A framework for measurement and control of risk –'Optimum Risk Decisions'

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Outline

- A Qualitative and Quantitative Approach to Risk
- Generic Representation of Risk
- Stochastic Programming and Risk Decisions
- Computational Study
- Future Considerations

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Overview

Risk Perception → Risk Identification → Risk Quantification → Risk Decisions

Sources of Risk → Risk Measures → Optimisation Model

Qualitative

Quantitative

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Risk Perception

- Social scientists and behavioural psychologists study human aspects / perception of risk.
- Economists refer to it as ‘appetite for risk’ or ‘risk aversion’.
- Reflects individual characteristics and/or external exigencies.
- Risk perception is closely related to utility theory…key for connecting decision making models with risk.

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Risk Identification / Sources of Risk

- Uncertainty translates into risk but uncertainty is not risk.
- Risk appears in a variety of business sectors and applications... Finance, Energy, Agriculture...
- In each of these industry sectors different factors lead to risk:
  - Market volatility
  - Liquidity
  - Credit worthiness
  - Regulatory requirements
  - People involved
- Depending on the nature of the applications the models should include such factors.

Risk Quantification

- The risk measure used should be relevant to our goals (risk targets)...should reflect and adequately capture the risk(s) we want to mitigate.
- Should satisfy the axioms of coherency especially when applied in an enterprise wide context.
- Symmetric or asymmetric?

Symmetric Measures of Risk

- Dispersion risk measures: Quantify risk in terms of probability-weighted dispersion of results around a specific reference point and are otherwise classified as *symmetric measures of risk*.
  - *Variance* or *standard deviation (*Markowitz (1952, 1959)).
  - *Mean Absolute Deviation (MAD) (*Atkison (1970), Konno and Yamasaki (1991)).

Asymmetric Measures of Risk

- Measures which quantify risk according to results and probabilities below reference points, selected either subjectively or objectively, and are otherwise classified as *asymmetric measures of risk*.
  - *Expected Value of Loss* (Domar and Musgrave (1944)).
  - *Safety First* (Roy (1952)).
  - *Semi-Variance* (Markowitz (1959)).
  - α-*t* criterion (Bawa (1975) and Fishburn (1977)).
  - *Value at Risk – VaR –* (JP Morgan (1993)) and its extension *Conditional VaR – CVaR –* (Artzner et.al (1997) and Uryasev (1999)).

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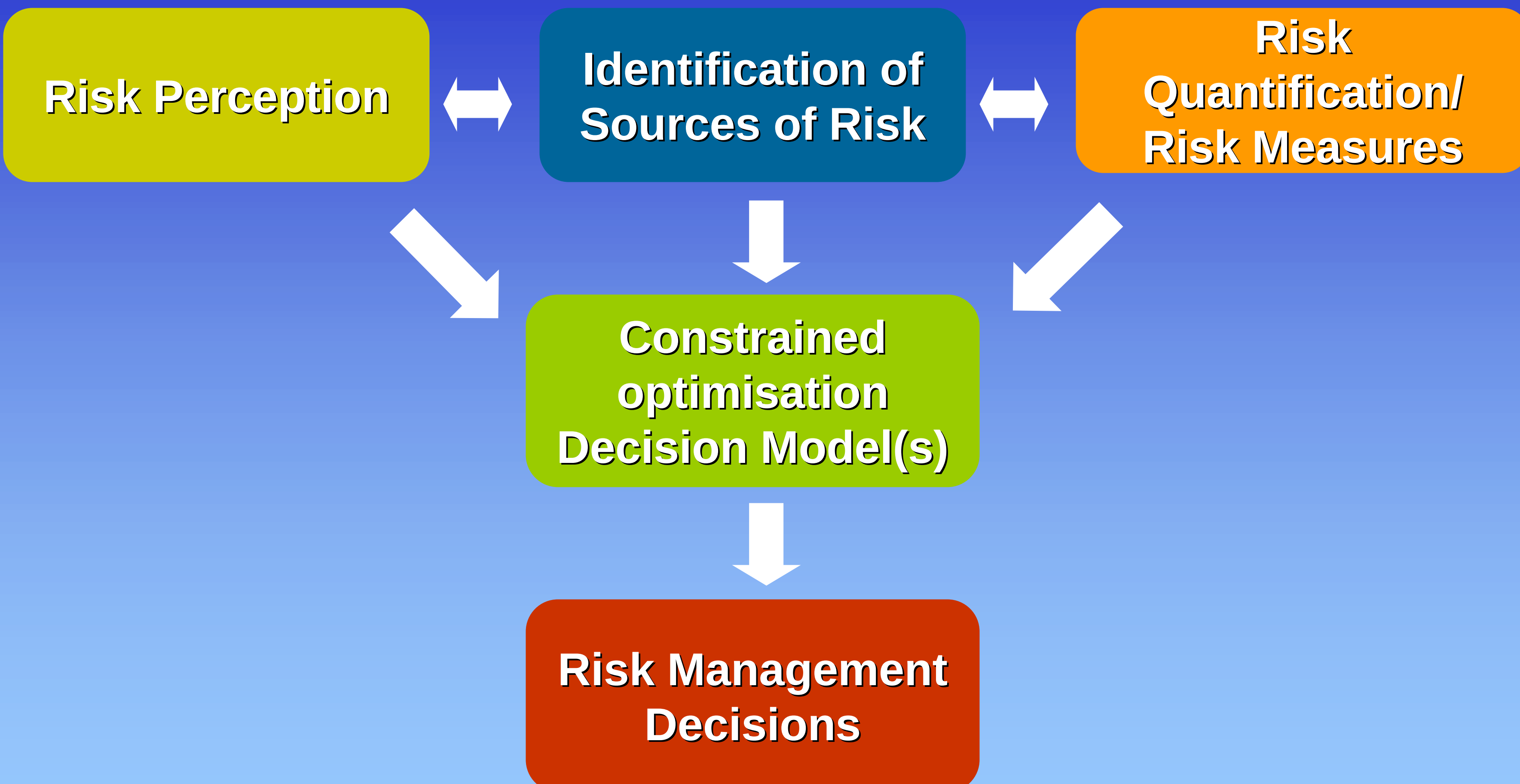
Risk Management Decisions



