

Value at Risk analysis of a multistage asset/liability management model using the Stochastic Programming Integrated Environment SPInE.

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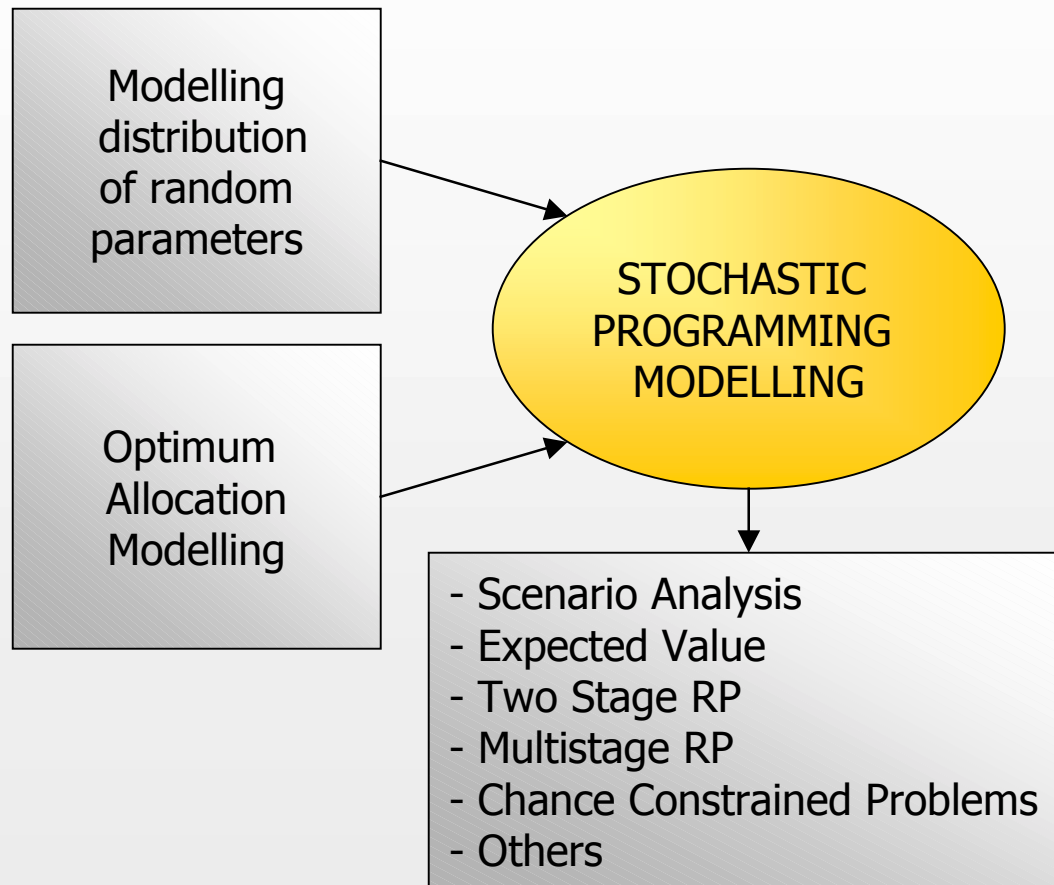


Outline

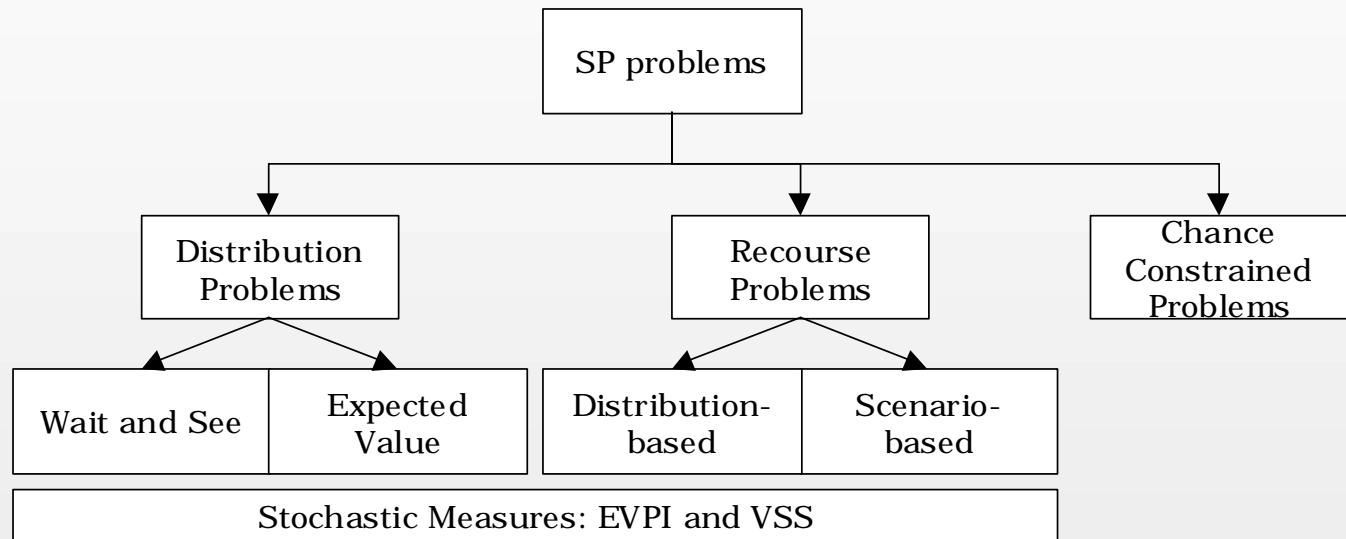
- Background
 - Modelling SP problems
 - Difficulties
 - Existing software for SP
- Asset/liability management model
- The SPInE system
- VaR analysis in SPInE
- Solver performances



