolios
- Index tracking in corporate bond markets
- ALM in the presence of credit risk

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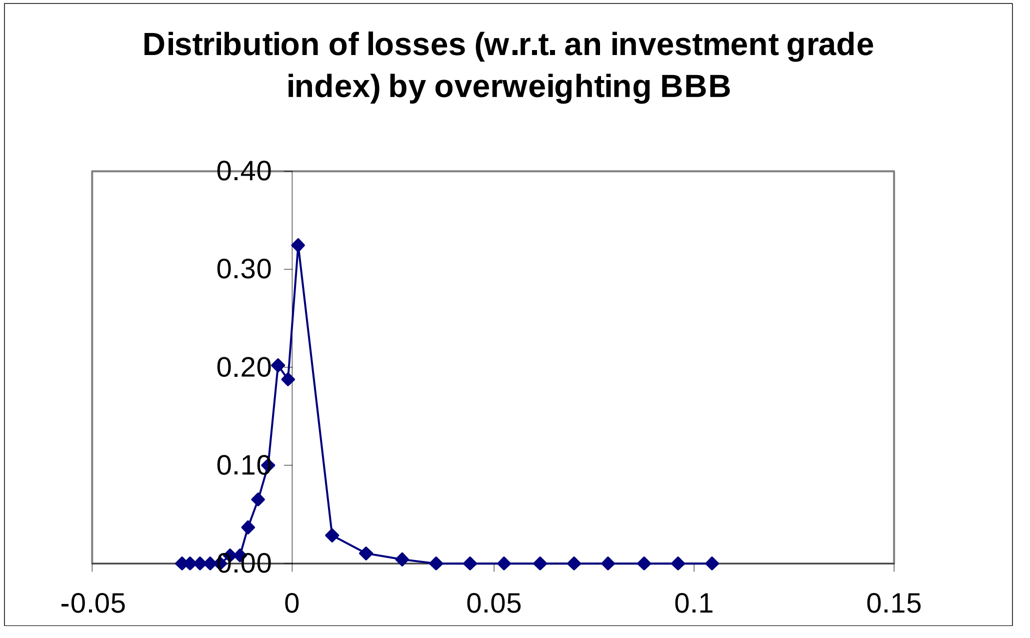
Credit Risk Optimisation III

A stochastic programming index tracking model:

Index tracking: risk usually measured by tracking error (std dev. of excess return)

Credit Risk: Loss distribution w.r.t the index is non-normal

Loss for a given simulation scenario is defined by the index value minus the portfolio value



Credit Risk Optimisation IV

A stochastic programming index tracking model (cont.)

VaR and **CVaR** are usually defined in terms of
Losses w.r.t. either the expected value or the intitial value

Index tracking in the presence of credit risk:

Losses are defined with respect to the (random) index

Basic model structure:

Maximise expected return
s.t.
Limiting the tail losses (w.r.t. the random index)

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Credit Risk Optimisation V

A stochastic programming index tracking model (cont.)

Formulation 1: Anticipative model (no recourse decisions)

- decisions based on future scenarios
- no corrective actions (rebalancing)

Ps2.pdf

Formulation 2: Recourse model

- decisions based on future scenarios AND
- future (corrective) rebalancing decisions

ps2_model2.pdf

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Numerical results: Backtesting

Index: Merril Lynch Euro Dollar Index

- contains 600 to 1000 bonds
- approximated by 16 asset classes:
  - 4 maturity buckets,
  - rating classes (AAA, AA, A, BBB)
- for bond picking results , see Jobst and Zenios (2001)
- transaction costs are incorporated

- 12000 scenarios sampled
- we allow for a CVaR of 1% at a 95% level, i.e. the average losses below VaR w.r.t. the index are at most 1%

- consider model over 3 timesteps (0, 3, 6 month)

