An Investigation comparing Mixed Integer Programming and Constraint Programming on a Generic Supply Chain Model

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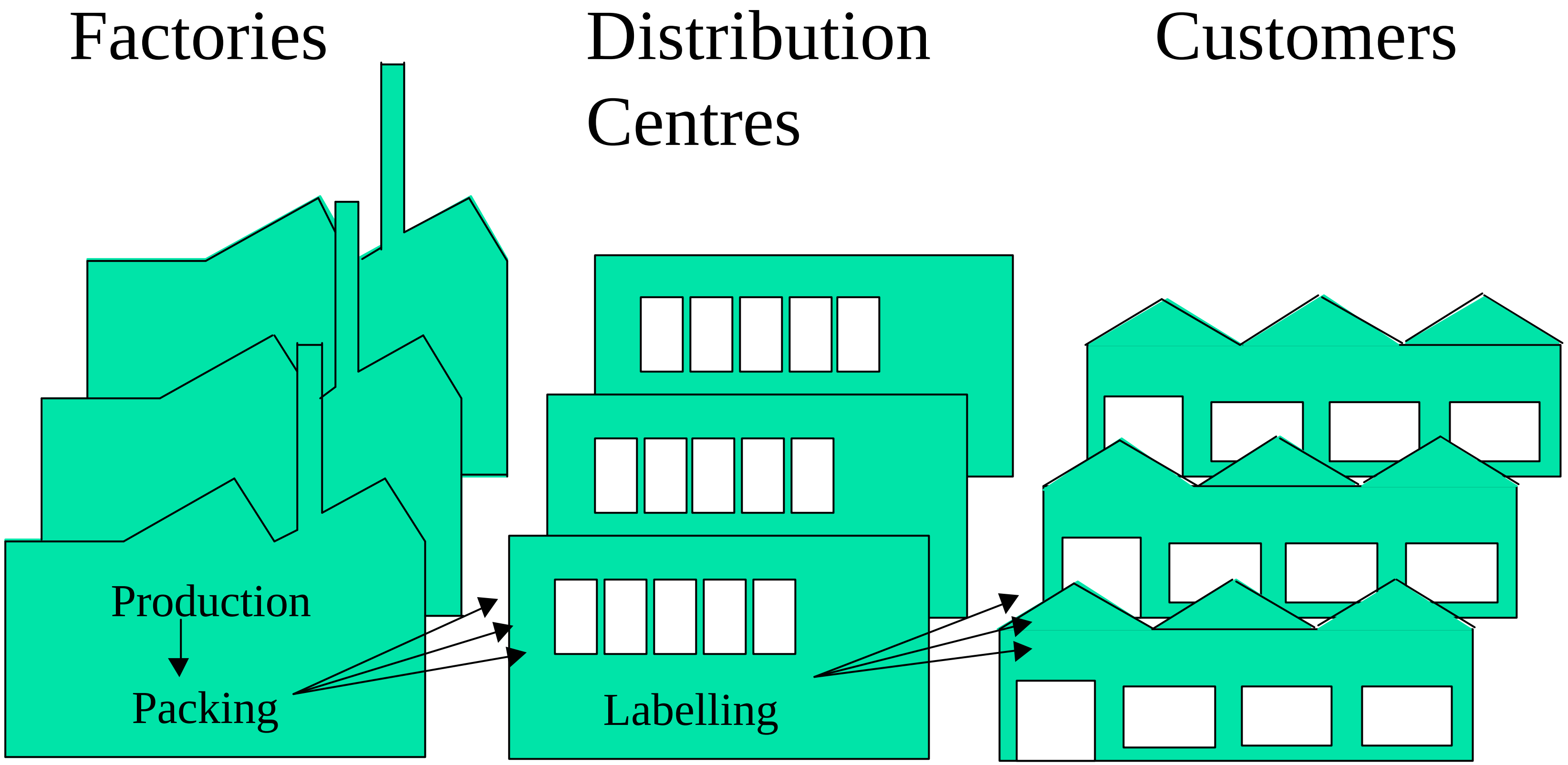
Outline

- The Supply Chain Problem
- The Generic Model
- MIP and CP Implementations
- Results