Modelling SP



SP models classification



Difficulties

- AMLs for LP are inadequate for SP
- Integration with scenario generators
- Close coupling solvers-modelling systems
- Compact representation formats (SMPS)
- Analysis of results requires connection to DBMS



Software tools for SP

Name	Affiliation	System Name
JJ Bisshop, et al.	Paragon Decision Technology	AIMMS
A Meeraus, et al.	GAMS	GAMS
R Fourer, et al.	Northwestern University	AMPL
MAH Dempster, et al.	Cambridge University	STOCHGEN
E Fragniere, et al.	University of Geneva	SETSTOCH
A King, et al.	IBM	OSL/SE
HI Gassmann, et al.	Dalhousie University	MSLiP
G Infanger et. al.	Stanford University	DECIS
P Kall, et al.	University of Zürich	SLP-IOR
G Mitra, et al.	Brunel University	SPIInE



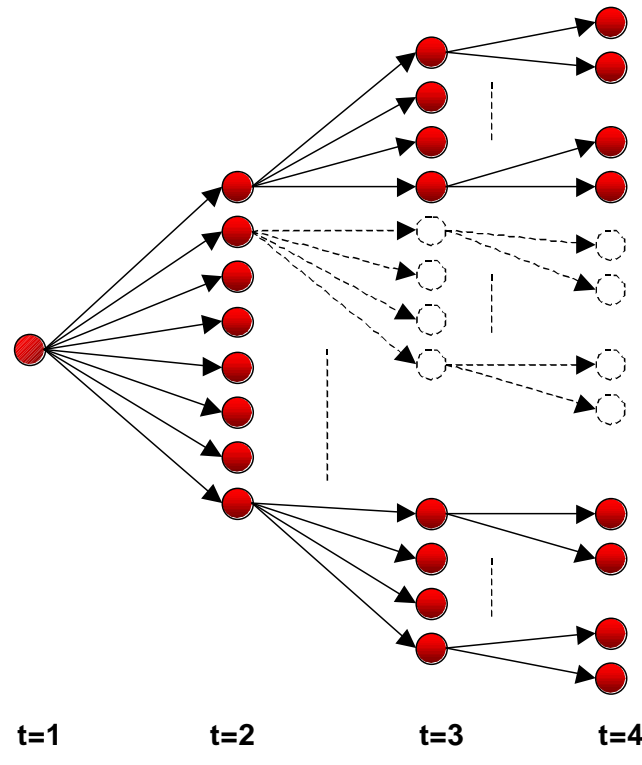
Asset/Liability Management Model

- Multistage SP with recourse (4 stages)
- Uncertain future asset prices
- 23 stocks
- 64 scenarios
- Downside Risk constraints
- Objective: maximise final portfolio wealth

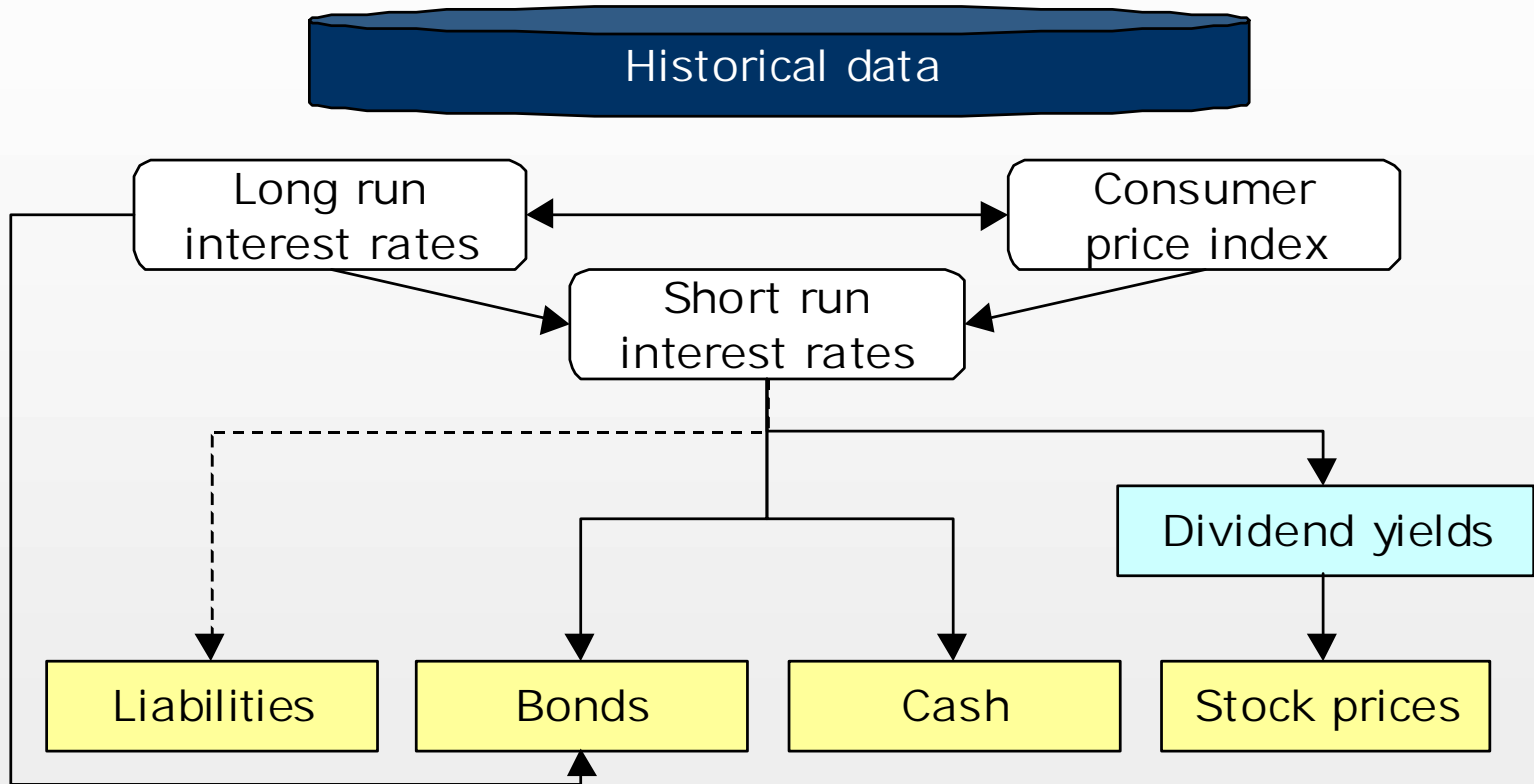


Event tree

- Historical data 1978 – 1996
- 1 year horizon divided in 4 quarters



Scenario Generation



ALM Model Formulation (1)