Model 1: only one decision at time 0

Model 2: two-stage model, decisions at time 0, 3 month

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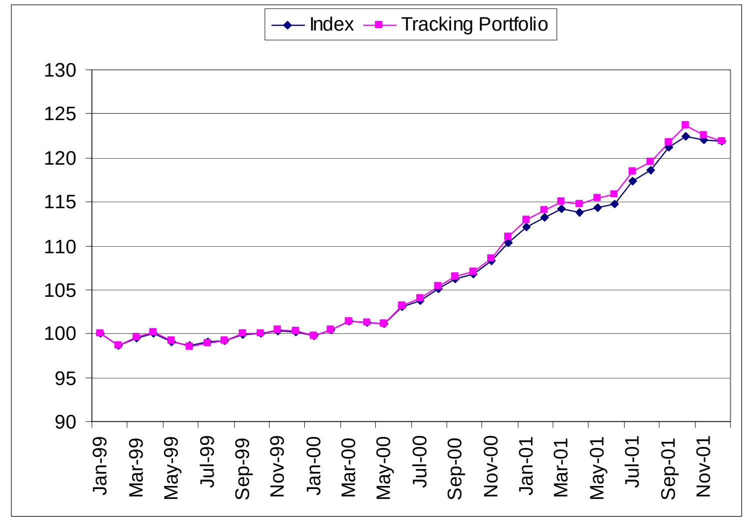
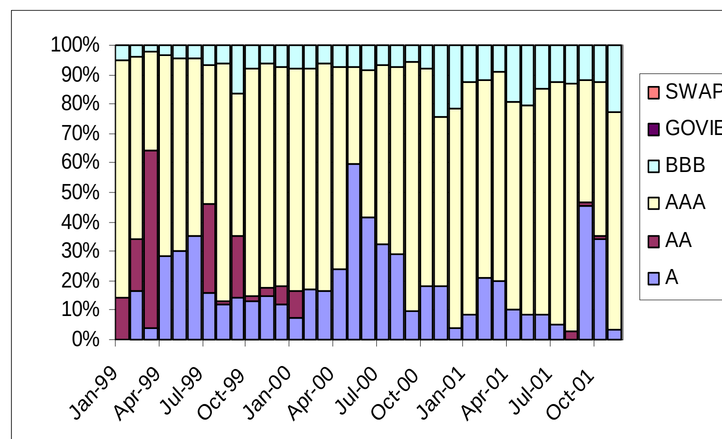
Numerical results: Backtesting

Index: Merril Lynch Euro Dollar Index

Model 1: - good tracking performance
- no outperformance
- holdings in all asset classes (similar to the index)





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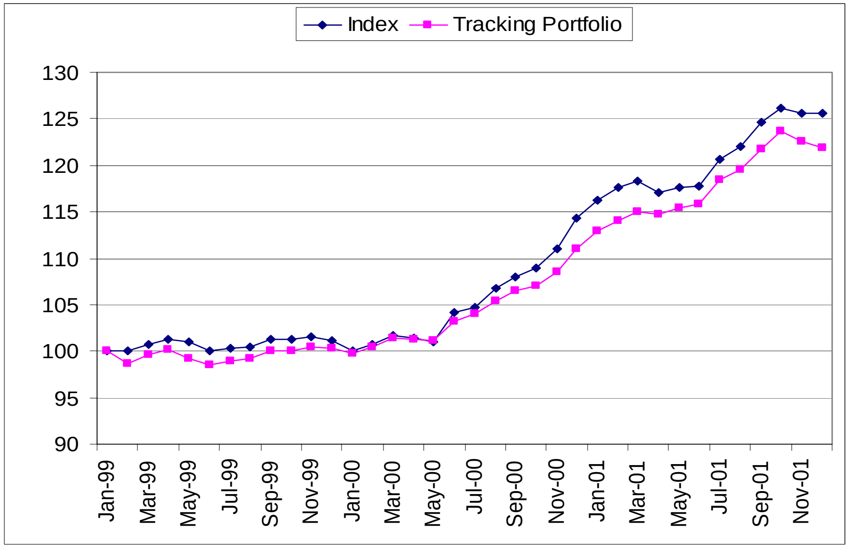
Numerical results: Backtesting

