Computational methods for processing two stage stochastic programming problems

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OUTLINE

1. Overview of 2-stage SP methods
2. Diagrammatic Presentation of Level Method
3. Theoretical Comparisons between Methods
4. Mathematical Description
5. Test Models
6. Parameter Variation
7. Solver Comparisons
8. Conclusions

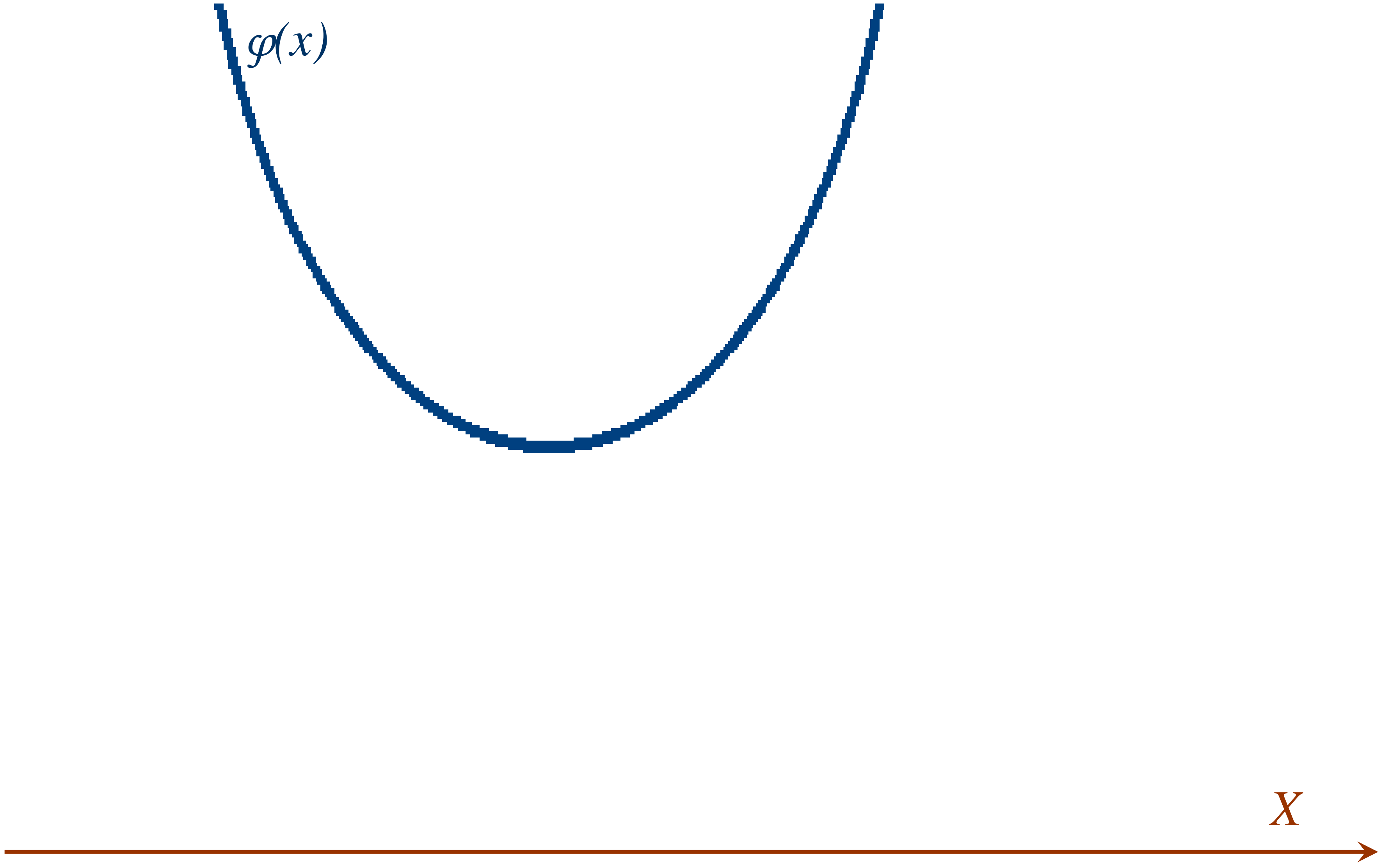
1) Overview of 2-stage SP methods

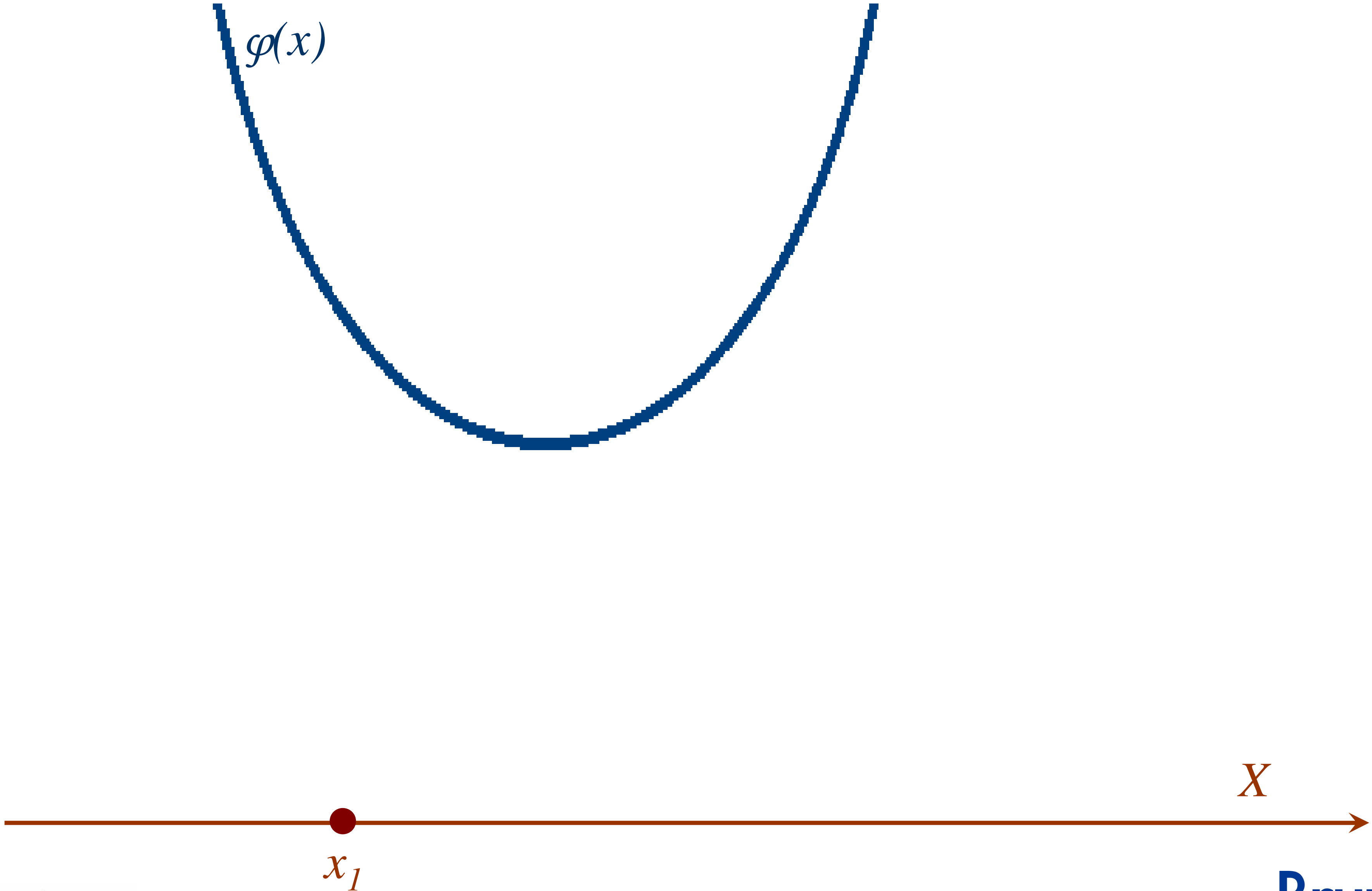
- L-shaped Method (LSM)
  - Van Slyke and Wets (1969)
- Regularised Decomposition (RDC)
  - Ruszczynski (1986)
- Stochastic Decomposition (SD)
  - Higle and Sen (1991)

1) Overview of 2-stage SP methods

- Level Method
  - Lemarechal, Nemirovskii, Nesterov (1995)
- Level Decomposition (LDC)
  - Fabian, Szoke (2007)