Sets and indices

$t = 1, \dots, T$ temporal horizon

$i = 1, \dots, I$ assets

$s = 1, \dots, Sc$ scenario index

Decision variables

H_{its} amount of security i held during $[t, t + 1)$ under scenario s

S_{its} amount of security i sold at the beginning of period t under scenario s

B_{its} amount of security i bought at the beginning of period t under scenario s

Parameters

$L_t \geq 0$ liability at time t

$price_{its} \geq 0$ price of asset i at time t under scenario s

$A_t \geq 0$ target for time t

$R \geq 0$ maximum deviation from target (in fraction)

$F_t \geq 0$ funding at time period t

$H0_i \geq 0$ initial holding for asset i

$g \geq 0$ transaction cost rate for buying



ALM Model Formulation (2)

Funds balance constraints

$$(1-g) \sum_{i=1}^I price_{its} S_{its} - L_t + F_t = (1+g) \sum_{i=1}^I price_{its} B_{its} \quad t = 1, \dots, T, s = 1, \dots, S$$

Stocks balance constraints

$$H_{its} = H_{it-1s} + B_{its} - S_{its} \quad i = 1, \dots, I, t = 1, s = 1, \dots, S$$

$$H_{its} = HO_i + B_{its} - S_{its} \quad i = 1, \dots, I, t = 2, \dots, T, s = 1, \dots, S$$

Downside risk constraints

$$A_t - \sum_{i=1}^I price_{its} H_{its} \leq A_t R \quad t = 2, \dots, T, s = 1, \dots, S$$

Objective function

$$\max \sum_{s=1}^{Sc} p_s \sum_{i=1}^I price_{iT} H_{iT}$$

Formulation in AMPL (1)

```
set I           := 1..23; #asset type
set T           := 1..4;  #time stages
set Sc          := 1..64; #scenarios

param g := 0.025;          # Transactions cost ratio
param R := 0.2;           # Risk level;
param L{T};              # Liabilities;
param H0{I};             # Initial portfolio;
param F{T};              # Funding
param A{T};              # Targets
param P{Sc};             # scenario probabilities
param price{T,I,Sc};     # asset prices

var S{T,I,Sc} >=0;
var H{T,I,Sc} >=0;
var B{T,I,Sc} >=0;

maximize wealth : sum{s in Sc, i in I} P[s]* H[4,i,s]* price[4,i,s];

subject to

assetholding1{i in I, s in Sc}:          H[1,i,s]=H0[i]+B[1,i,s]-S[1,i,s];
assetholding2{i in I, t in 2..4,s in Sc}: H[t,i,s]=H[t-1,i,s]+B[t,i,s]-S[t,i,s];

fundbalance{t in T,s in Sc}:  sum {i in I} B[t,i,s]*price[t,i,s]*(1+g) -
                               sum {i in I} S[t,i,s]*price[t,i,s]*(1-g) = F[t]-L[t];

zeta{ t in 2..4, s in Sc}:  A[t]- sum {i in I} H[t,i,s]*price[t,i,s] <= R*A[t];
```



Formulation in AMPL (2)

```
##### NON ANTICIPATIVITY CONSTRAINTS #####  
  
nah11{i in I,s in 2..64}      :      H[1,i,s]= H[1,i,s-1];  
  
nah21{i in I,s in 2..8}      :      H[2,i,s]= H[2,i,s-1];  
nah22{i in I,s in 10..16}:    H[2,i,s]= H[2,i,s-1];  
...  
nah28{i in I,s in 58..64}:    H[2,i,s]= H[2,i,s-1];  
  
nah31{i in I,s in 2..4}      :      H[3,i,s]= H[3,i,s-1];  
nah32{i in I,s in 6..8}      :      H[3,i,s]= H[3,i,s-1];  
...  
nah316{i in I,s in 62..64}:   H[3,i,s]= H[3,i,s-1];  
  
#### same for variables S and B ####
```