The Supply Chain Problem



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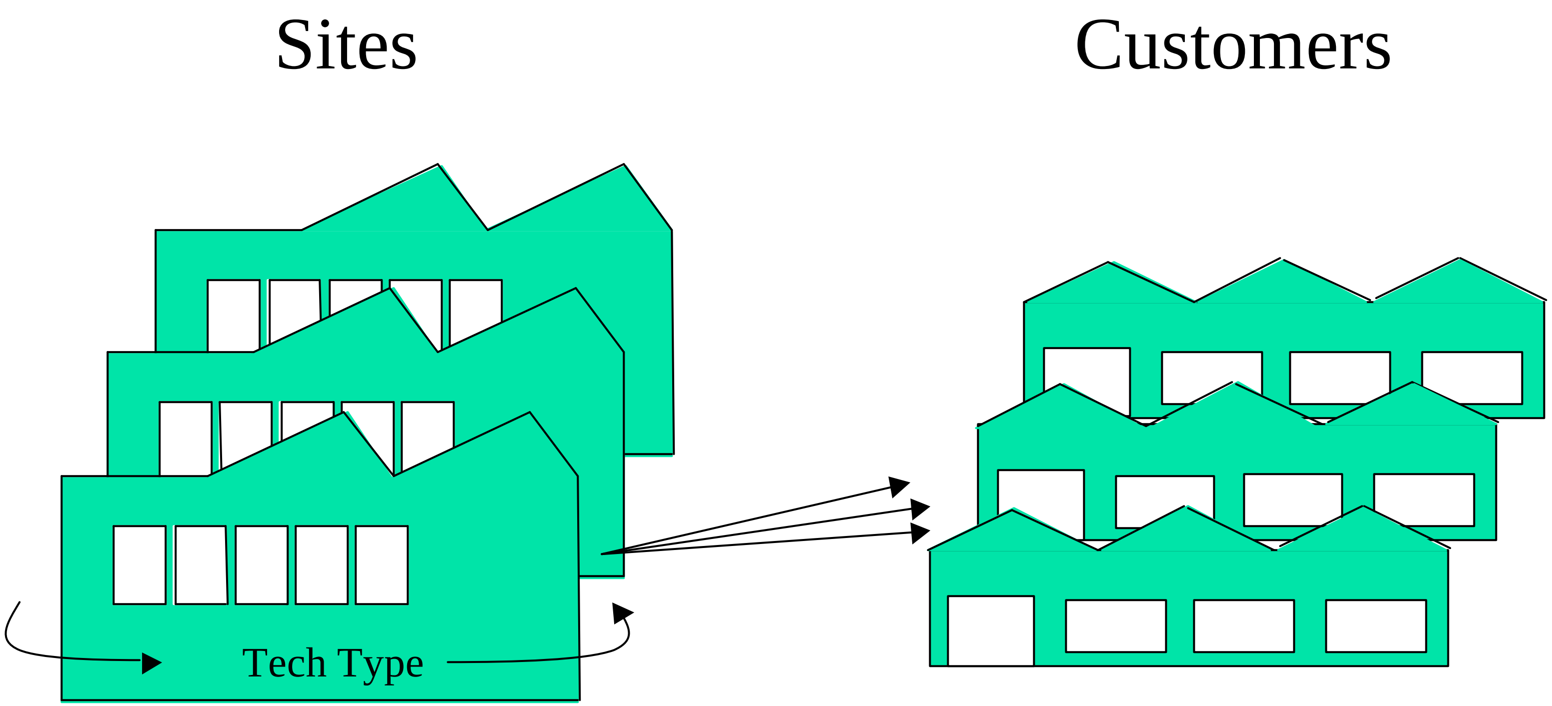
The Generic Model