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Evolution of Risk Measures until mid 70's

Expected Downside

Domar and Musgrave (1944)

Safety First

Roy (1952)

Variance

Markowitz (1952)

Semi- Variance

Markowitz (1959)

α-t model

Bawa (1975) and Fishburn (1977)

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Generic Representation - *α-t* model

- α: a parameter specifying the moment of the return distribution. In some cases α indicates different attitudes towards risk.
- τ: a predefined target level of the investment return
- *F(x):* the cumulative probability distribution function of the investment with return *x*.
- The LPM of order α for a given τ defines the α-*t model* and has the following form:

$$F_{\alpha}(\tau) \equiv LPM_{\alpha}(\tau; x) = \int_{-\infty}^{\tau} (\tau - x)^{\alpha} f(x)\, dx = E\{(\max[0, \tau - x])^{\alpha}\} \;, \alpha > 0$$

Variance

- For $\tau = \bar{x}$ and α=2

$$\sigma^2 \equiv LPM_2(\bar{x};x) = \int_{-\infty}^{+\infty} (\bar{x} - x)^2 f(x)dx = E\{(\bar{x} - x)^2\}$$

- For target return τ:

$$\sigma^2 \equiv LPM_2(\tau;x) = \int_{-\infty}^{+\infty} (\tau - x)^2 f(x)dx = E\{(\tau - x)^2\}$$

MAD

- For $\tau = \bar{x}$ and α=1, the MAD

$$MAD \equiv LPM_1(\bar{x};x) = \int_{-\infty}^{+\infty} |\bar{x} - x|^1 f(x)dx = E\{(|\bar{x} - x|)^1\}$$

- For target return τ:

$$MAD \equiv LPM_1(\tau;x) = \int_{-\infty}^{+\infty} |\tau - x|^1 f(x)dx = E\{(|\tau - x|)^1\}$$

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Asymmetric Measures of Risk (I)

- Safety First : $\alpha \to 0$

$$SF \equiv LPM_{\alpha\to 0}(\tau; x) = F_{\alpha\to 0}(\tau) = \int_{-\infty}^{\tau} (\tau - x)^{\theta\to 0} f(x)dx \;\; = E\{(\max[0, \tau - x])^{\alpha\to 0}\}$$

- Expected Downside: $\alpha = 1$

$$\overline{D} \equiv LPM_1(\tau; x) = F_1(\tau) = \int_{-\infty}^{\tau} (\tau - x)^1 f(x)dx = E\{(\max[0, \tau - x])^1\}$$

- For $\tau = \bar{x}$ special case of MAD

$$MAD^{-} \equiv LPM_1(\bar{x}; x) = F_1(\bar{x}) = \int_{-\infty}^{\bar{x}} (\bar{x} - x)^1 f(x)dx = E\{(\max[0, \bar{x} - x])^1\}$$

Asymmetric Measures of Risk (II)

- Semi-Variance: $\alpha = 2$

$$\sigma^{-2} \equiv LPM_2(\tau; x) = F_2(\tau) = \int_{-\infty}^{\tau} (\tau - x)^2 f(x) dx = E\{(\max[0, \tau - x])^2\}$$

- Worst Case Scenario: $\alpha \rightarrow +\infty$

$$WCS \equiv F_{\alpha \to +\infty}(\tau) = LPM_{\alpha \to +\infty}(\tau; x) = \int_{-\infty}^{\tau} (\tau - x)^{\alpha \to +\infty} f(x) dx = E\{(\max[0, \tau - x])^{\alpha \to +\infty}\}$$

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Asymmetric Measures of Risk (III)

- VaR of a portfolio at the β probability level is the left quantile of the losses of the portfolio, i.e, the lowest possible value such that the probability of losses less than VaR exceeds $\beta \times 100\%$. The VaR is given as:

$$VaR(x, \beta) = \theta$$

- The corresponding LPM is:

$$LPM_0(\theta; x) = F_0(\theta) = \int_{-\infty}^{\theta} (\theta - x)^0 f(x) dx = 1 - \beta$$

- CVaR: $\alpha = 1$, and the expected downside risk is divided by (1-β)

$$CVaR \equiv LPM_1(\theta; x) = F_1(\theta) = \frac{\int_{-\infty}^{\theta} (\theta - x)^1 f(x) dx}{1 - \beta} = \frac{E\{(\max[0, \theta - x])^1\}}{1 - \beta}$$