Cutting-plane method


$\varphi(x)$
X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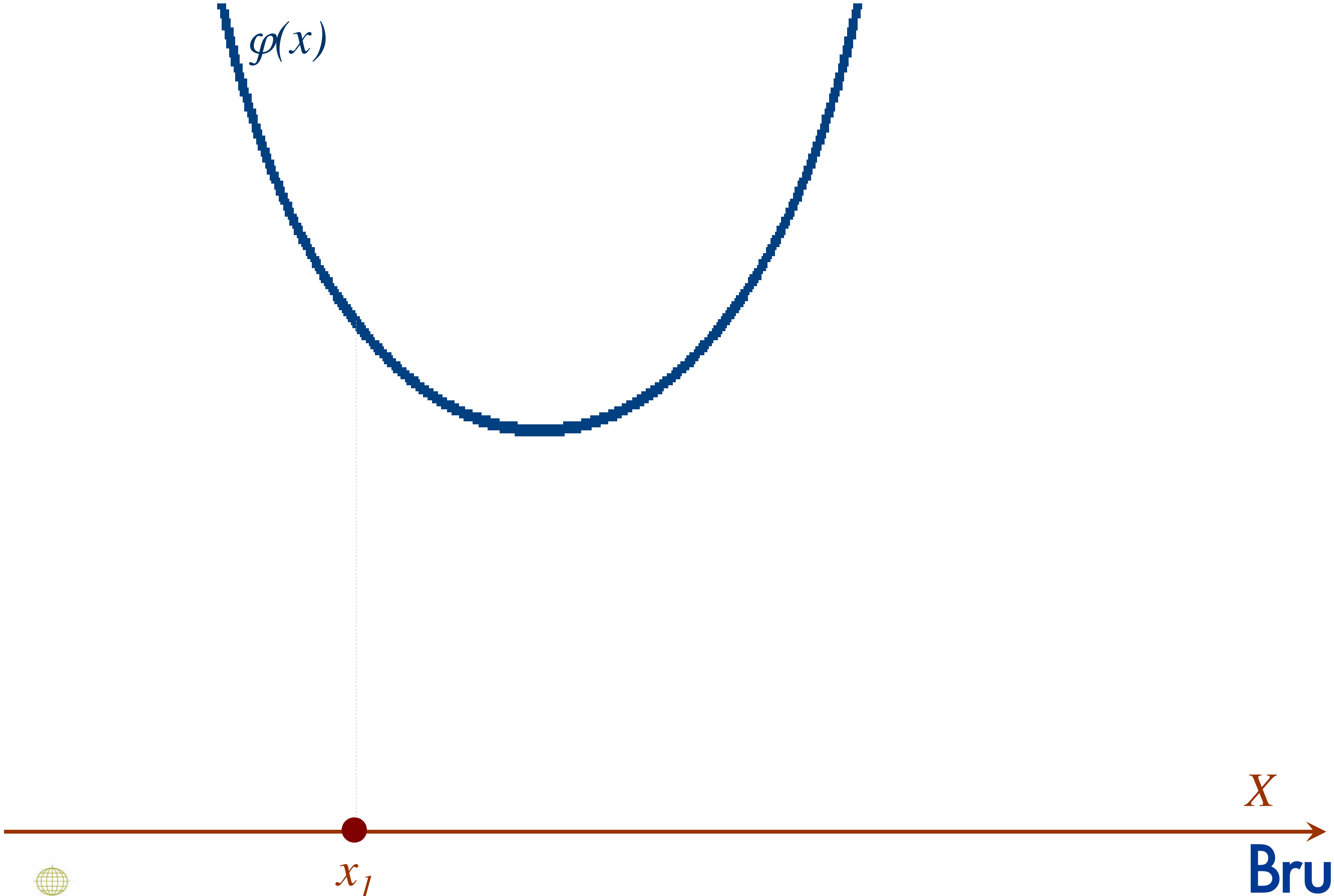
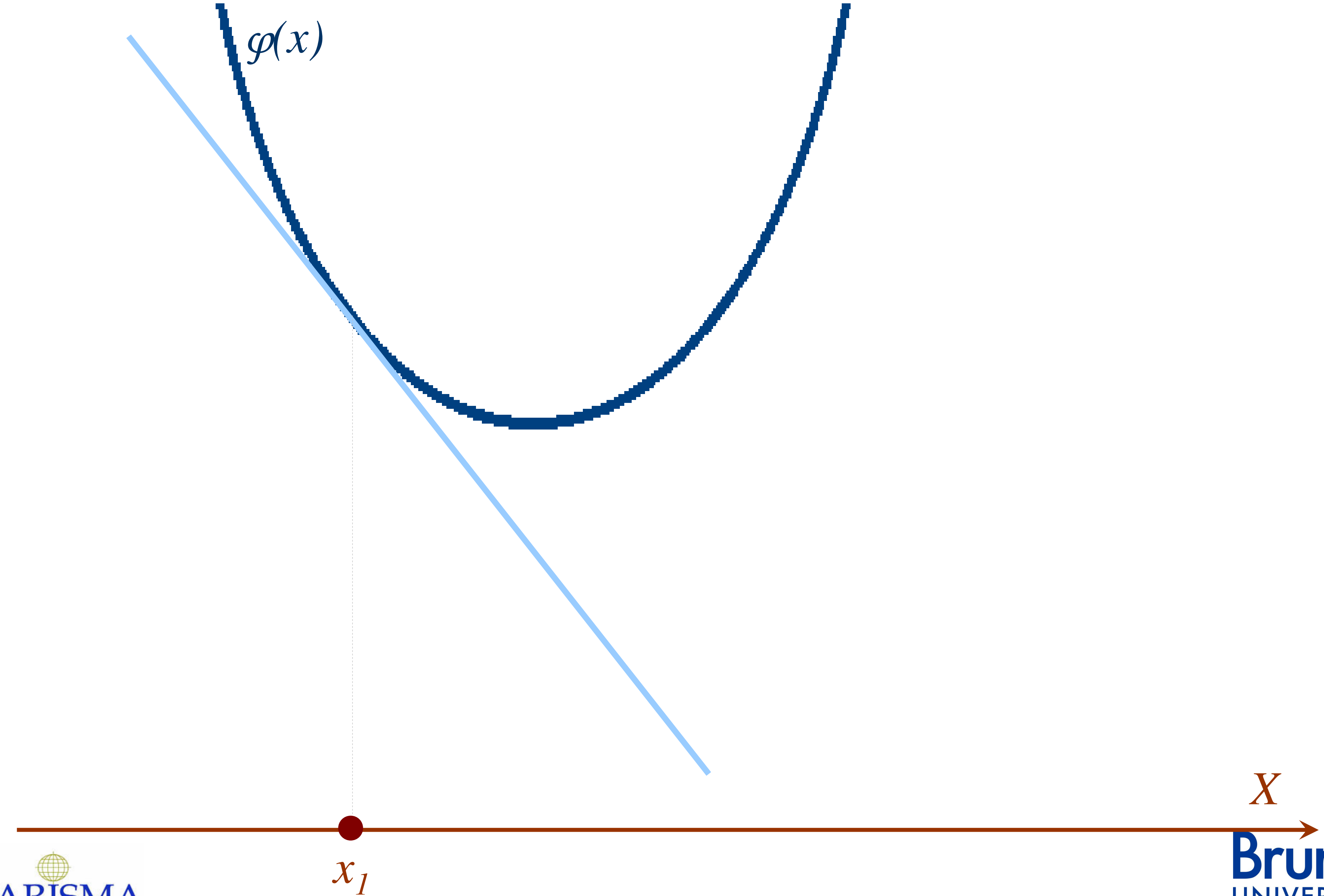
X
x1