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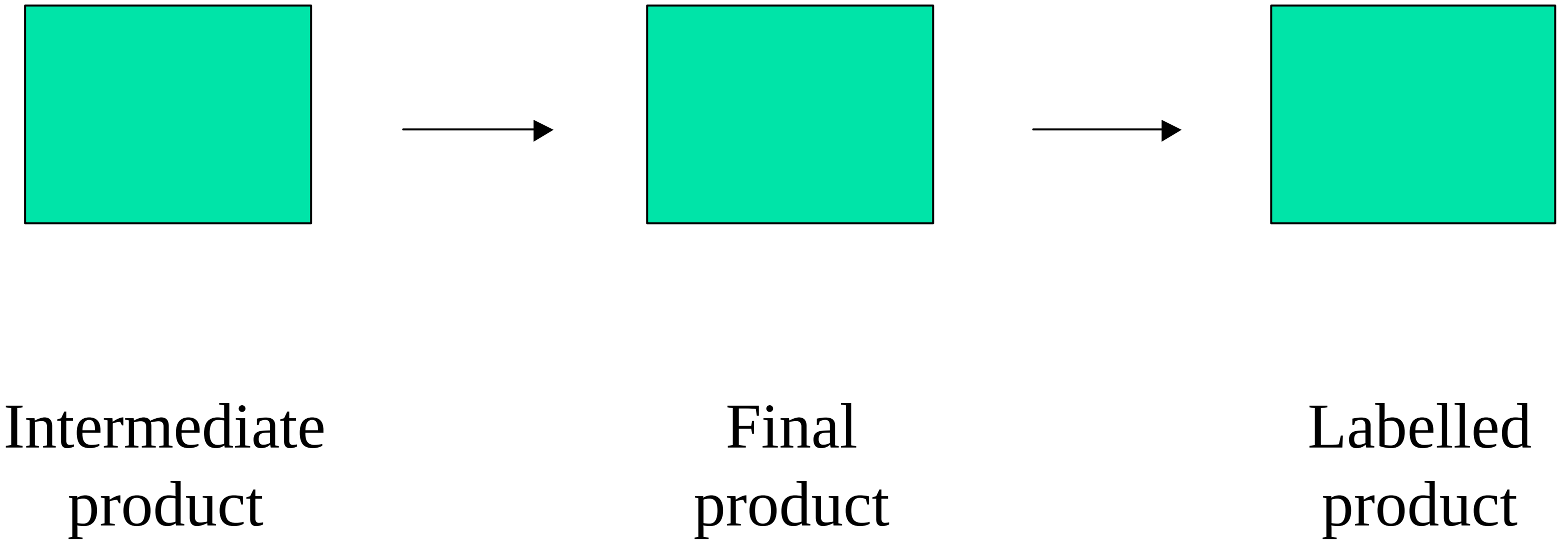
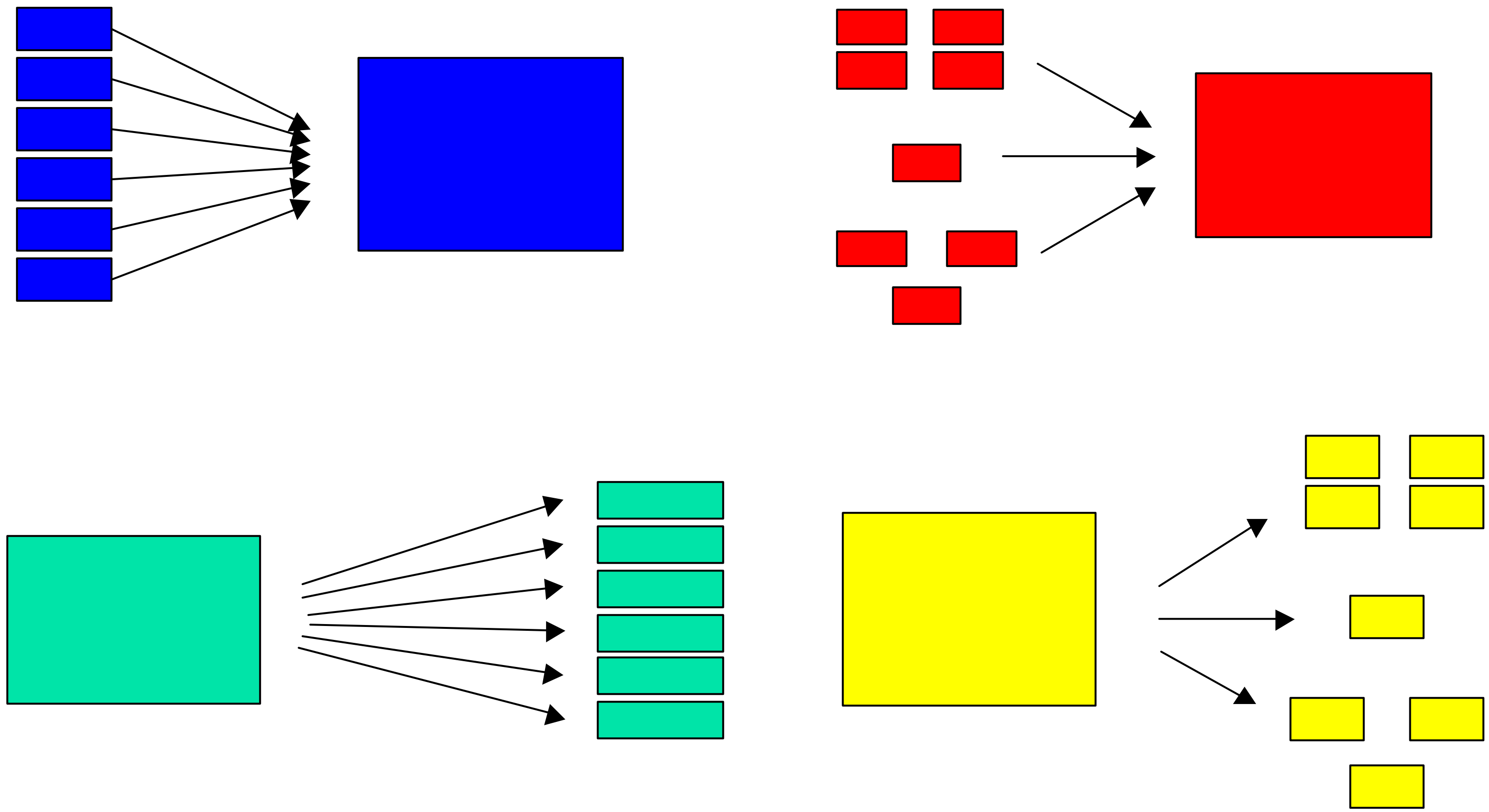
Production Flow