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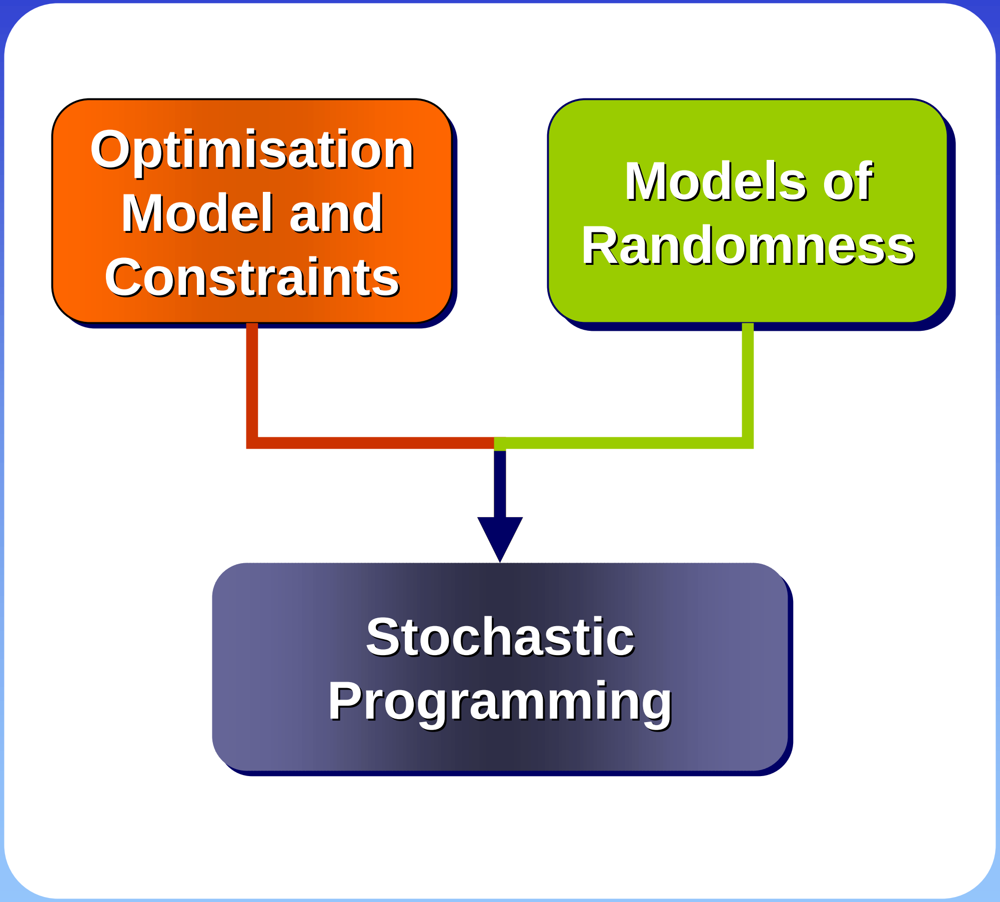
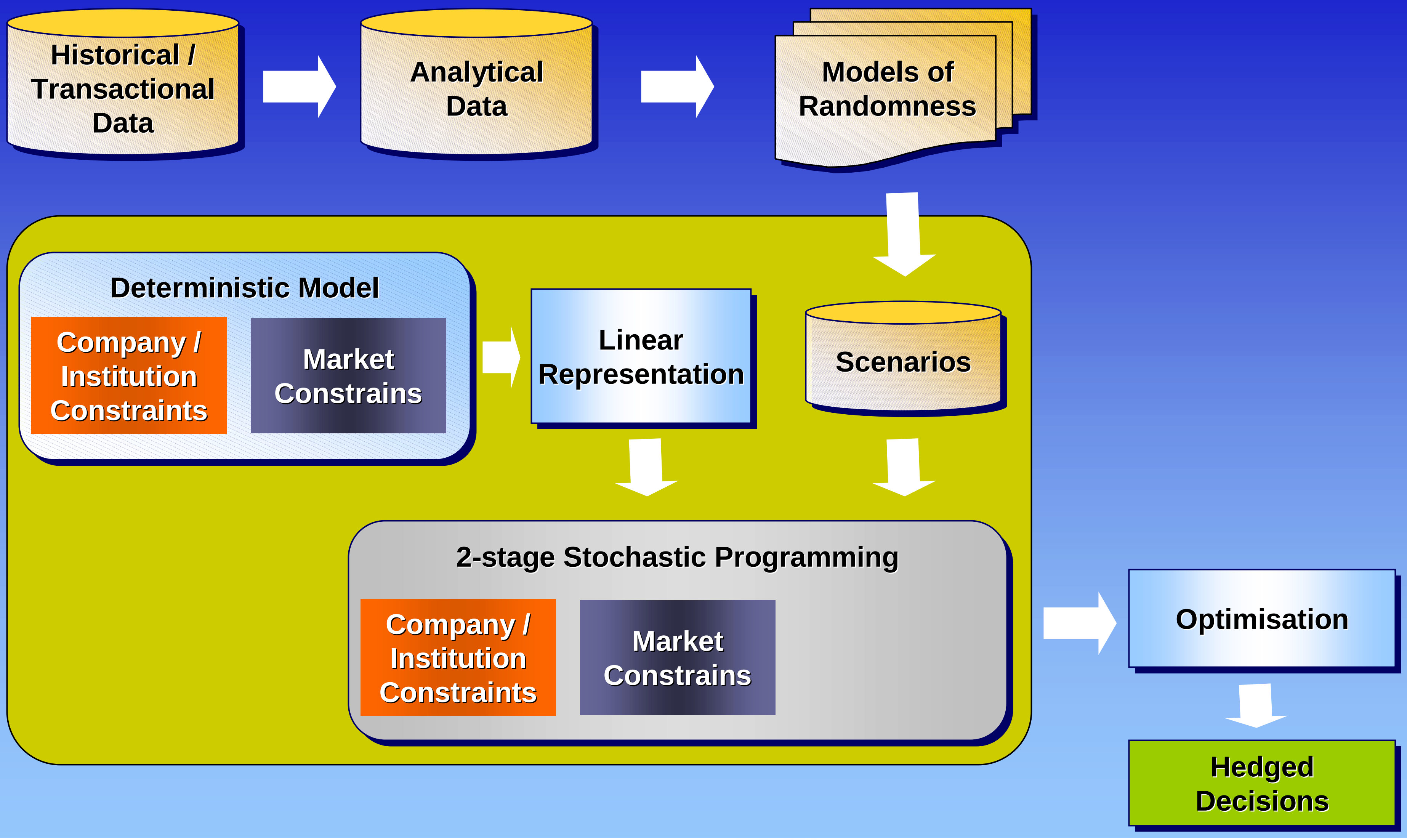
Why Stochastic Programming?