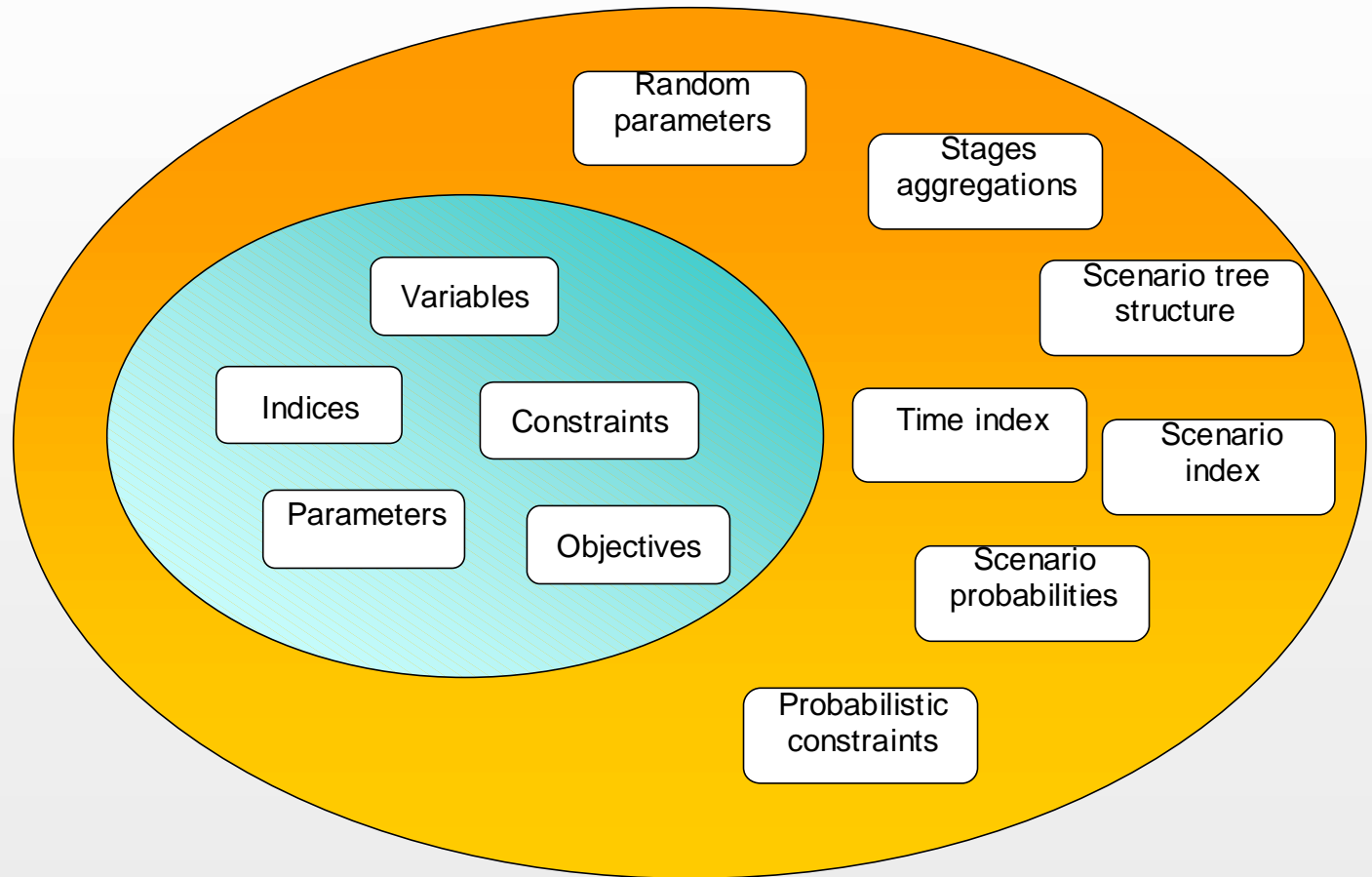


Extended Syntax for AMLs

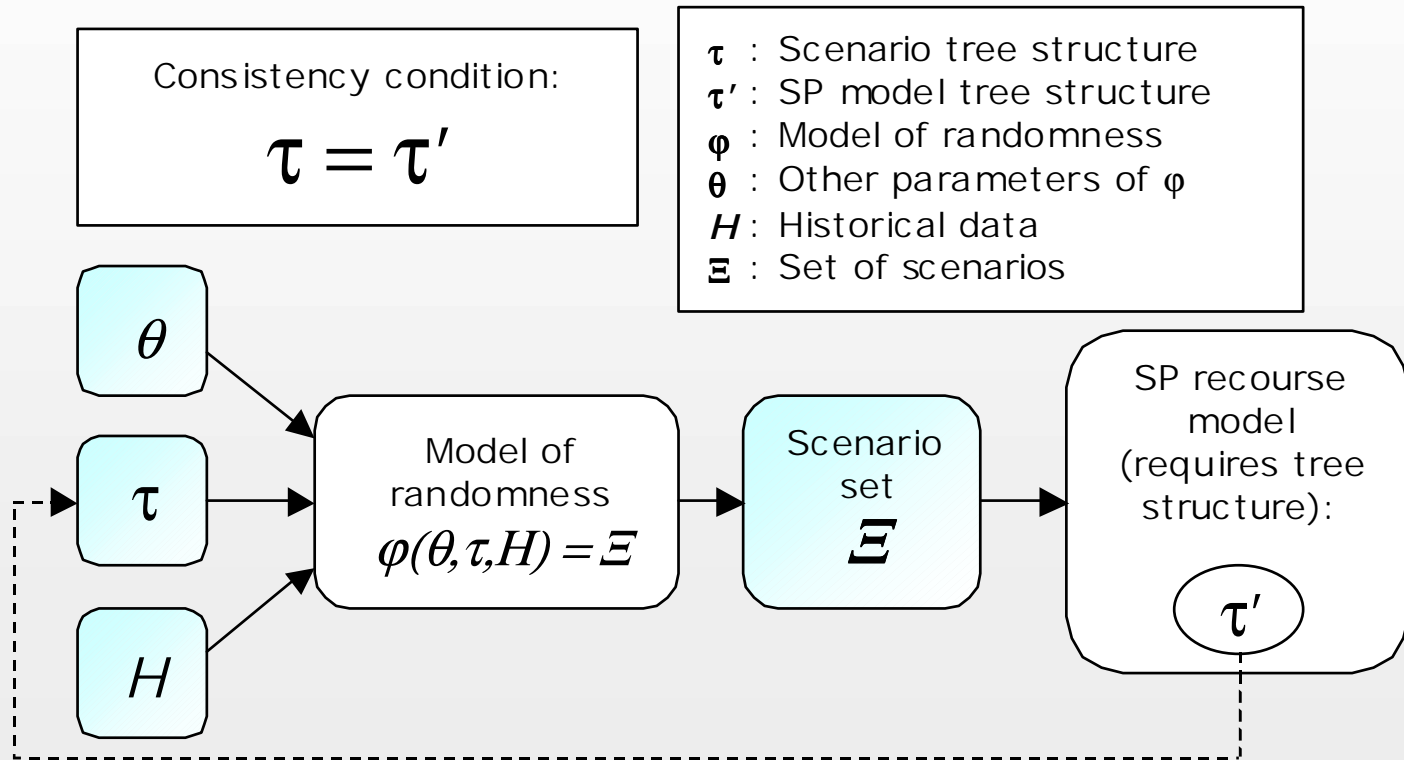
- Consider SP models as refinement of deterministic problems by introduction of uncertainty
- SP models identify:
 - An underlying deterministic model (core)
 - Information related to the model of randomness (stochastic framework)