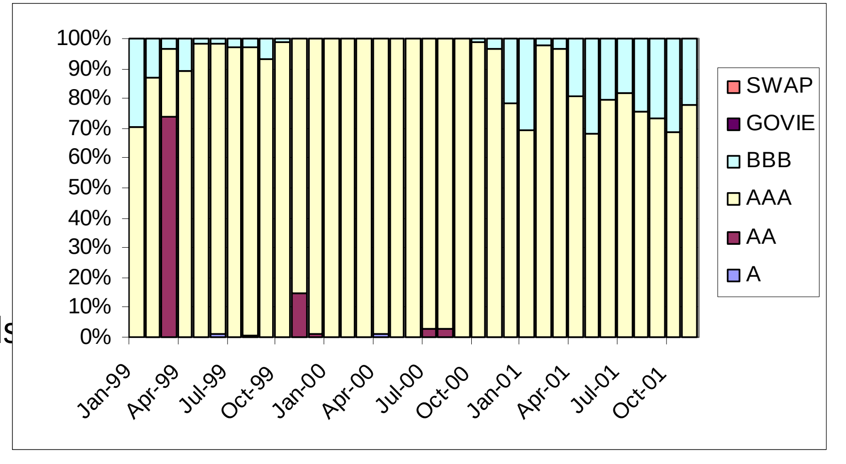
Index: Merril Lynch Euro Dollar Index



Model 2: - better performance

Especially interesting:

Over the period of spread widening (9/99 to 10/00) the model reduces holdings in BBB rated bonds



Period of spread tightening afterwards: Model increases holdings in BBBs

Overall: The model does what a portfolio Manager should have done, knowing the the model can correct initial decisions leads to portfolios that differ more from the index

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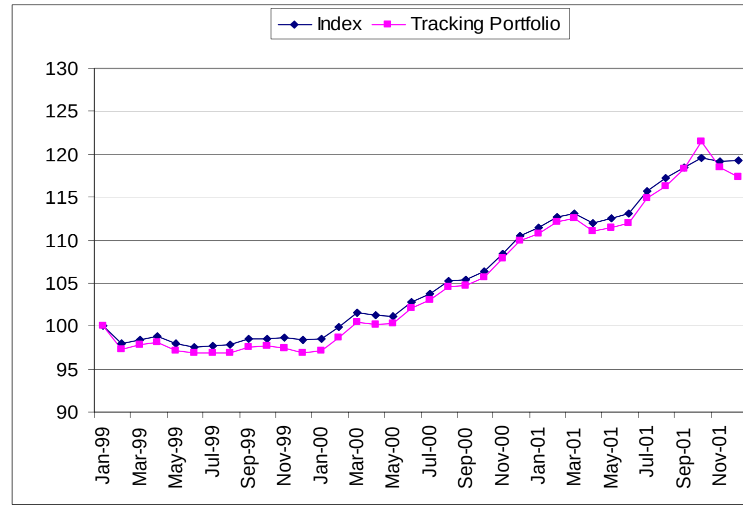
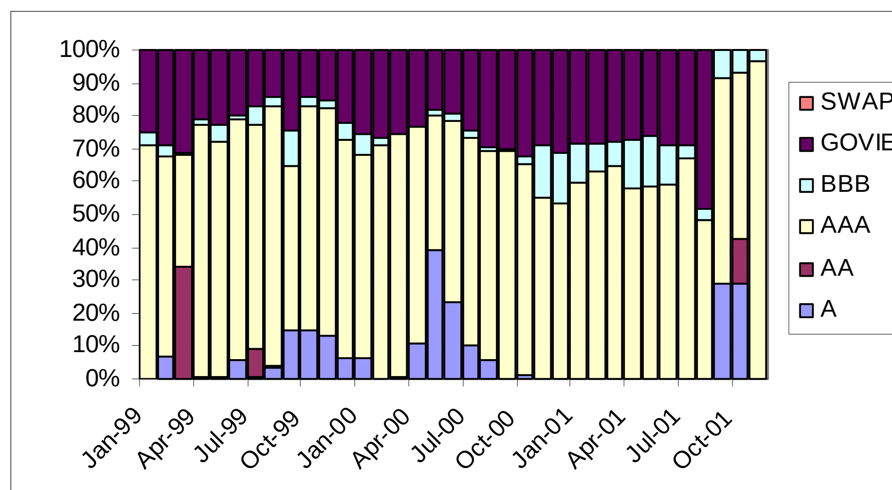
Numerical results: Backtesting