φ(x)
X
x1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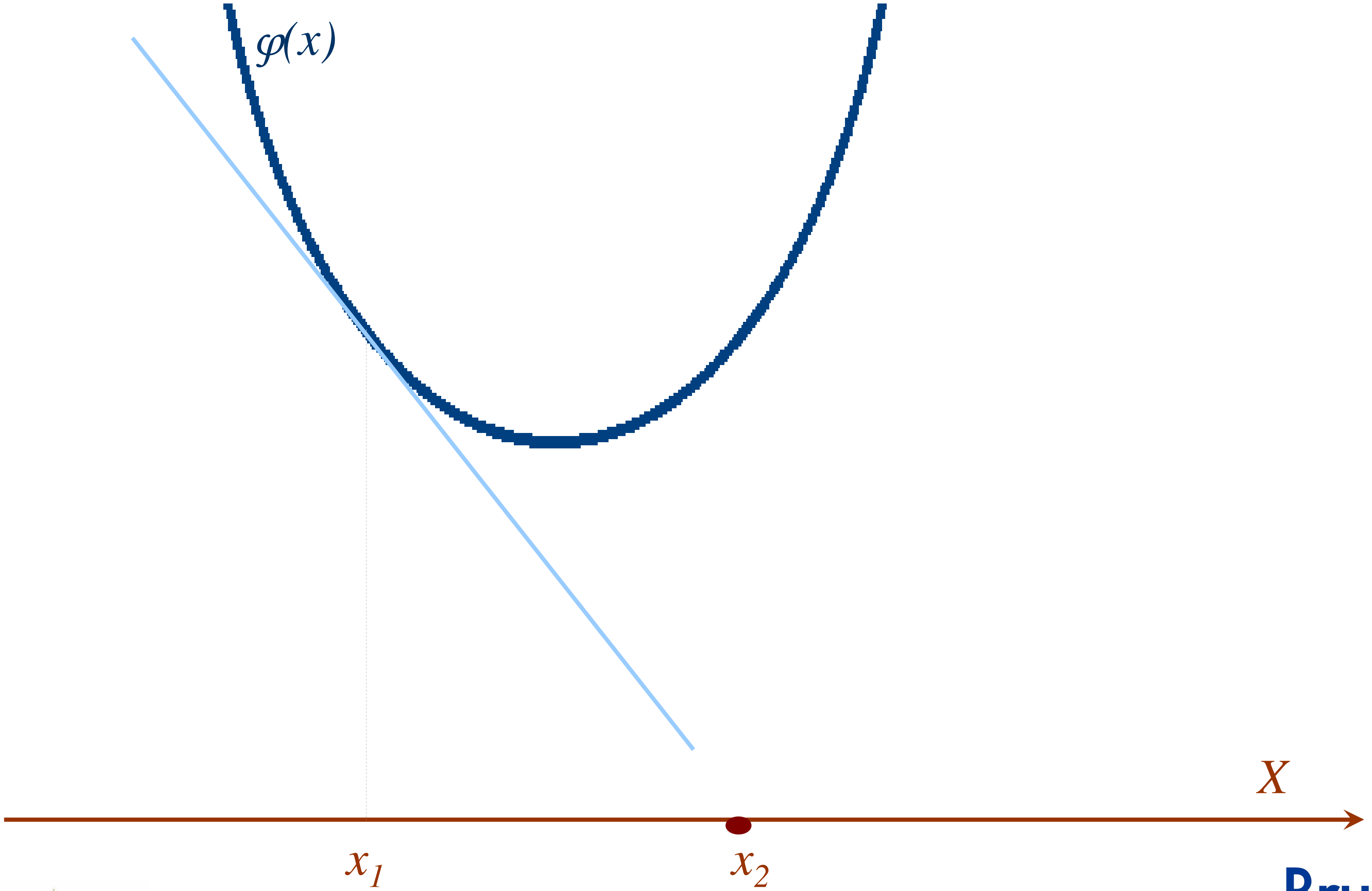
X
x1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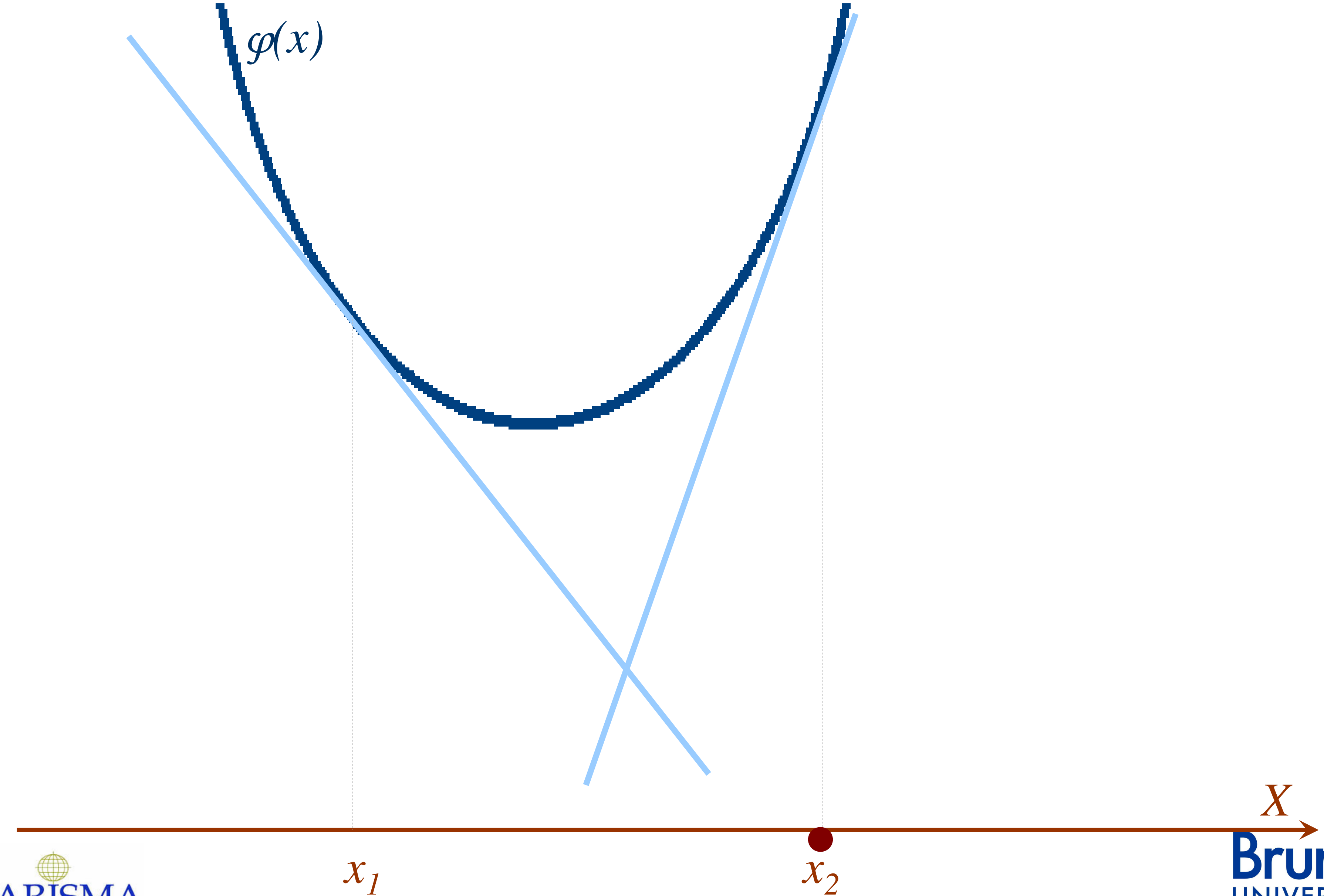
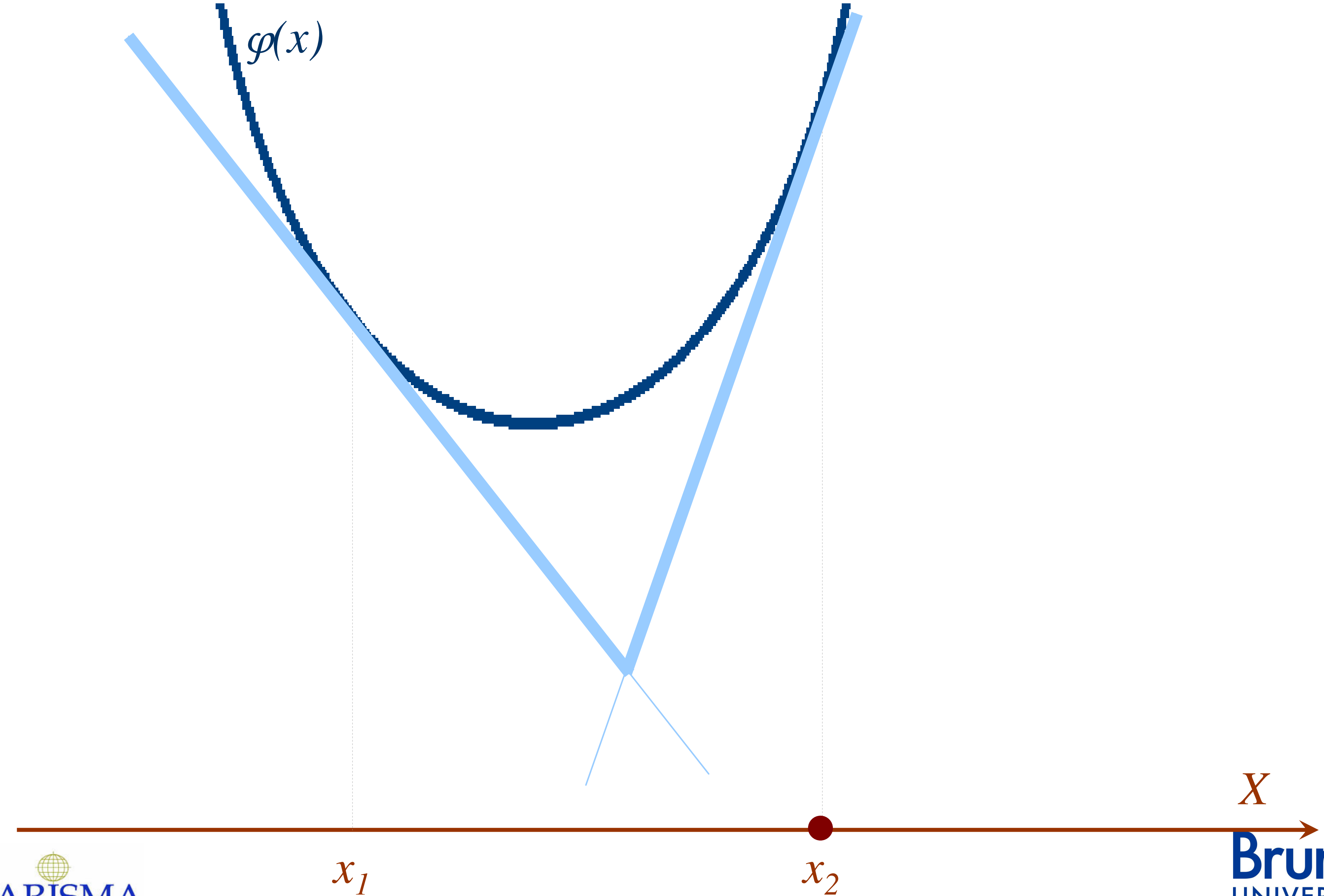
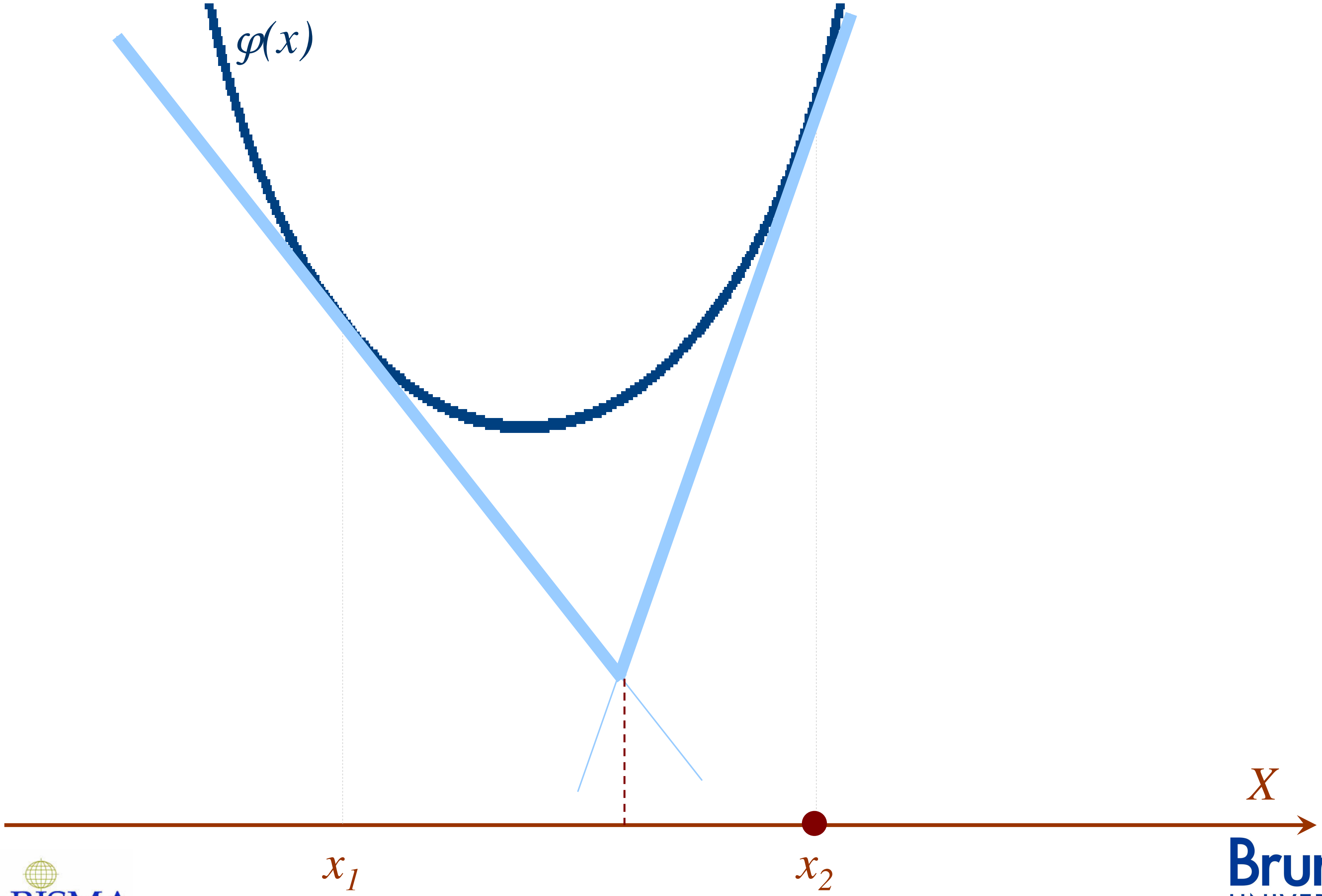
x1
x2