SP Modelling Constructs



Scenario Generation and SP Modelling



Formulation in SAMPL (Core)

```
set T      := 1..4;           #time horizon
set I      := 1..23;         #asset type

param g := 0.025;           # Transactions cost rate
param R := 0.2;             # Risk level;
param L{T};                 # Liabilities;
param H0{I};                # Initial portfolio;
param F{T};                 # Funding
param A{T};                 # Targets
param price{T,I};          # Asset price (scenario dependent!)

var S{T,I} >=0;
var H{T,I} >=0;
var B{T,I} >=0;

maximize wealth : sum{i in I} H[4,i]*price[4,i];

subject to

assetholding1{i in I}:      H[1,i]=H0[i]+B[1,i]-S[1,i];
assetholding2{i in I, t in 2..4}:  H[t,i]=H[t-1,i]+B[t,i]-S[t,i];

fundbalance{t in T}:        sum {i in I} B[t,i]*price[t,i]*(1+g) -
                             sum {i in I} S[t,i]*price[t,i]*(1-g) = F[t]-L[t];

zeta{ t in 2..4}:           A[t]-sum {i in I} H[t,i]*price[t,i] <=R[t]*A[t];
```



Formulation in SAMPL (Stochastic Framework)

```
class SBRP;  
  
timeset T;  
  
scenarioset Sc;  
  
probability all_equal;  
  
random param price;  
  
stages partition one_to_one;  
  
tree multibranch{8,4,2};
```



SPI nE Controls

The image displays the SPI nE software interface. The main window has a menu bar with 'File', 'View', 'Window', 'Options', 'Run', and 'Model'. The 'Options' menu is open, showing 'Generator Settings...', 'Solver Settings...', 'Parse', 'Generate SMPS', 'Solve Current', 'Solve', and 'Export Results'. The 'Run' menu is also open, showing 'View Tree' and 'Edit Controls'. A 'Solver Controls' dialog box is open, showing 'Solution models' (Here and Now, Wait and See, Expected Value), 'Stochastic Measures' (EVPI, VSS), and 'Algorithms control' (Algorithm type: Benders, Lagr. Scale Factor: 0.07, Sec. Stage Breakup: 0.2, Output filter: stage: 1). The 'SMPS generation options' dialog box is also visible, showing 'Model Stochasticity' (RHS, Cost vector, Technology Matrix, Bounds) and 'Generation Controls' (Assume Constant Core, Compact SMPS).



ALM model in SPInE: solution

```

SPInE - [D:\models\SMPL\models\AssetLiability\alm_PAPER.tul]
File View Window Options Run Model

-----
SPInE SOLUTION REPORT
-----
EVPI: 146019.017003
VSS: INFINITE

HERE AND NOW SOLUTION
Objective value: 195006.06553           Algorithm: BENDERS

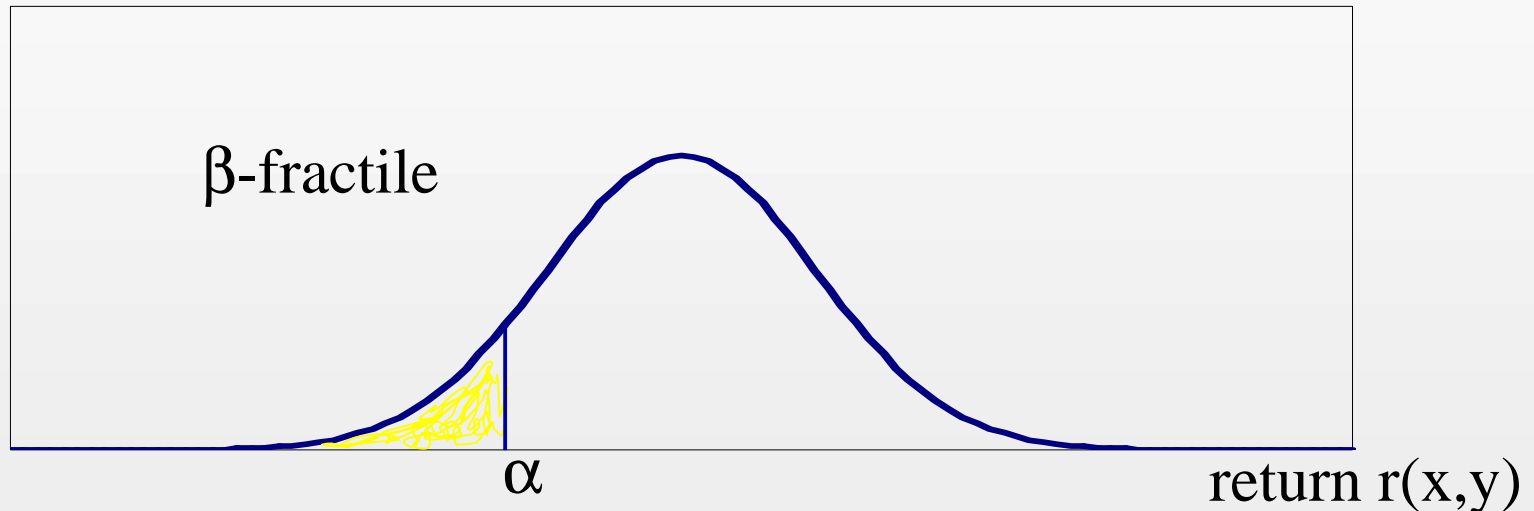
VARIABLES VECTOR: S
-----
t      i      Activity      Reduced Cost      Lower Bound      Upper Bound
VARIABLES VECTOR: H
-----
t      i      Activity      Reduced Cost      Lower Bound      Upper Bound
1      1      2890.243000      0                  0                  1e+036
1      17     385.924135102    0                  0                  1e+036
VARIABLES VECTOR: B
-----
t      i      Activity      Reduced Cost      Lower Bound      Upper Bound
1      1      2890.243000      0                  0                  1e+036
1      17     385.924135102    0                  0                  1e+036
CONSTRAINTS VECTOR: assetholding
-----
Status: 08/08/2001 14:57
    
```