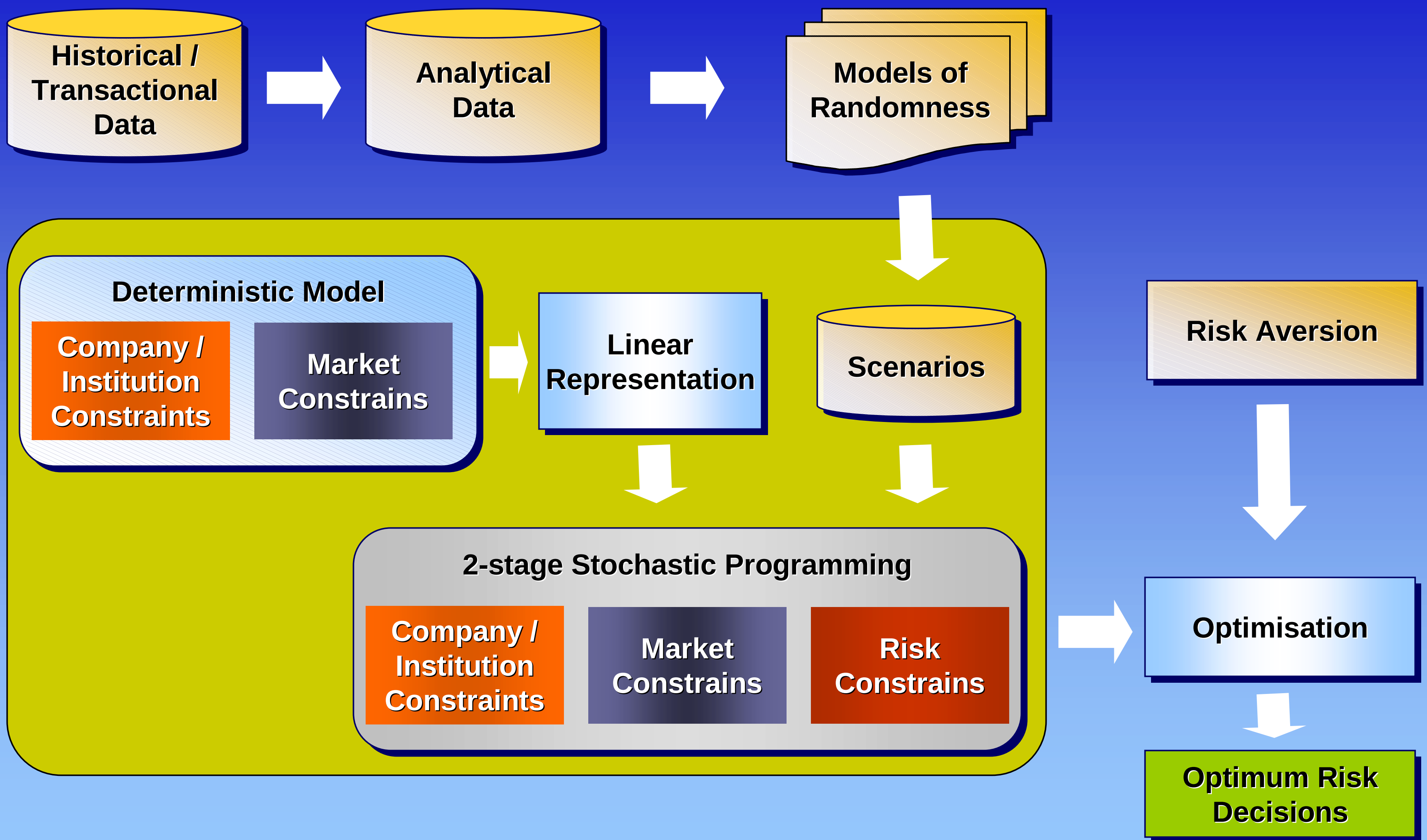
- Planning for the Future connected with uncertainty.
- Optimisation (Linear/Integer Programming) models assume exact (deterministic) states of nature.
- Dynamic and time dependent nature of investment decisions.
- Decisions need to be made today, “Here and Now.”
- Translates into modelling flexibility and thus allows consideration of realistic constraints.



Integrated View…Hedged Decisions



Integrated View...Optimum Risk Decisions



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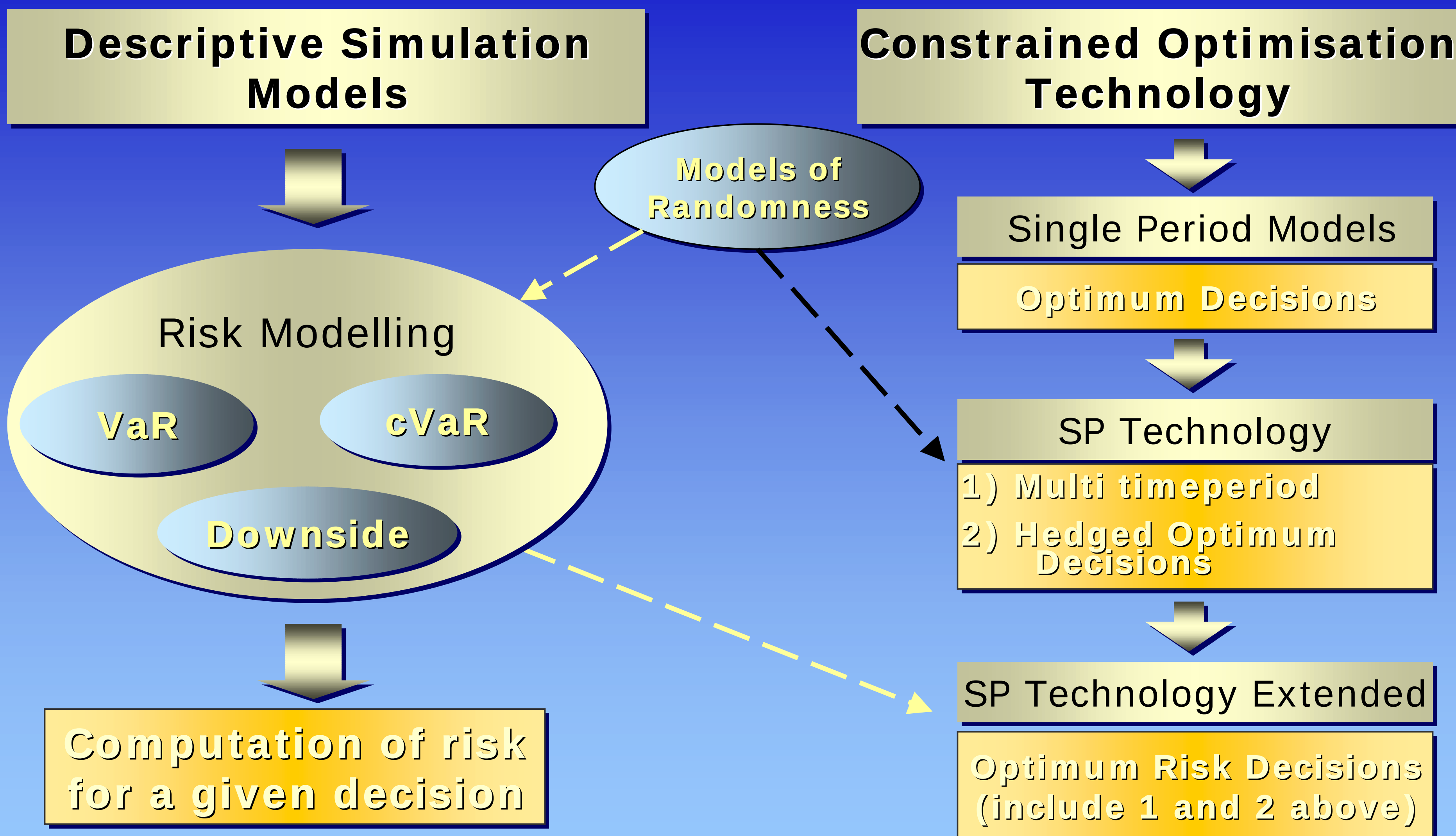
Risk Decisions



