ON MARKOV ERROR-CORRECTION MODELS

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Abstract

This paper considers Markov error-correction (MEC) models in which deviations from the long-run equilibrium follow a Markov-switching autoregressive process which is nonstationary in one state of nature and mean-reverting in the other. In order to establish the existence of MEC adjustment, we suggest testing for cointegration first and then examining the equilibrium error or the error-correction model for signs of Markov-switching behaviour. Monte Carlo experiments demonstrate that standard cointegration tests based on linear models remain useful when the error-correction representation is subject to Markov switching. Furthermore, tests for parameter instability and neglected nonlinearity are capable of revealing the invalidity of the assumption of linear adjustment. The performance of a model selection procedure based on an information criterion is also investigated. As an empirical illustration of these ideas, we analyze the long-run properties of US stock prices and dividends.

Key Words: Cointegration; Error correction; Markov chain; Model selection; Nonlinearity; Parameter instability.

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1 Introduction

Following the seminal work of Engle and Granger (1987), a phenomenal amount of theoretical and empirical research has been devoted to the analysis of systems of cointegrated variables and the related issue of error-correction mechanisms. The latter may be thought of as providing an adjustment process through which deviations from a long-run equilibrium relationship (or an attractor) are corrected for. In the great majority of research papers, this adjustment towards equilibrium is implicitly assumed to be linear, which implies that it takes place at all times and is a constant portion of the disequilibrium error.

More recently, however, a number of authors have recognized that the assumption of linear adjustment is likely to be inappropriate for a variety of economic situations (e.g., in the presence of transaction costs, lumpy costs of adjustment, or policy interventions), and have discussed cointegration models in which disequilibrium adjustment occurs in a nonlinear manner. Examples include the nonlinear error-correction models discussed in Granger and Swanson (1996), threshold cointegration models (van Dijk and Franses, 1996; Balke and Fomby, 1997), smooth transition error-correction models (Anderson, 1997; Hansen and Kim, 1996; Michael et al., 1997; van Dijk and Franses, 2000), neural network error-correction models (Haefke and Helmenstein, 1996), Markov switching error-correction models (Hall et al., 1997), regime-sensitive cointegration models (Siklos and Granger, 1997), and bilinear error-correction models (Peel and Davidson, 1998).

This paper takes up the idea of Markov-type discontinuous disequilibrium adjustment introduced by Hall et al. (1997) in their study of the dynamics of house prices in the UK. Thus, we consider situations where deviations from the long-run equilibrium are stationary (so that cointegration holds) but they occasionally tend to follow a nonstationary path. During the latter periods the system behaves as if cointegration has been "switched off" and disequilibrium adjustment does not take place. Such behaviour is modelled by letting deviations from equilibrium follow a nonlinear process which is second-order stationary but subject to discrete Markov switching between two regimes that are respectively characterized by stationary and unit-root dynamics. We shall refer to this type of disequilibrium adjustment as 'Markov error correction' (MEC).

The motivation behind the notion of MEC derives from the economically plausible situation where major events, or changes in technology, in government policy or in some important institutional feature of the economy, can interrupt temporarily the adjustment towards an underlying long-run equilibrium. Like bilinear error-correction mechanisms, MEC models provide a flexible stochastic structure for describing situations involving fairly abrupt changes or bubble-like behaviour (cf. Hall et al., 1997), situations which might not be adequately described by models with smooth transitions or threshold effects. MEC is also related to the concept of regime-sensitive cointegration proposed by Siklos and Granger (1997) which allows for an equilibrium relationship to exist but not over the whole sample period of interest. Our specification of the mechanism governing transitions between the stationary and unit-root regimes as a Markov process has the obvious advantage of requiring no a priori information about the location of the shifts in regime, letting the data select when and where these shifts occur.

The aim of the present paper is to study MEC models in detail and, in particular, to propose ways in which the presence or otherwise of MEC adjustment in the data may be established in practice. The next section of the paper introduces the MEC model and discusses some of its probabilistic properties. In Section 3, Monte Carlo experimentation is used to examine whether a multi-step procedure, based on a combination of tests for cointegration, parameter instability and neglected nonlinearity, is capable of detecting the presence of MEC adjustment. The properties of a model selection procedure based on a popular complexity-penalized likelihood criterion are also investigated. In Section 4, we discuss an empirical application which examines whether the long-run relationship between historical US stock prices and dividends can be described by a MEC model. Finally, in Section 5, we present our conclusions and suggest some other areas where we expect MEC models to be useful.