Production Flow Flexibility



Modelling the Production Flow

Data inputs required

- Predecessor Requirement
- Line Throughput
- Transport Cost Internal
- Transport Cost Customer

The Model Implementations

The two implementations

- Mixed Integer Program (MIP)
- Constraint Program (CP)

Both implemented using

- ILOG OPL Studio
- CPLEX
- ILOG Solver and CPLEX

The Continuous (Tactical) Variables

Production Quantity $_{\text{tp, site, line type, prod}}$

Consumed Quantity $_{\text{tp, site, prod}}$

Transport Internal Quantity $_{\text{tp, site1, site2, prod}}$

Transport Customer Quantity $_{\text{tp, site, cust, prod}}$

Shortage Quantity $_{\text{tp, site, cust, prod}}$

The Discrete (Strategic) Variables

MIP

sets of binary variables

- OpenSiteOpen
- PotentialSiteOpen
- ExistingLineOpen
- NewLineOpen

$\forall sites, time\ periods$

sets of subsidiary variables

- OpenSiteNewlyClosed
- PotentialSiteNewlyOpen
- ExistingLineNewlyClosed
- NewLineNewlyOpen

$\forall sites, time\ periods$

CP

sets of discrete variables

- OpenSiteCloses
- PotentialSiteOpens
- ExistingLineCloses
- NewLineOpens

$\forall\ sites$

sets of subsidiary binary variables

- OpenSiteDisused
- PotentialSiteUsed
- ExistingLineDisused
- NewLineNewlyUsed

$\forall\ sites$

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The CP Discrete Variables

- OpenSiteCloses = 1 .. NoTp + 1
- PotentialSiteOpen = 1 .. NoTp + 1
- ExistingLineCloses = 1 .. NoTp + 1
- NewLineOpens = 1 .. NoTp + 1

Where 1 .. NoTp indicate the time period where the site or line opens or closes and NoTp + 1 indicates that the site or line does not open or close.