EV	WS	HN	VSS	EVPI
151072.86	341025.08	195006.06	Infinite	146019.02



Value at Risk

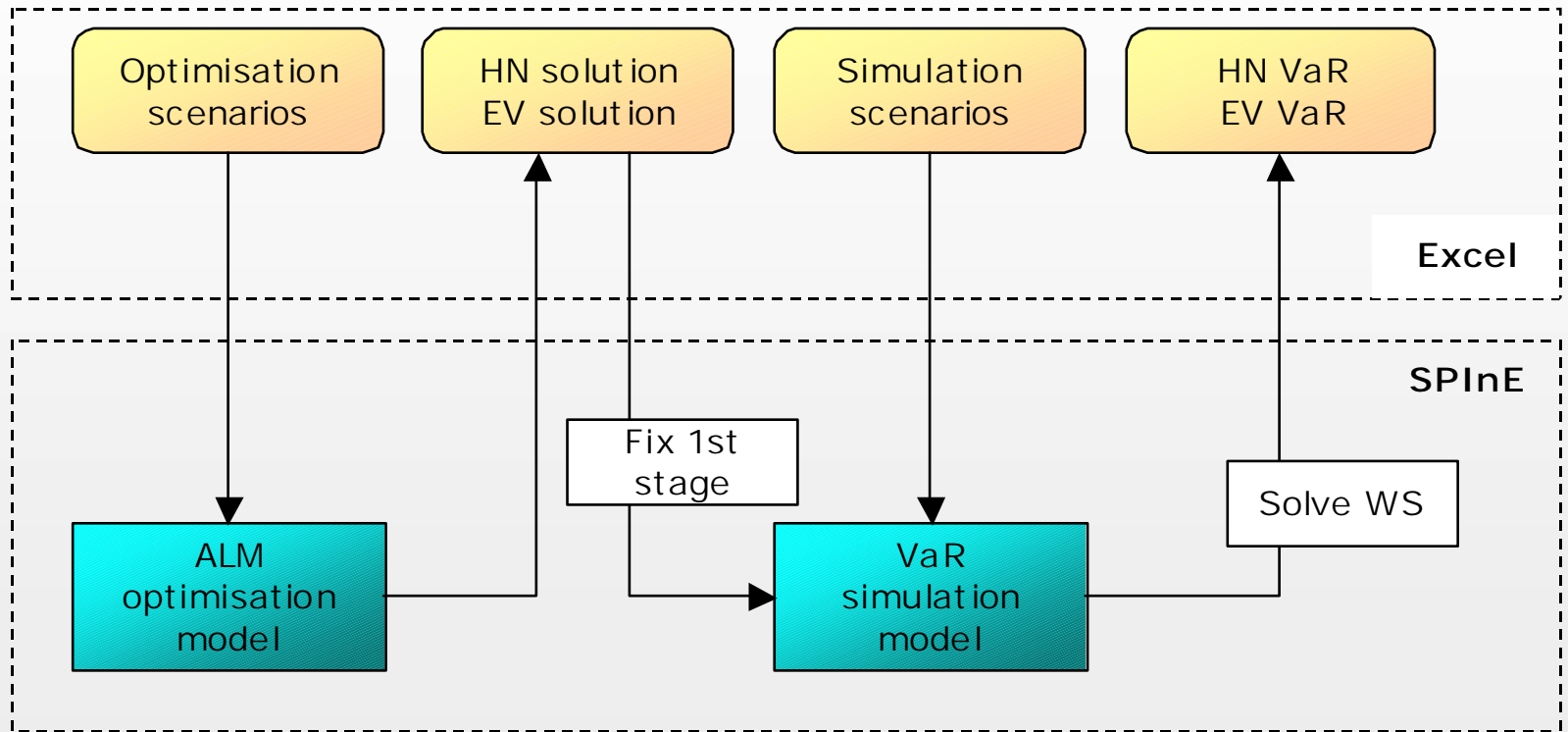
- Finance industry has introduced Value at Risk (VAR) also known as the β -var.