2 Markov Error Correction

2.1 A simple model

To illustrate the concept of MEC, consider the following simple model for the bivariate time series $\{(y_t, x_t)^\intercal : t \in \mathbb{N}\}$,

$$y_t + \alpha x_t = z_t, \qquad z_t = \phi_{s_t} z_{t-1} + \varepsilon_{1t}, \tag{1}$$

$$y_t + \beta x_t = u_t, \qquad u_t = u_{t-1} + \varepsilon_{2t}, \tag{2}$$

where $\alpha \neq 0$, $\beta \in \mathbb{R}$, $\phi_{s_t} \in (-1, 1]$, $\{\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, \varepsilon_{2t})^\intercal\}$ is a white-noise process with zero mean and positive-definite covariance matrix, and $\{s_t\}$ are latent random variables on $\{0, 1\}$, independent of $\{\boldsymbol{\varepsilon}_t\}$, which indicate the state (or regime) that the system is in at date t. This is a simple cointegrated system where $\{y_t\}$ and $\{x_t\}$ are both integrated processes of order 1 and equation (1) defines a cointegrating relationship between y_t and x_t, provided that $\{z_t\}$ is stationary (cf. Engle and Granger, 1987).

Like Balke and Fomby (1997) and Hall et al. (1997), we consider a situation where adjustment towards the long-run equilibrium determined by $y + \alpha x = 0$ does not necessarily takes place at each date t but only during periods that are associated with one of the two regimes. It is assumed that nature selects state at date t with a probability that depends on what state the system was in at date $t - 1$. In particular, it is assumed that

$$\phi_{s_t} = \phi_0 + (\phi_1 - \phi_0)s_t, \qquad |\phi_0| < 1, \qquad \phi_1 = 1, \tag{3}$$

where $\{s_t\}$ is a homogeneous, irreducible, and aperiodic Markov chain of order 1 with state space $\mathbb{S} = \{0, 1\}$ and transition probabilities

$$p_{ij} = \Pr\{s_t = j | s_{t-1} = i\}, \qquad i, j \in \mathbb{S}. \tag{4}$$

The time series $\{z_t\}$ obeys, therefore, a model which allows the dynamic behaviour of the series to be governed by either a stable first-order stochastic difference equation or a random walk scheme, depending on the realized value of the state indicator s_t. Consequently, deviations from equilibrium tend to decay to the mean level of zero as long as $s_t = 0$; otherwise, z_t behaves like a nonstationary process, and there is no tendency for the system in (1)–(2) to move towards equilibrium.

An alternative characterization of such state-dependent behaviour may be given in terms of the error-correction representation of the system in (1)–(2). The latter can be written as

$$\Delta y_t = \beta(\alpha - \beta)^{-1}(1 - \phi_{s_t})z_{t-1} + v_{1t}, \tag{5}$$

$$\Delta x_t = -(\alpha - \beta)^{-1}(1 - \phi_{s_t})z_{t-1} + v_{2t}, \tag{6}$$

where $v_{1t} = (\alpha\varepsilon_{2t} - \beta\varepsilon_{1t}) / (\alpha - \beta)$, $v_{2t} = (\varepsilon_{1t} - \varepsilon_{2t}) / (\alpha - \beta)$, and Δ is the differencing operator defined by $\Delta w_t = w_t - w_{t-1}$. In (5)–(6), $z_{t-1} = y_{t-1} + \alpha x_{t-1}$ represents deviation from long-run equilibrium at date $t-1$, while the coefficients on z_{t-1} measure the strength of short-run disequilibrium adjustment. Clearly, correction for past disequilibrium only takes place when $s_t = 0$ (and $\alpha > 2\beta$).