Index: US Government bond index

Investment universe extended by Eurodollar Bonds (corporate bonds)

Model 1: - good tracking performance
- on average, 70 – 80% holdings in high quality corporate bonds
- consistent with market practice
- the model switches completely to corp. at the end of the backtesting period (when corporate bonds fell considerably)





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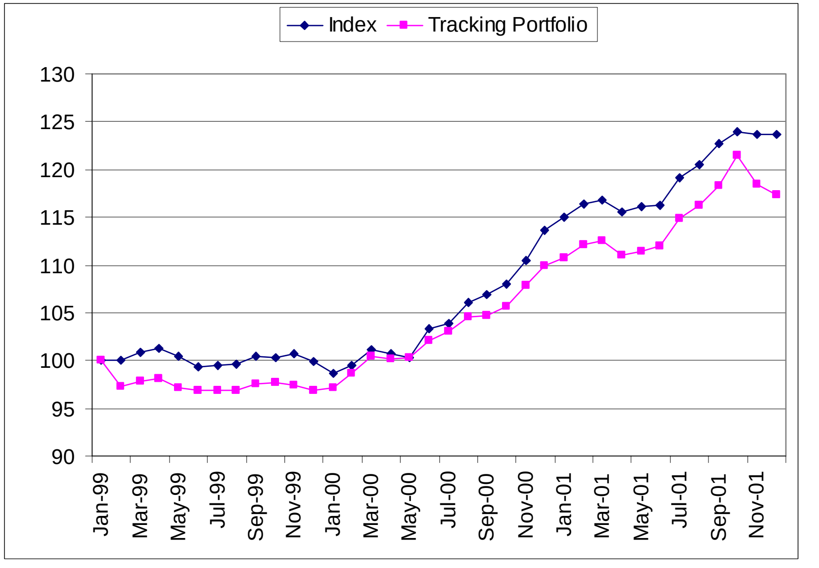
Numerical results: Backtesting

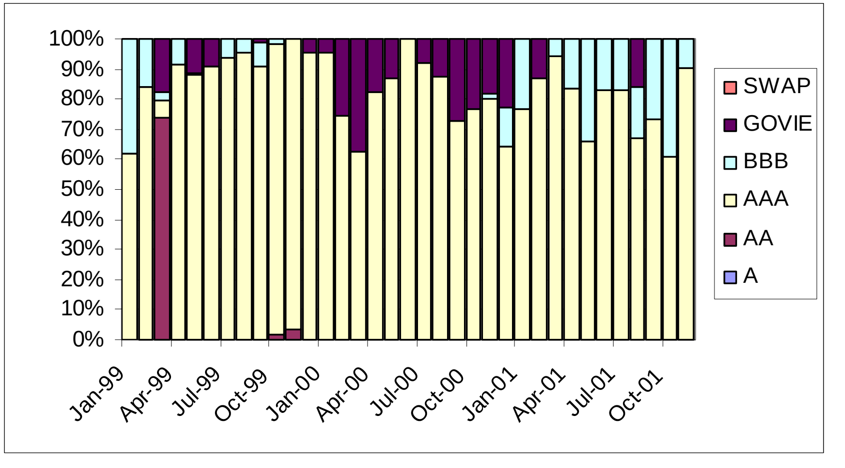
Index: US Government bond index

Model 2:

- flexibility seems to pay off again

- Especially interesting:
  Spread widening period, the model invests in Government bonds, that’s how it should be





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Future research

- include swap products and credit derivatives
    - take a pure view on credit
    - asset swaps
    - credit spread options
    - ALM: default protection through derivatives

- stochastic programming to develop dynamic hedging strategies for OTC-credit derivatives

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Thank you for your attention

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