probability function : $\Psi(x, \alpha) = \int_{r(x,y) \leq \alpha} p(x, y) dy$

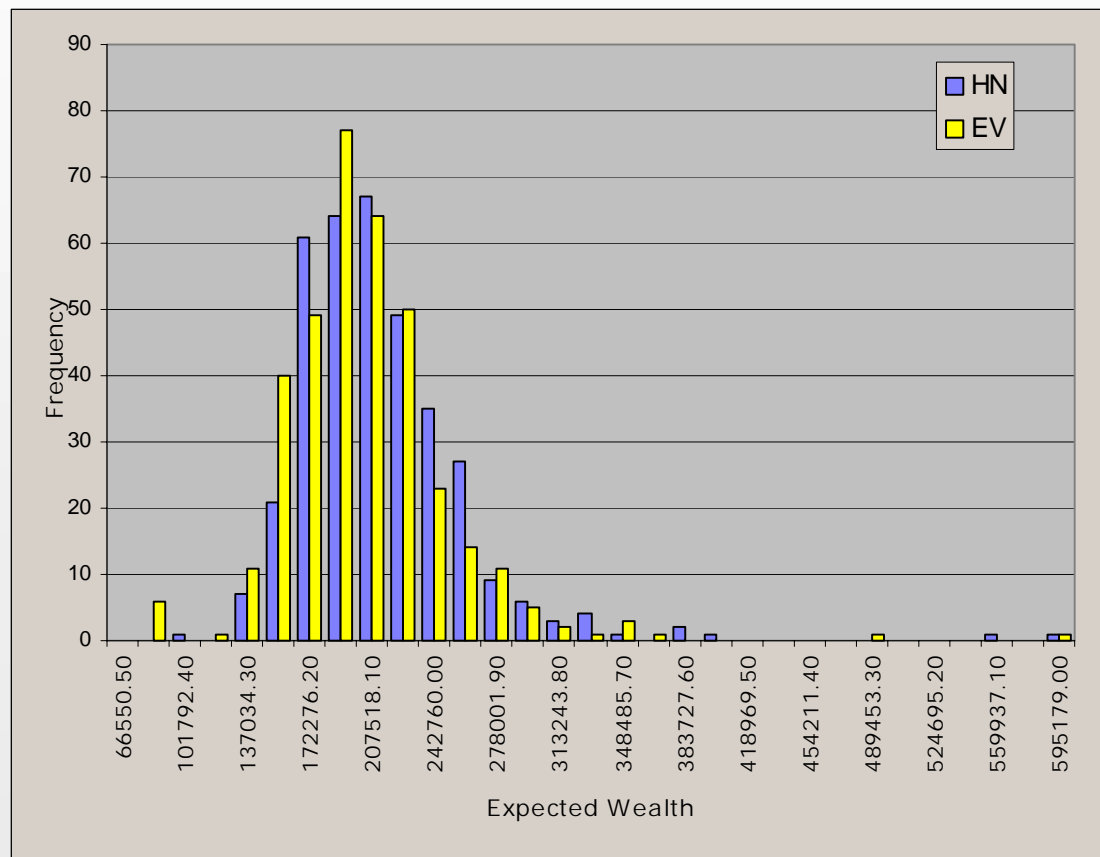
quantile function : $\alpha(x, \beta) = \min\{\alpha : \Psi(x, \alpha) \geq \beta\}$

VaR Computation



VaR Results

Implemented Solution	VaR
HN	131638
EV	82565



SP Solver in SPInE

- DEQ explicit: model processed by IPM
- DEQ implicit: model processed by SSX
- (Nested) Benders decomposition: the master and subproblems processed by SSX,
- Wait-and-See(WS): The individual problems processed by SSX.
- Expected Value(EV): The expected value LP problem processed by SSX.

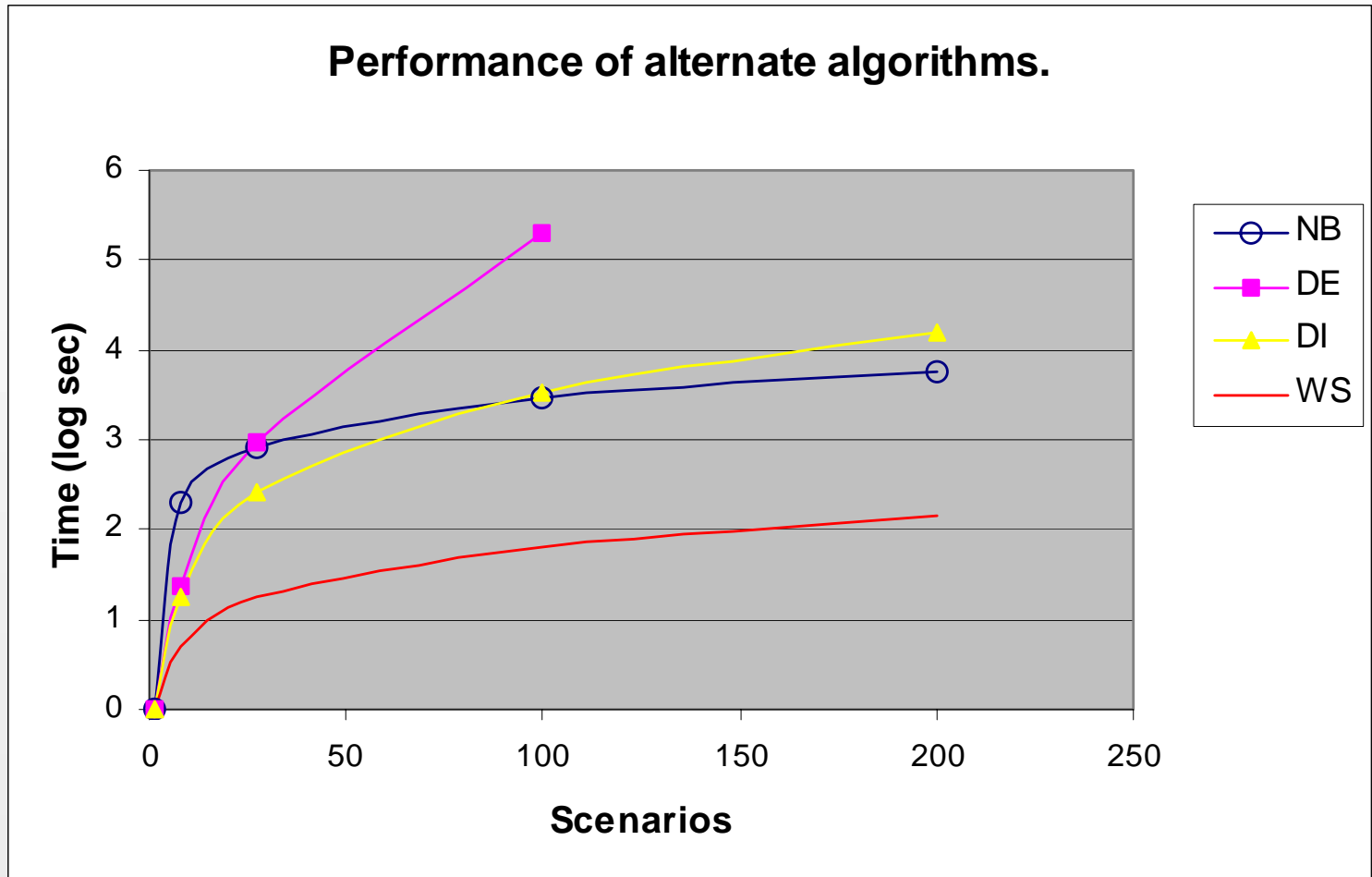


SP Solver Benchmarks (1)