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Our Investigation

Model	Objective	Risk	Methodology
Two-stage multi-time period SP	Maximise Terminal Wealth	Downside VaR	Rolling & Backtesting
Mean Variance	Maximise Single Period Wealth	Variance	Rolling & Backtesting

Model Specification

- Planning Horizon: 12 months
- Initial Capital: $1,000,000
- Additional Inflow at the time of rebalance: $50,000
- Liabilities: Inflation Related
- Rebalance Strategy: Twice at months 2 and 6
- 59 Assets from S&P100
- Number of scenarios: 300
- 10 years Backtesting / Rolling (1993-2002)

Maximise (Multiobjective Function)

Terminal Wealth - Risk_t

Subject to:

Initial Holdings = 0

Wealth = investment * return - Liabilities

Fund balance constraint

Asset holding constraint

Transaction costs

Risk : Expected Downside, CVaR, and Variance

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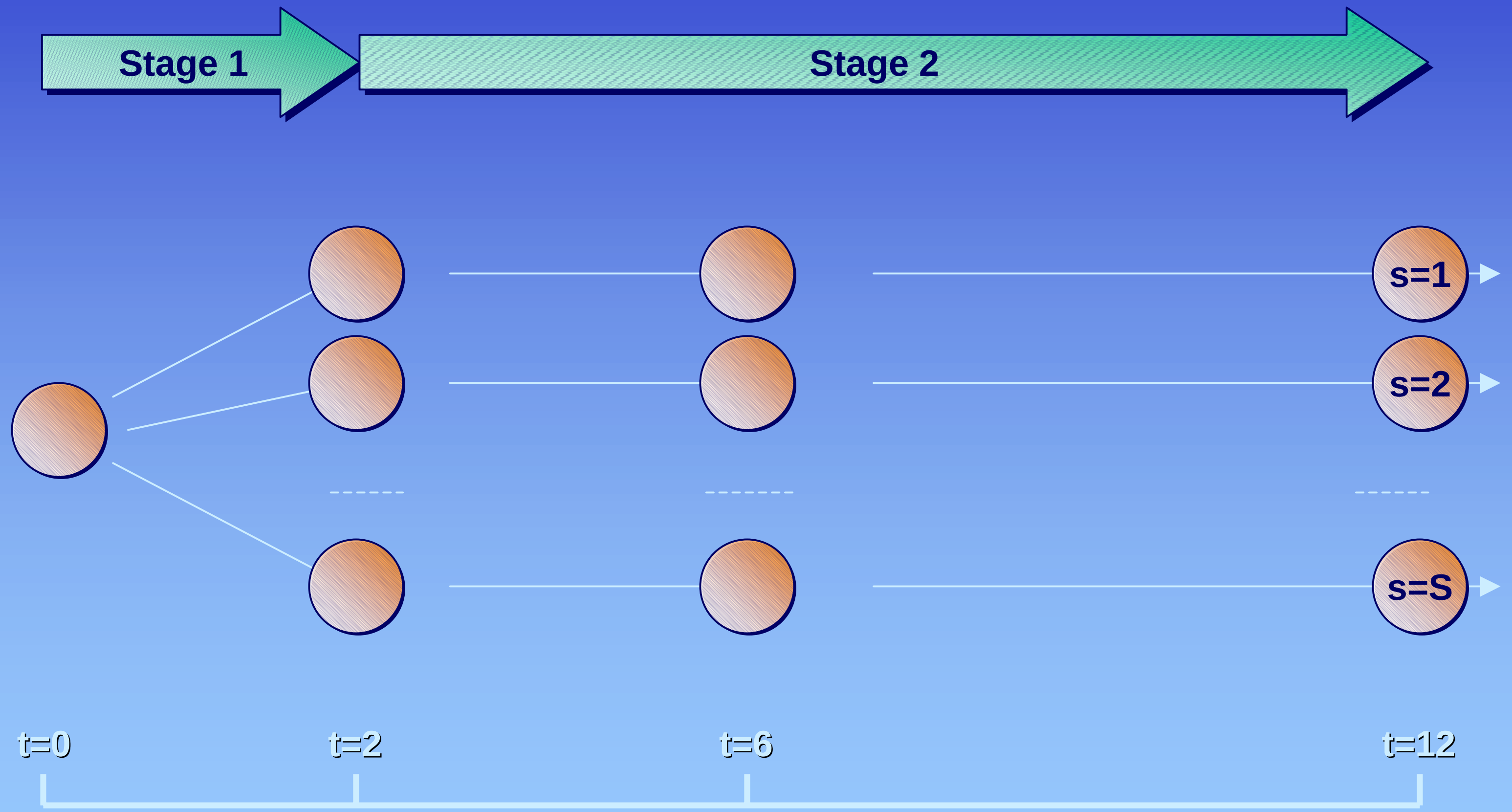
2-stage Stochastic Tree