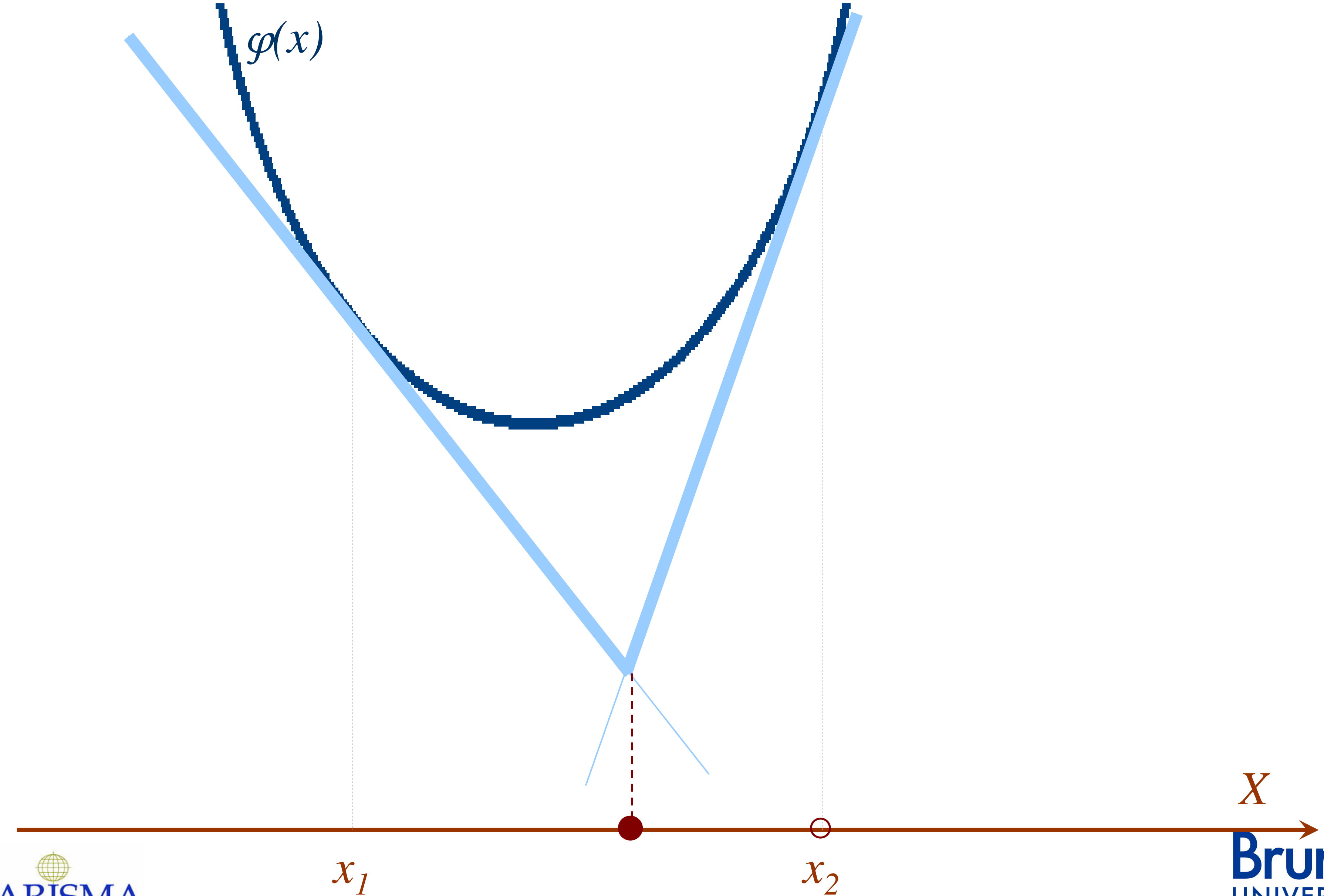


φ(x)
X
x1
x2



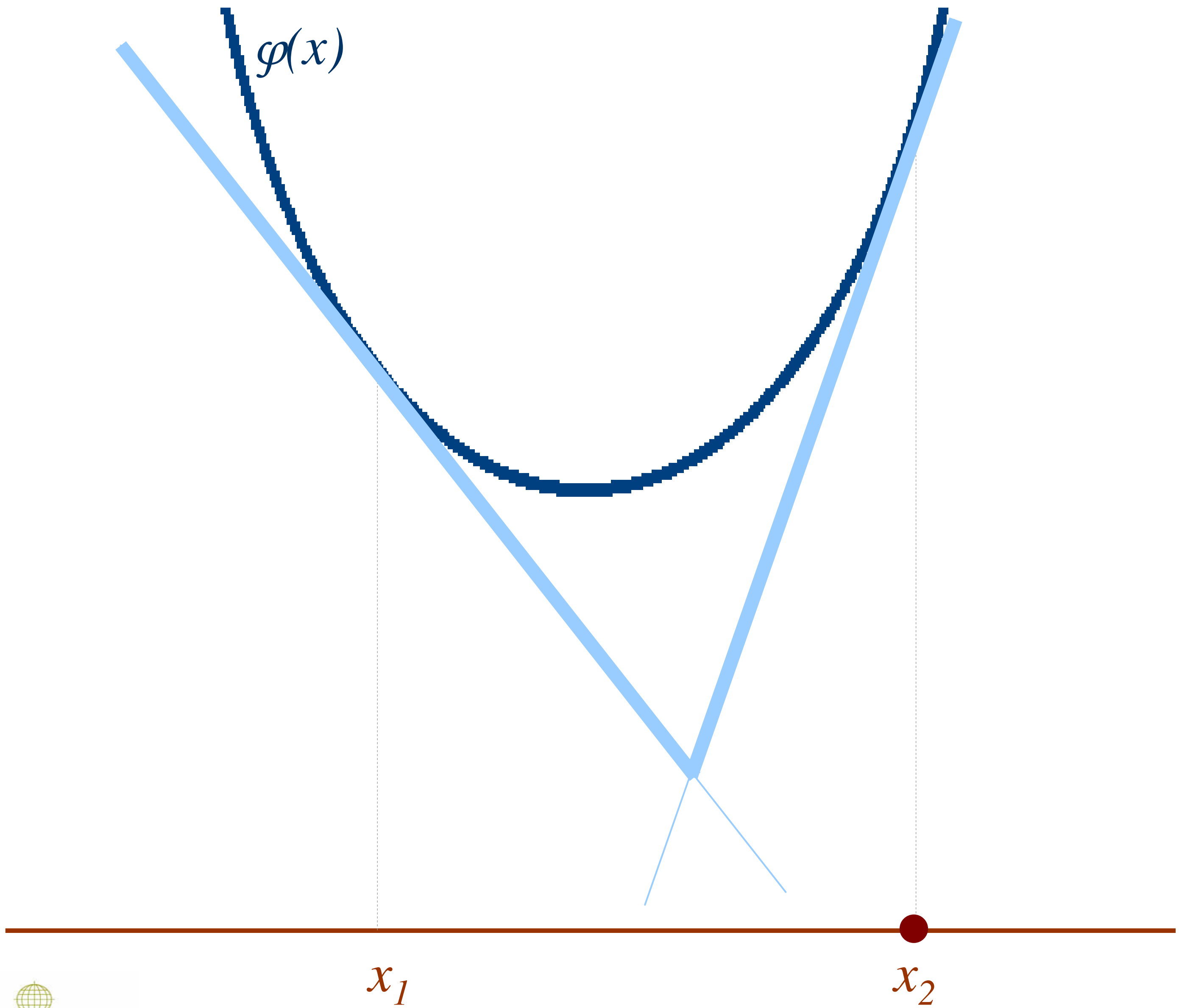
φ(x)
X
x1
x2


φ(x)
X
x_1
x_2

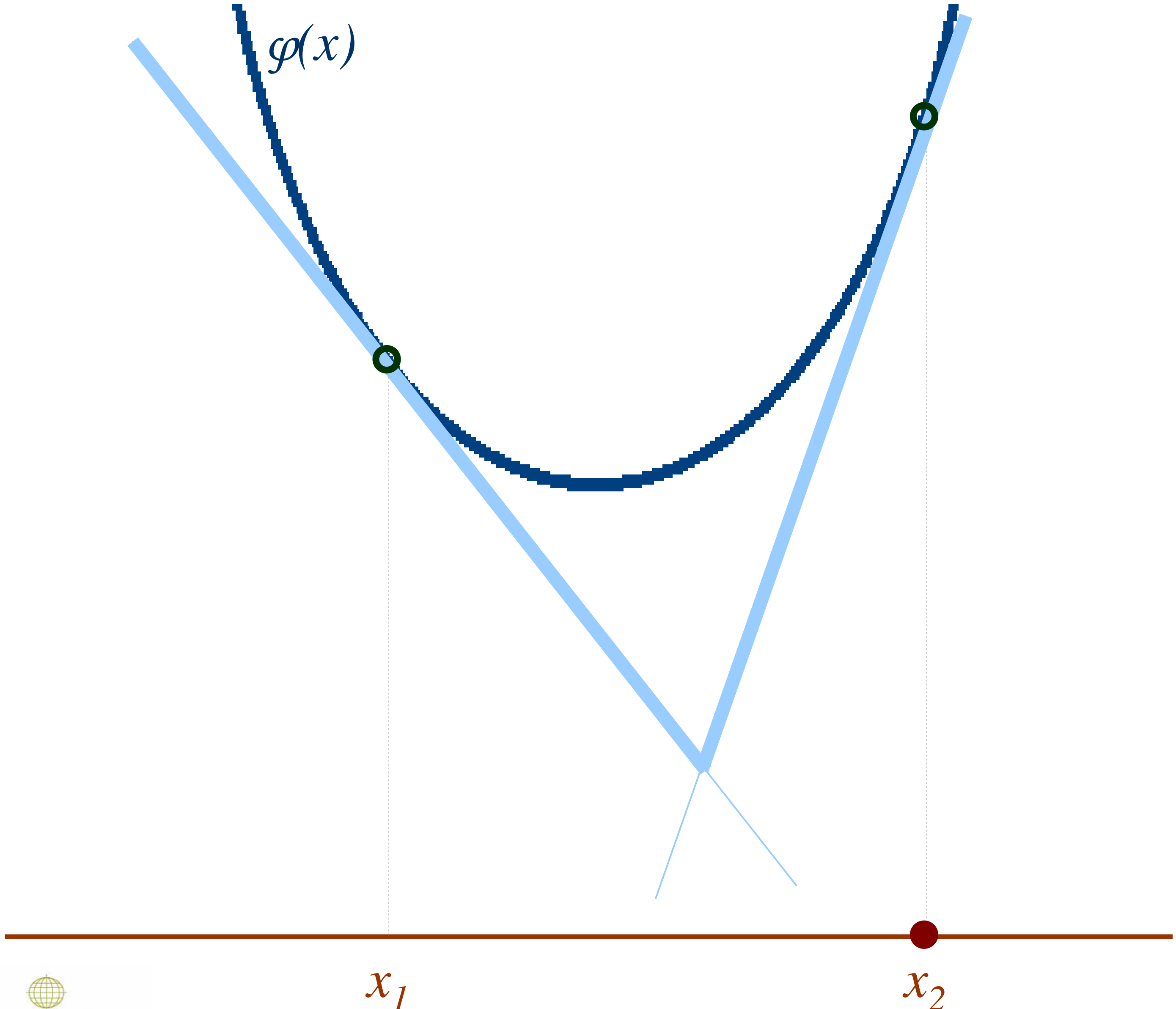