- STORM model (Mulvey and Ruszczyński 1995)
 - 2 stage freight scheduling SP model
 - Up to 1000 scenarios
 - Core: 713 rows, 1380 columns 4037 non-zeroes
 - Solved on a Pentium III 500 MHz, 128 Mbyte RAM
 - FortMP as LP solver



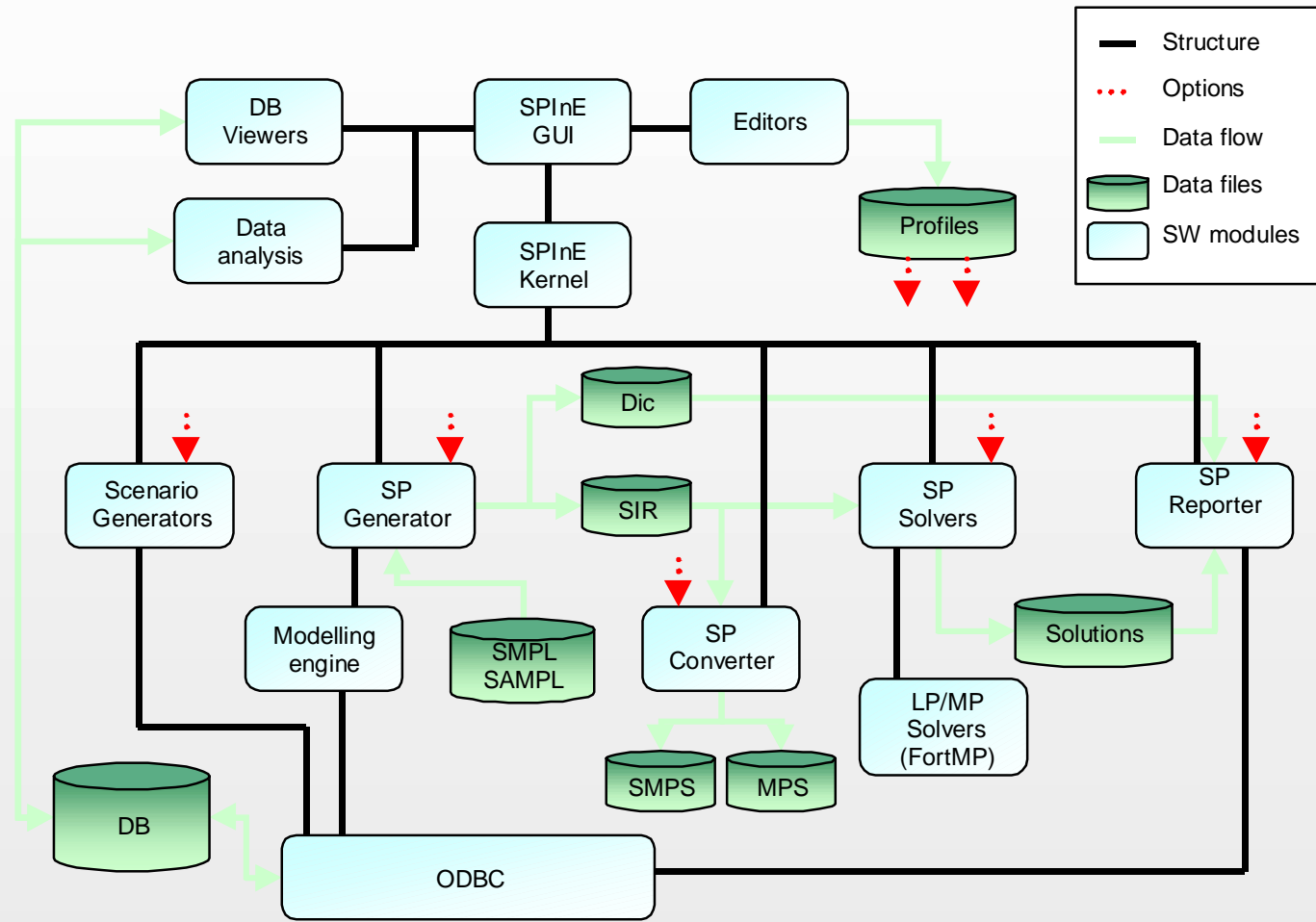
SP Solver Benchmarks (2)



SP Solver Benchmarks (3)

Scenarios	NB	DE	DI	WS
1s	1s	1s	1s	1s
8s	198s	24s	18s	5s
10s	310s	74s	30s	7s
25s	781s	1293s	200s	17s
27s	839s	913s	255s	18s
50s	1342s	5533s	930s	32s
70s	1601s	29719s	1893s	51s
100s	2912s	M	3375s	65s
200s	5559s		15742s	145s
550s	14410s		159446s	345s
700s	15202s		M	441s
1000s	18222s			628s

Spine System Architecture



Conclusions

- The SAMPL/SMPL extensions enable rapid development of SP models
- Models of randomness and algebraic optimisation models are brought together
- Integration with DMBS allows advanced analysis of the results (such as VaR)
- SP solver embeds several algorithms and scalability is addressed by the decomposition method