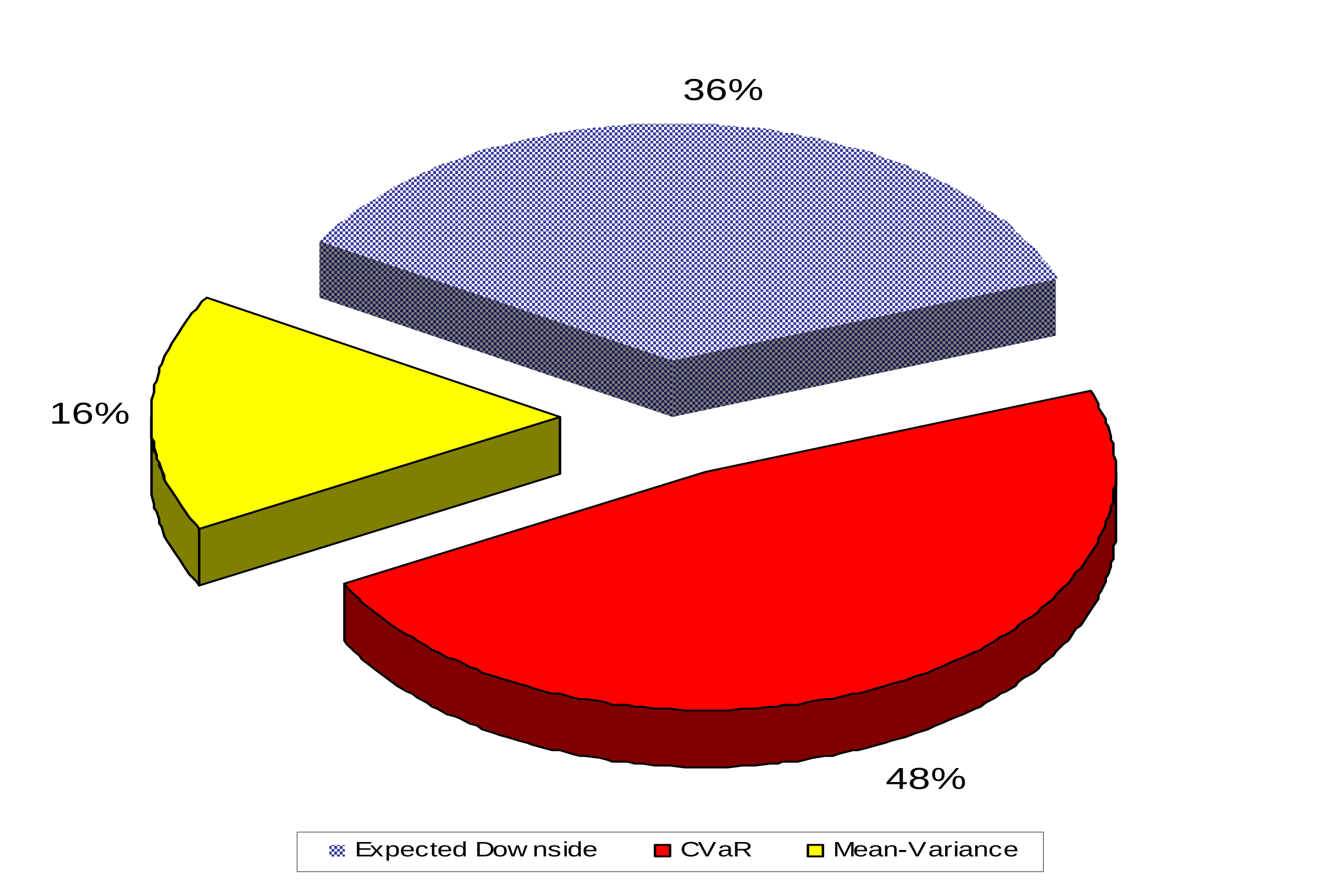
Risk Profiles

Profile	Appetite for Risk	Scale
MinRisk	Lowest	0
MeLRisk	Medium Low	25
MedRisk	Medium	50
MeHRisk	Medium High	75
MaxRisk	Highest	100

Risk Performance Mapping

Dates	MinRisk	MeLRisk	MedRisk	MeHRisk	MaxRisk
1992-1993	Exp. Downside	CVaR	CVaR	Exp. Downside	CVaR
1993-1994	Mean-Variance	Mean-Variance	Mean-Variance	Mean-Variance	Mean-Variance
1994-1995	CVaR	Exp. Downside	Exp. Downside	Exp. Downside	Exp. Downside
1995-1996	CVaR	CVaR	Exp. Downside	CVaR	Exp. Downside
1996-1997	Exp. Downside	CVaR	Exp. Downside	Exp. Downside	CVaR
1997-1998	CVaR	CVaR	CVaR	CVaR	Exp. Downside
1998-1999	CVaR	Exp. Downside	CVaR	Exp. Downside	Exp. Downside
1999-2000	Mean-Variance	Exp. Downside	CVaR	CVaR	Exp. Downside
2000-2001	CVaR	CVaR	Mean-Variance	Exp. Downside	Mean-Variance
2001-2002	CVaR	CVaR	CVaR	CVaR	CVaR