φ(x)
X
x1
x2

Level method


φ(x)
x1
x2

OptiRisk SYSTEMS



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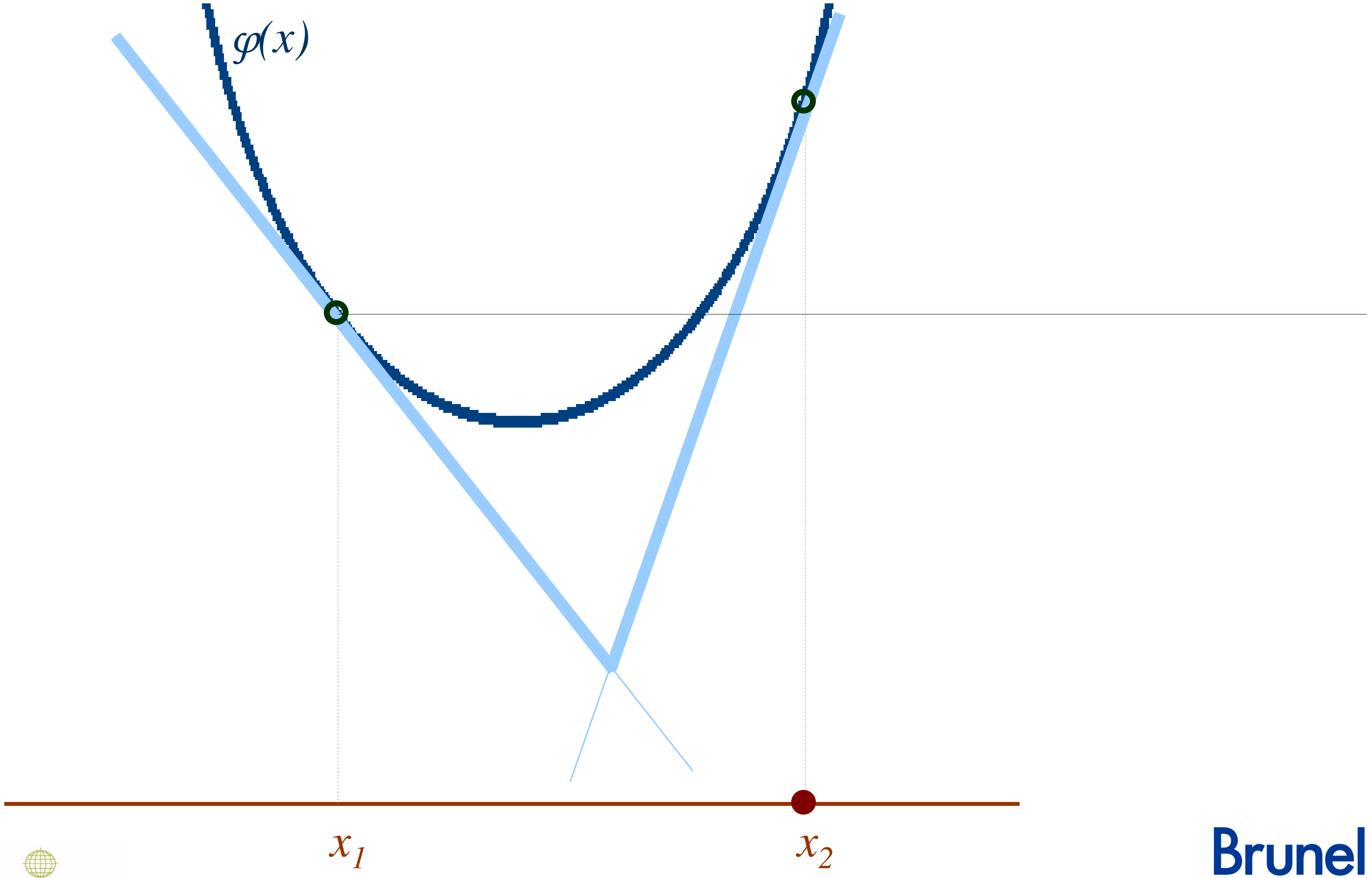