Potential Site Constraints

In the MIP

$$SiteOpen_{tp,\, site} \geq SiteNewlyOpened_{tp'\, site} \qquad tp \geq tp' \qquad \forall site$$

$$SiteOpen_{tp,\, site} \leq \sum_{tp',site} SiteNewlyOpened \qquad tp \leq tp' \qquad \forall site$$

$$\sum_{tp} siteNewlyOpened_{tp,\, site} \leq 1 \qquad \forall site$$

In the CP

$$(NoTp \geq siteOpens_{site}) \Leftrightarrow (siteUsed_{site} = 1)$$

$$\forall potentialSites$$

Maximum Site Capacity Constraints

- In the MIP

$$\sum \text{existingLineOpen} + \sum \text{newLineOpen}$$
$$\leq \text{maxCapacity} \times \text{siteOpen} \qquad \forall tp, \forall sites$$

- In the CP

$$\sum (\text{existingLineCloses} > t) + \sum (\text{newLineOpens} \leq t)$$
$$\leq \text{maximumCapacity} \times (\text{siteOpens} \leq \text{t}) \qquad \forall t, potentialSites$$

$$\sum (\text{existingLineCloses} > t) + \sum (\text{newLineOpens} \leq t)$$
$$\leq \text{maximumCapacity} \times (\text{siteCloses} > \text{t}) \qquad \forall t, openSites$$

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Model Statistics for Reduced Data Set

	MIP	CP
Decision Variables	2260	1612
Constraints	1533	1283

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Solutions

The MIP model

- Optimum solution of 200939 in just over 30 seconds.

The CP model

- No solutions
  - hours of run time
  - variety of search procedures (eg Depth First Search, Limited Discrepancy Search, Best First Search etc)
- Validated
  - using solutions from MIP

Search Procedures

Shortage Quantity $_{\text{tp, site, cust, prod}}$ — try all {0, 1}
Open Site Disused $_{\text{site}}$
Potential Site Opens $_{\text{site}}$
Existing Line Closes $_{\text{site}}$
New Line Opens $_{\text{site}}$

$\forall$ tp, sites, cust, prod
or $\forall$ sites,

Open Site Closes $_{\text{site}}$
Potential Site Opens $_{\text{site}}$
Existing Line Closes $_{\text{site}}$
New Line Opens $_{\text{site}}$

try all {1,2,3,4,5}
{5,4,3,2,1}
{5,1,2,3,4}

$\forall$ tp, sites, cust, prod

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Search Results 1

	Try	First solution	Time first solution	Subsequent solutions
shortage	{0,1}	117430 (58%)	34s	None in an hour
open sites potential sites existing lines new lines	{5,4,3,2,1} {1,2,3,4,5} {5,4,3,2,1} {1,2,3,4,5}	4105 (2%)	65s	4765 in 370s

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Search Results 2

	Try	First solution	Time first solution	Subsequent solutions
shortage open sites potential sites	{0,1} {1,2,3,4,5} {1,2,3,4,5}	60497 (30%)	58s	None in an hour
shortage open sites potential sites	{0,1} {5,4,3,2,1} {5,4,3,2,1}	114009 (57%)	49s	None in an hour
shortage open sites potential sites	{0,1} {5,1,2,3,4} {5,1,2,3,4}	114009 (57%)	53s	None in an hour
shortage open sites potential sites	{0,1} {5,4,3,2,1} {1,2,3,4,5}	117430 (58%)	72s	None in an hour

Search Results 3

	Try	First solution	Time first solution	Subsequent solutions
shortage open sites disused potential sites used	{0, 1} {0, 1} {0, 1}	114009 (57%)	36s	None in an hour
shortage existing lines disused new lines used	{0, 1} {0, 1} {0, 1}	182486 (91%)	25s	Improving very slowly
shortage open sites disused potential sites used existing lines disused new lines used	{0, 1} {0, 1} {0, 1} {0, 1} {0, 1}	184647 (92%)	23s	Improving very slowly

Search Results 4

	Try	First solution	Time first solution	Subsequent solutions
shortage potential sites open sites new lines existing lines	{0,1} {5,4,3,2,1} {1,2,3,4,5} {5,4,3,2,1} {1,2,3,4,5}	200939 (100%)	20s	

200939 is 98% of LP solution 204138

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Conclusions

Constraint Programming

- Can find good solutions in comparable times.
- Allows control of type of solution found.
- Allows comparison of management strategies
- LP solution of MIP as measure of solution quality

Further work

Larger data sets

- Scalability

Different problems

- Flexibility of model
- Applicability of meaningful search

Thank you!