Analysis of Results



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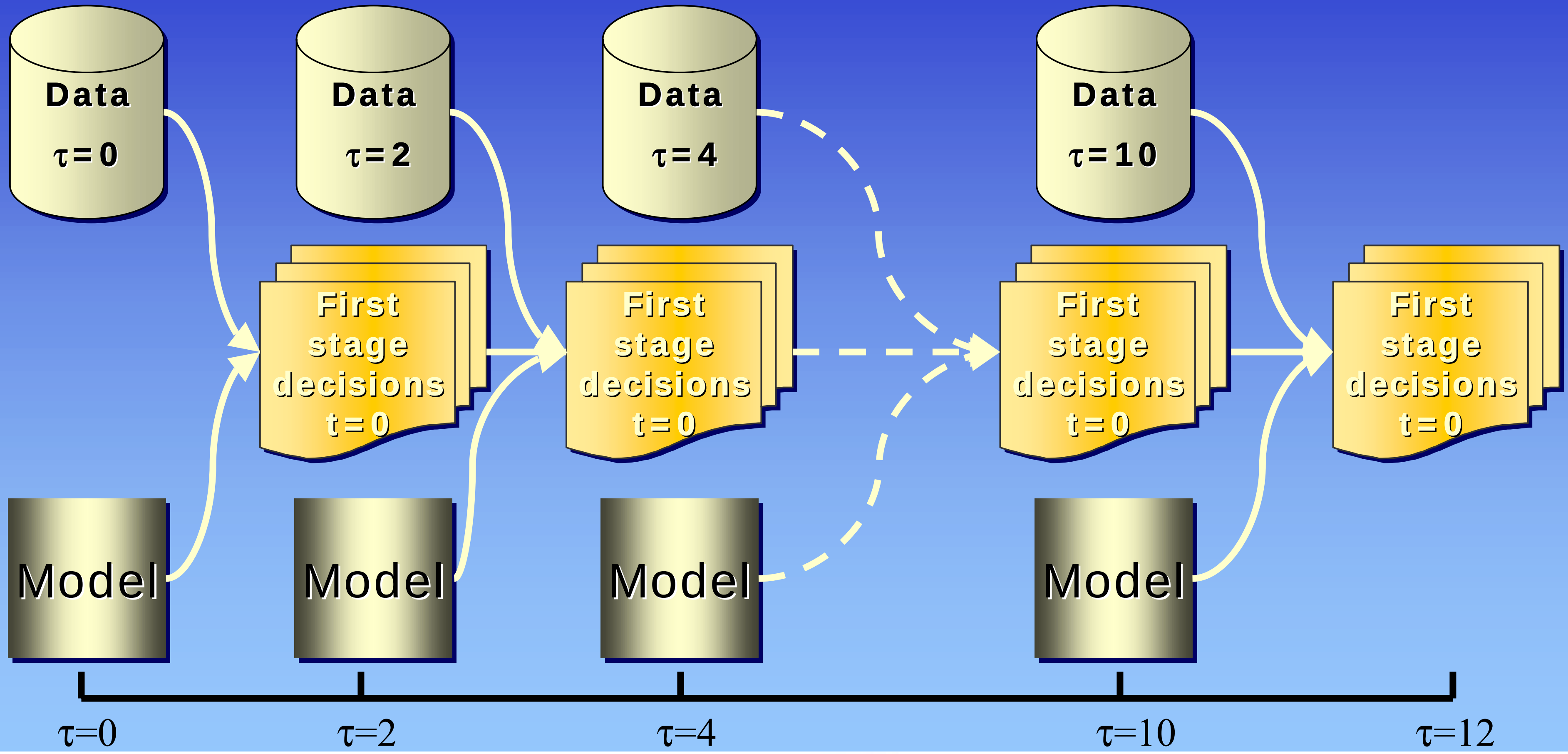
Future Consideration

Close coupling between descriptive pre-analytics and decision models. Benefits:

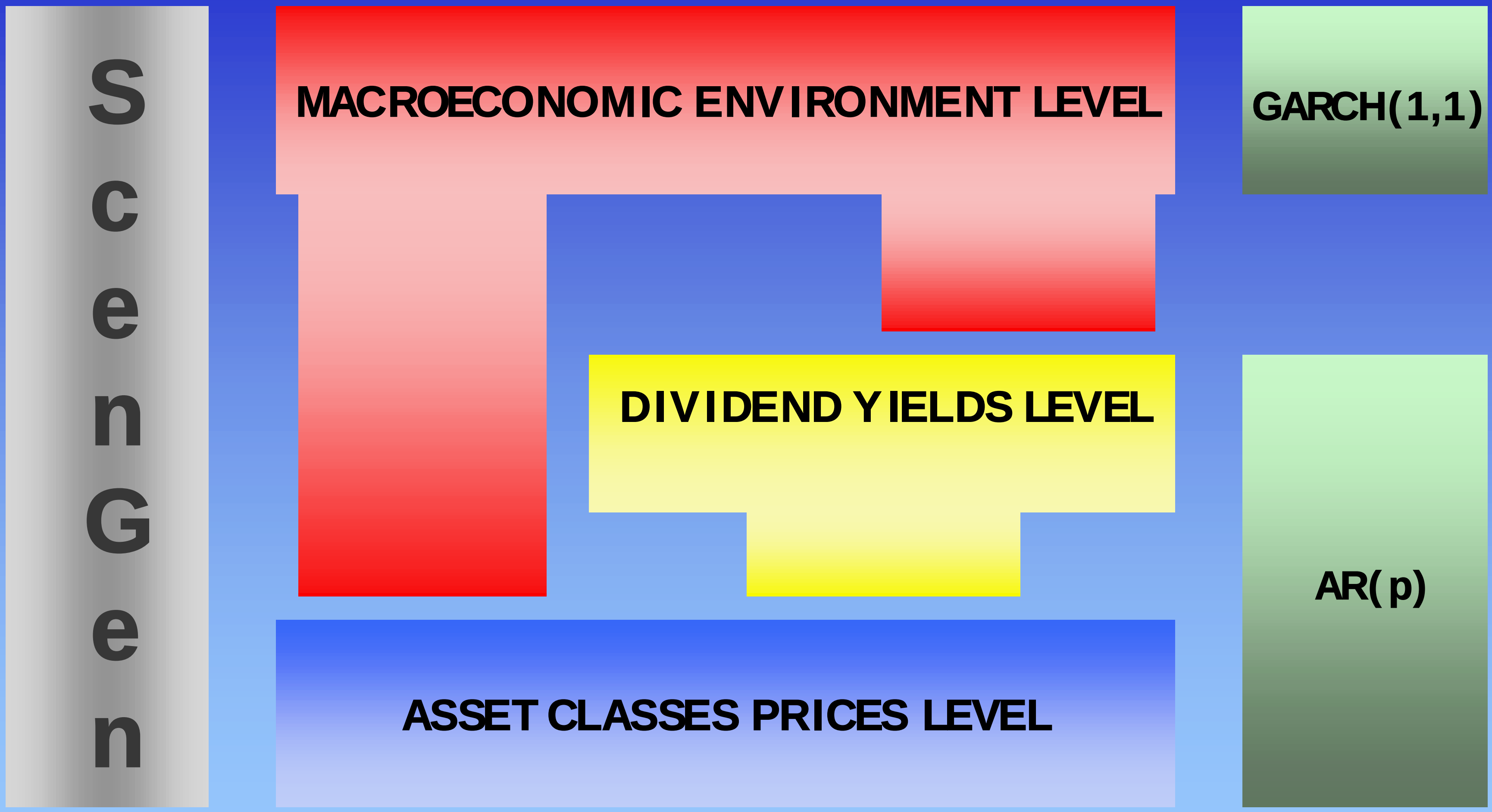
- Decision based on one model can be put through alternative risk analytics
- The framework can be used for backtesting and out of sample testing
- Convergence of simulation and scenario based models.
- Can such a tool be used to elicitate the risk perception of the decision makers...?