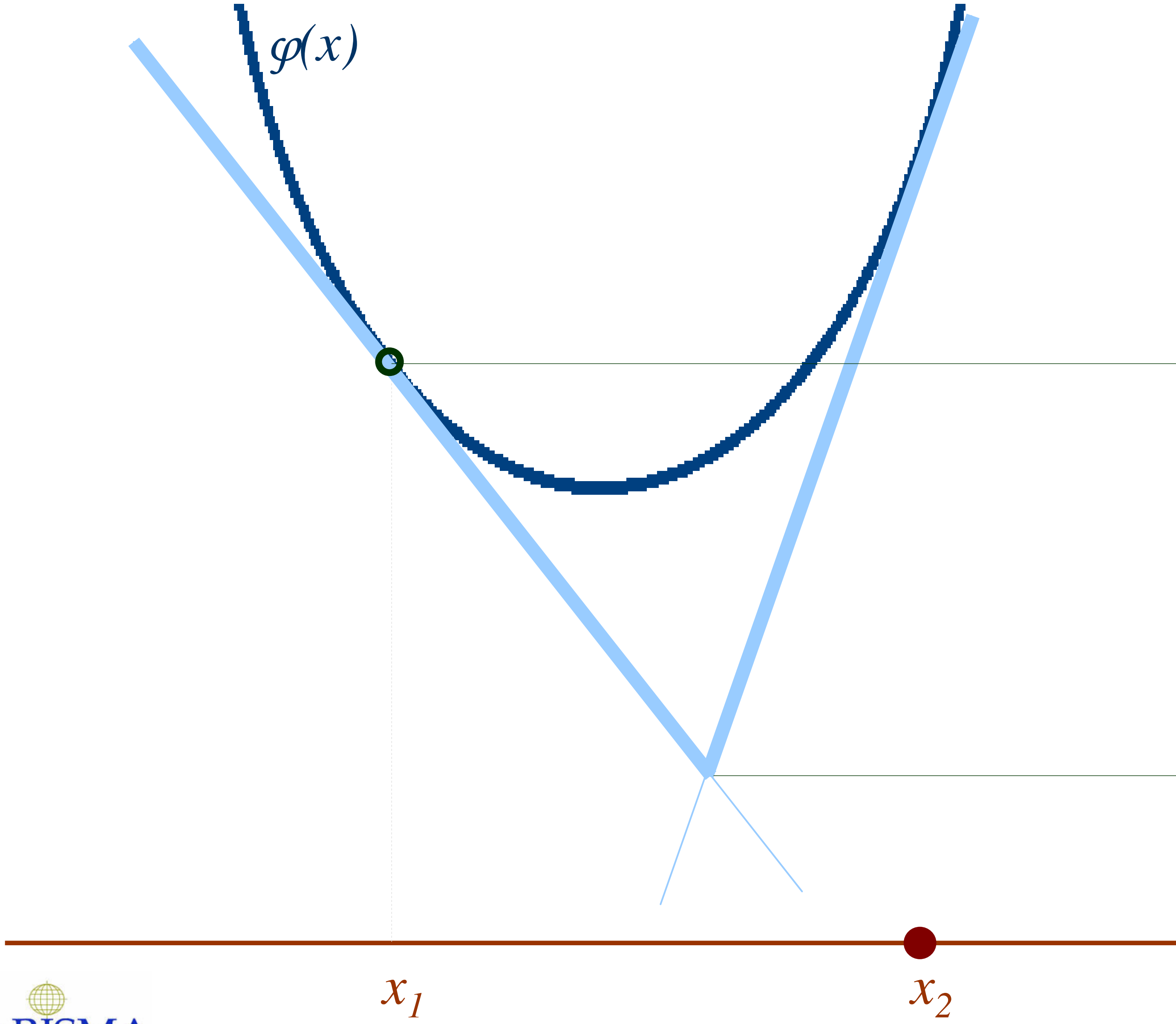
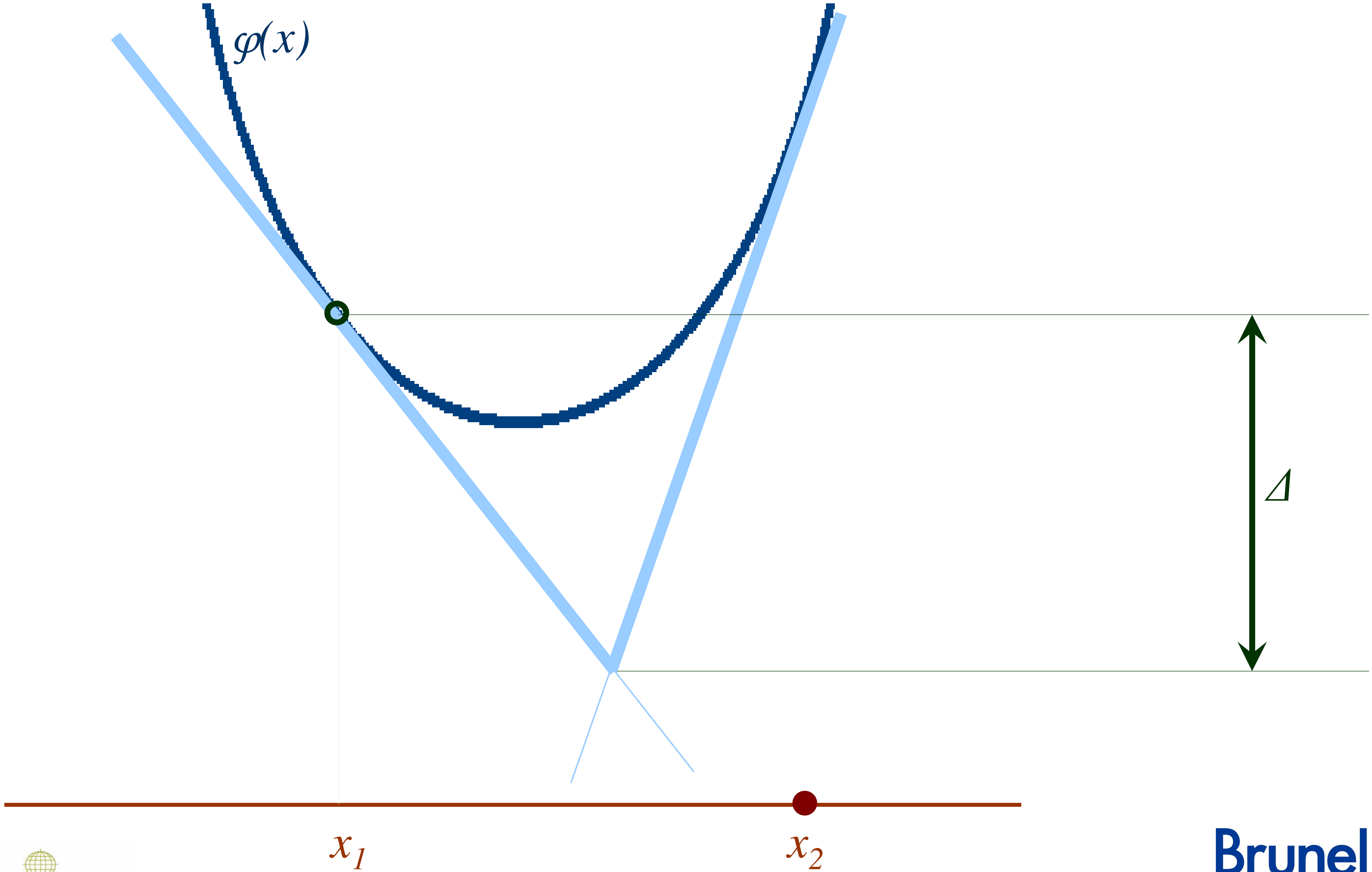
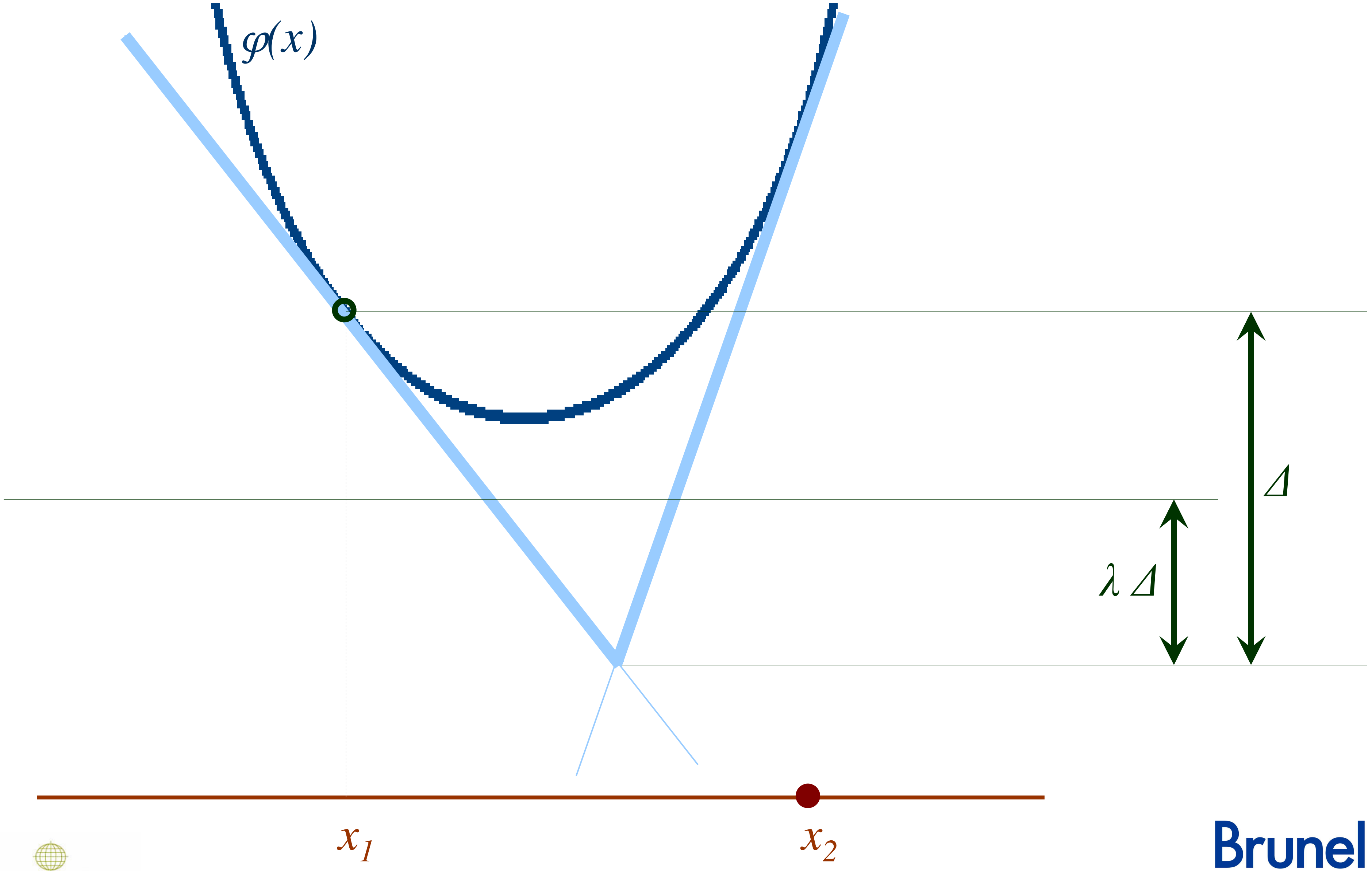
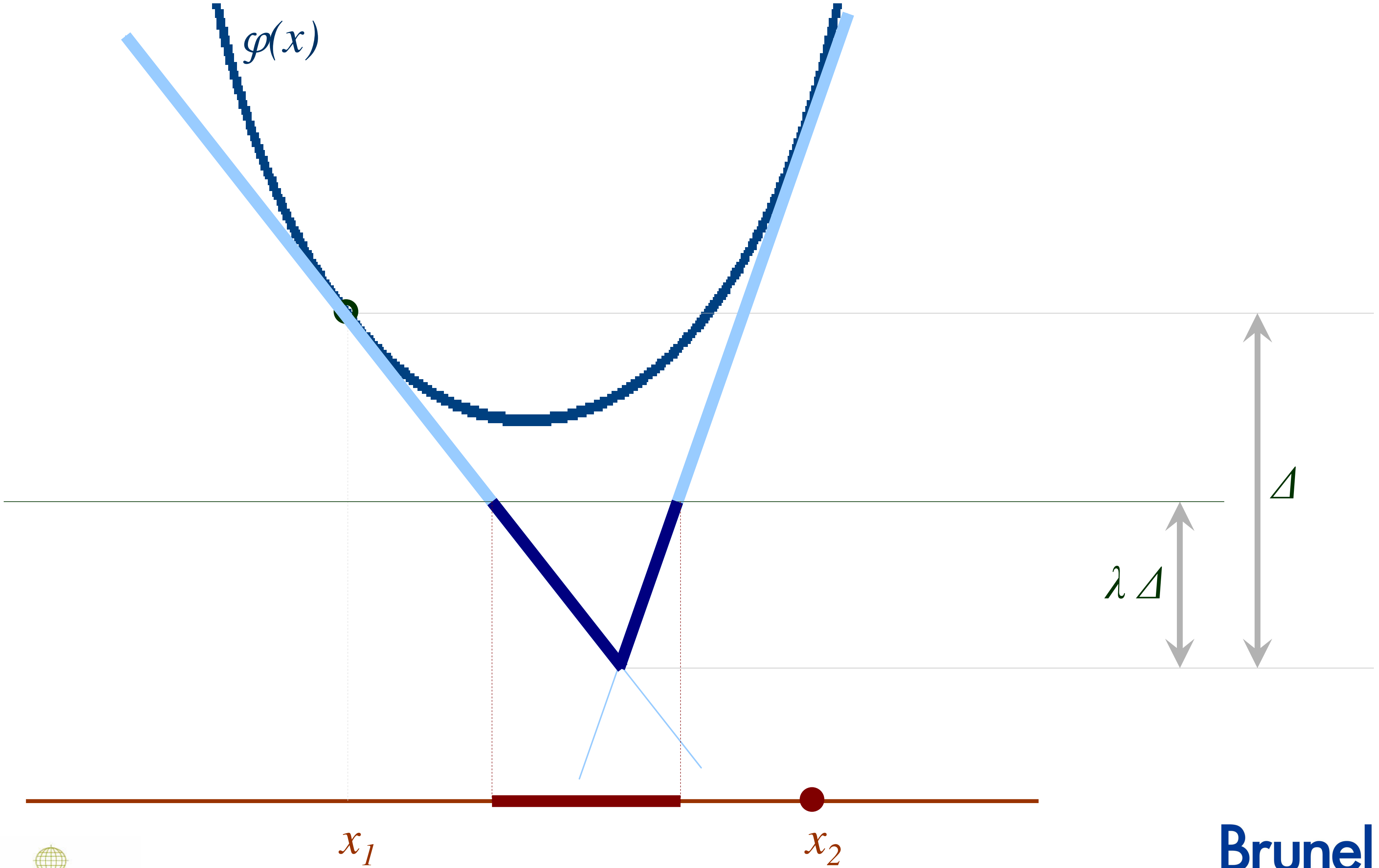

φ(x)
x1
x2

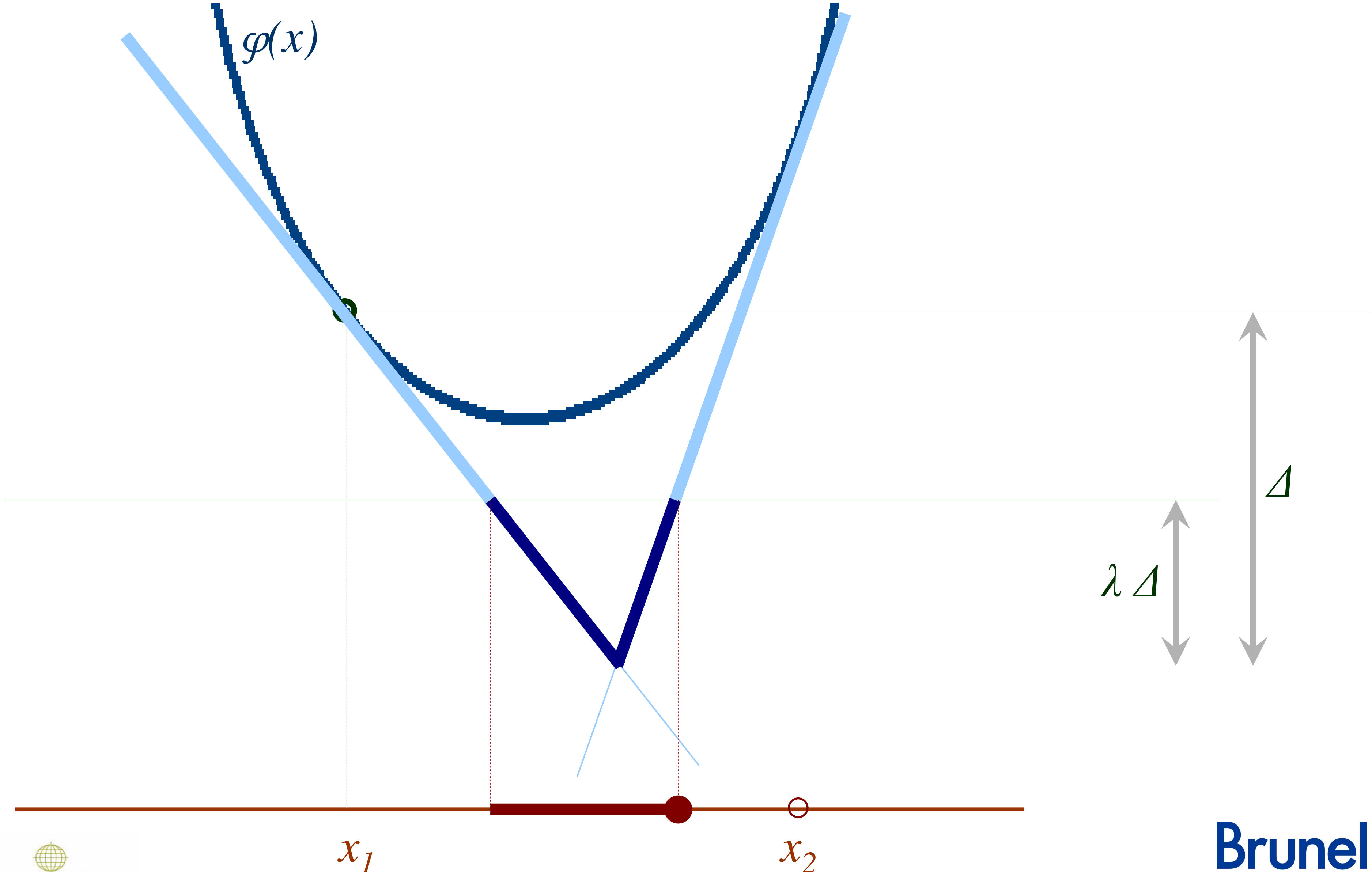

$\varphi(x)$
x_1
x_2


φ(x)
Δ
x1
x2


φ(x)
Δ
λ Δ
x_1
x_2



φ(x)
Δ
λ Δ
x1
x2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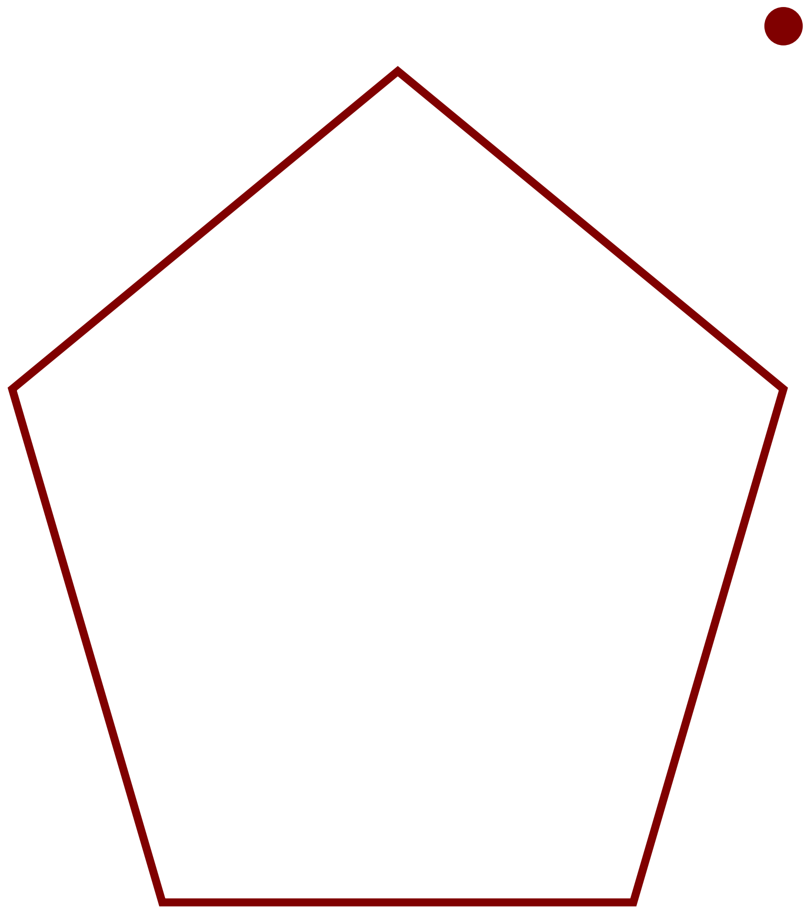