Rolling Decisions

- Explanation of the framework



ScenGen - Overview



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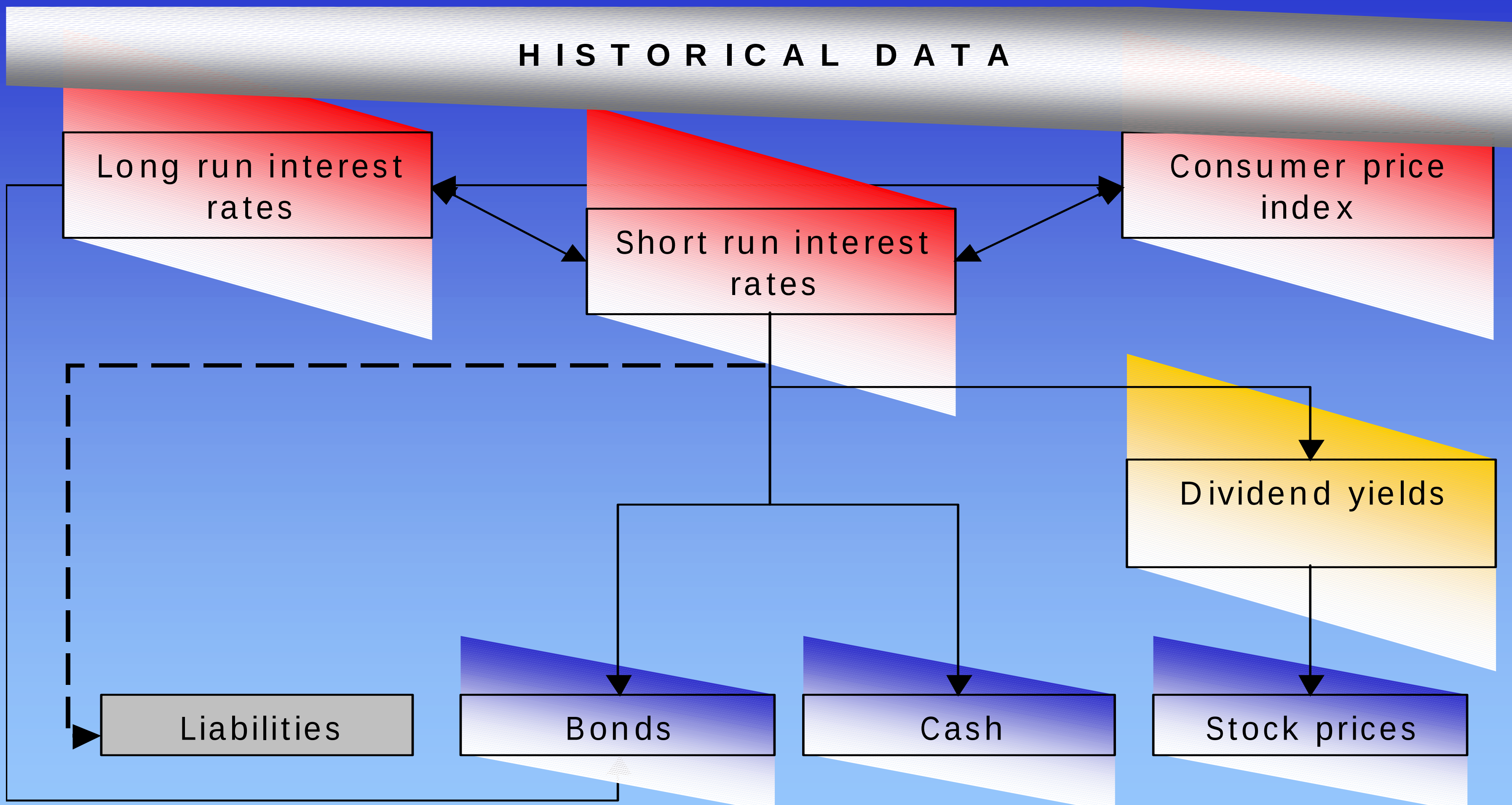
Scen-Gen



HISTORICAL DATA
Long run interest rates
Short run interest rates
Consumer price index
Dividend yields
Liabilities
Bonds
Cash
Stock prices