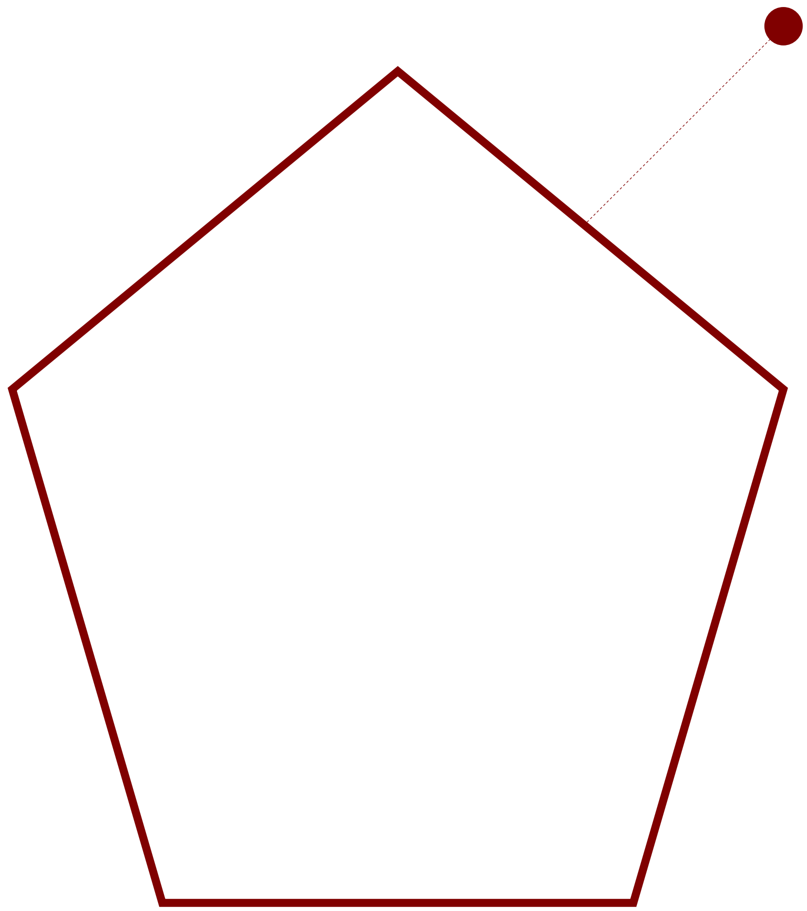

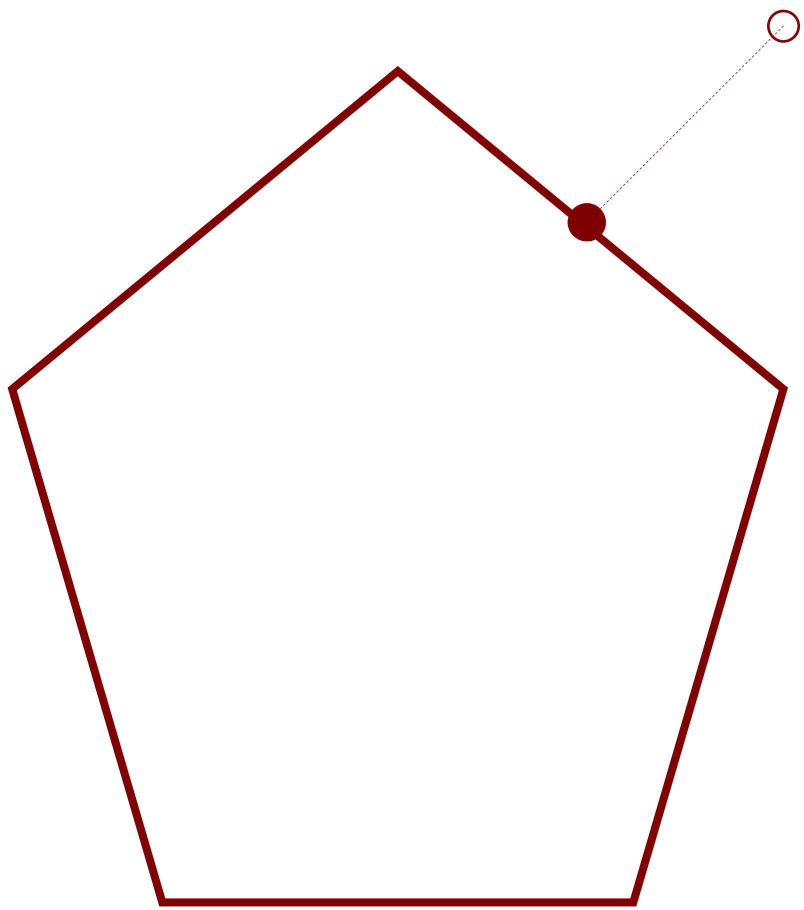

3) Theoretical Comparisons between Methods

- Convergence:-
  - LSM/RDC:- Finite
  - LDC:- Bounded
- Confidence:-
  - SD:- Not high, But may be good enough
  - LSM/RDC/LDC:- Epsilon-gap (accurate)
- Performance:-
  - SD:- Can tackle unlimited no of scenarios
  - LSM:- Patchy, best for small-to-medium no of scenarios
  - RDC:- Better than LSM on the whole
  - LDC:- Not so good for small, otherwise compares well with LSM

4.1) Mathematical Description

- First Stage Problem is:-

  Min $f(x) \equiv c^T x + E[Q(x,\xi)]$

  s.t.:- $x \in X$ (polyhedral region)

- Model Problem (MP) is:-

  Min $c^T x + \theta$

  s.t.:- $x \in X$

  $Dx + \theta \geq d$ (cuts)

4.2) Mathematical Description

Let x^0 be current iterate (from last pass)

Let ***UB***, ***LB*** be current upper, lower bounds (as in LSM)

LB is the minimum of the current MP

UB is the best value of f known so far

Let λ be a parameter:- $0 \leq \lambda < 1$

4.3) Mathematical Description

Then the new iterate x^1 is the optimal solution to:-

$$\text{Min} \quad \tfrac{1}{2}(x-x^0)^T(x-x^0)$$

$$\text{s.t.:-} \quad x \in X$$

$$Dx + \theta \geq d \quad \text{(cuts)}$$

$$c^T x + \theta \leq \lambda UB + (1-\lambda)LB$$

5) Test Models

	No.rows	No.cols	Non.zros	Scen.s
4node256	91	238	958	256
cep1	17	23	66	216
envlrge	97	98	378	8,232
FXM3T_16	331	457	2,612	256
FXM4T_16	331	457	2,612	4,096
pgp2	10	20	60	576
phone	25	93	324	32,768
pltxpAT3_16	271	732	1,830	256
pltxpAT4_16	375	1,004	2,496	4,096
pltxpAT5_6	479	1,276	3,162	1,296
pltxpAT5_16	479	1,276	3,162	65,536
pltxpAT6_6	583	1,548	3,828	7,776
pltxpAT7_6	687	1,820	4,494	46,656
stormG2_1000	714	1,380	5,045	1,000

6.1) Parameter Variation

Level decomposition
epsilon = 1e-6

	lamba = 0.25		lambda = 0.5		lamba = 0.75	
	Time	Iters	Time	Iters	Time	Iters
4node256	2.16	11	3.70	19	7.78	40
cep1	1.11	11	2.11	21	4.76	48
envlrge	28.91	2	29.02	2	28.89	2
FXM3T_16	10.06	18	12.59	23	21.02	39
FXM4T_16	144.78	18	184.81	23	304.25	39
pgp2	7.81	31	6.81	27	8.58	34
phone	70.95	4	89.50	5	145.22	8
pltxpAT3_16	2.34	2	2.30	2	2.33	2
pltxpAT4_16	65.13	2	65.47	2	65.05	2
pltxpAT5_6	29.39	2	29.36	2	29.34	2
pltxpAT6_6	957.00	5	919.08	5	724.81	4

6.2) Parameter Variation

Level decomposition
lambda = 0.5

	epsilon = 1e-4		epsilon = 1e-5		epsilon = 1e-6	
	Time	Iters	Time	Iters	Time	Iters
4node256	2.75	14	3.13	16	3.70	19
cep1	1.42	14	1.83	18	2.11	21
envlrge	28.91	2	28.94	2	29.02	2
FXM3T_16	8.45	15	11.03	20	12.59	23
FXM4T_16	120.63	15	160.98	20	184.81	23
pgp2	4.80	19	6.06	24	6.81	27
phone	89.59	5	89.53	5	89.50	5
pltxpAT3_16	2.30	2	2.30	2	2.30	2
pltxpAT4_16	65.19	2	65.09	2	65.47	2
pltxpAT5_6	29.38	2	29.36	2	29.36	2
pltxpAT6_6	923.76	5	920.92	5	919.08	5
stormG2_1000	137.45	18	143.76	24	185.08	31

6.3) Parameter Variation

L-shaped method

	epsilon = 1e-4		epsilon = 1e-5		epsilon = 1e-6	
	Time	Iters	Time	Iters	Time	Iters
4node256	1.01	5	1.03	5	1.01	5
cep1	0.33	3	0.33	3	0.33	3
envlrge	28.89	2	28.91	2	28.94	2
FXM3T_16	6.09	11	8.14	15	8.66	16
FXM4T_16	84.91	11	118.00	15	126.66	16
pgp2	8.22	33	8.47	34	8.61	34
phone	160.72	9	160.80	9	160.89	9
pltxpAT3_16	2.34	2	2.30	2	2.30	2
pltxpAT4_16	65.11	2	65.53	2	65.11	2
pltxpAT5_6	29.19	2	29.20	2	29.20	2
pltxpAT6_6	960.44	5	976.70	5	956.67	5
stormG2_1000	190.47	32	275.70	46	359.06	52

7) Solver Comparisons

lambda = 0.5
epsilon = 1e-6

	L-shaped method		Level Decomposition		Stochastic Decomp.	
	Time	Iters	Time	Iters	Time	Iters
4node256	1.01	5	3.70	19	2.39	300
cep1	0.33	3	2.11	21	1.16	300
envlrge	28.94	2	29.02	2		
FXM3T_16	8.66	16	12.59	23	6.20	304
FXM4T_16	126.66	16	184.81	23	6.33	306
pgp2	8.61	34	6.81	27	1.27	300
phone	160.89	9	89.50	5	0.89	301
pltxpAT3_16	2.30	2	2.30	2	5.36	300
pltxpAT4_16	65.11	2	65.47	2	7.34	300
pltxpAT5_6	29.20	2	29.36	2	11.09	300
pltxpAT6_6	956.67	5	919.08	5	14.95	300
stormG2_1000	359.06	52	185.08	31	10.53	300

Conclusions

- LDC shows better times than than LSM – especially for larger/harder problems
- LDC offers extensions to:-
  - Regularise infeasibility cuts
  - Progressive approximation of the distribution
- LDC offers better solution confidence than SD – but SD is faster & can tackle indefinite nos of scenarios