At this point, some remarks on the properties of the 'equilibrium error' $\{z_t\}$ are in order. It is clear from the Markov specification in (3)–(4) that $\{z_t\}$ is 'locally' nonstationary since $\phi_{s_t} = 1$ in the state characterized by $s_t = 1$. However, second-order stationarity of an autoregressive process with Markov regimes does not necessarily require the characteristic polynomial of the process to have all its zeros lying on the open unit disk. Hence, despite the occasional nonstationary behaviour of $\{z_t\}$ (when $s_t = 1$), the equilibrium error can be 'globally' stationary, provided that p_{00}, p_{11}, ϕ_0 and ϕ_1 satisfy appropriate restrictions. Unfortunately, the determination of necessary and sufficient conditions for second-order or strict stationarity of an autoregressive process with Markov switching parameters is still an open question. When the underlying Markov chain is strictly stationary, a sufficient condition for second-order stationarity is given in Karlsen (1990) and Holst et al. (1994). For the equilibrium error $\{z_t\}$ that evolves according to (1), (3) and (4), this stationarity condition requires that the eigenvalues of the matrix

$$\mathbf{\Lambda}_1 = \begin{bmatrix} p_{00}\phi_0^2 & p_{10}\phi_0^2 \\ p_{01}\phi_1^2 & p_{11}\phi_1^2 \end{bmatrix}$$

lie on the open unit disk. This is equivalent to:

$$\begin{aligned} p_{00}\phi_0^2 + p_{11}\phi_1^2 - (p_{00} + p_{11} - 1)\phi_0^2\phi_1^2 &< 1, \\ -p_{00}\phi_0^2 - p_{11}\phi_1^2 - (p_{00} + p_{11} - 1)\phi_0^2\phi_1^2 &< 1, \\ |(p_{00} + p_{11} - 1)\phi_0^2\phi_1^2| &< 1. \end{aligned} \tag{7}$$

It can be easily verified that, for an irreducible and aperiodic Markov chain $\{s_t\}$, the conditions in (7) are always satisfied when $|\phi_0| < 1$ and $\phi_1 = 1$, in which case $\{(y_t, x_t)^\intercal\}$ admits a MEC representation.[1] It follows, of course, that our characterization of MEC models implies that cointegration between y_t and x_t is a global property that is guaranteed by the second-order stationarity of the equilibrium error $\{z_t\}$. Yet, as long as $s_t = 1$, y_t and x_t do not respond to deviations from the long-run equilibrium. These are properties that the MEC model shares with the threshold model of Balke and Fomby (1997), although in the latter transitions between regimes are governed by the threshold variable z_{t-1} rather than by an exogenous Markov process.

[1] Note also that these conditions are much weaker than the naive condition $\max\{|\phi_0|, |\phi_1|\} < 1$ which excludes processes that occasionally follow nonstationary or explosive paths.

2.2 More General Models

The simple model of the previous subsection may be generalized by specifying richer dynamics for the equilibrium error $\{z_t\}$. For example, we could allow $\{z_t\}$ to evolve according to a Markov switching autoregression where the intercept, coefficients and innovation variance are all state-dependent, i.e.,

$$z_t = \mu_{s_t} + \sum_{j=1}^{m} \phi_{s_t}^{(j)} z_{t-j} + \sigma_{s_t}\eta_t, \tag{8}$$

$\{\eta_t\}$ being a white-noise process, independent of $\{s_t\}$, with $\mathsf{E}(\eta_t) = \mathsf{E}(\eta_t^2 - 1) = 0$. The parameters in (8) can be such that a second-order stationary solution exists even though $1 - \sum_{j=1}^{m} \phi_{s_t}^{(j)} = 0$ for $s_t = 1$, so that $(1, -\alpha)$ would still be a cointegrating vector for $(y_t, x_t)^\mathsf{T}$. This would indeed be true if all the eigenvalues of the $2m^2 \times 2m^2$ matrix

$$\mathbf{\Lambda}_m = \begin{bmatrix} p_{00}(\mathbf{\Phi}_0 \otimes \mathbf{\Phi}_0) & p_{10}(\mathbf{\Phi}_0 \otimes \mathbf{\Phi}_0) \\ p_{01}(\mathbf{\Phi}_1 \otimes \mathbf{\Phi}_1) & p_{11}(\mathbf{\Phi}_1 \otimes \mathbf{\Phi}_1) \end{bmatrix}$$

lay on the open unit disk, where

$$\mathbf{\Phi}_i = \begin{bmatrix} \phi_i^{(1)} & \phi_i^{(2)} & \cdots & \phi_i^{(m-1)} & \phi_i^{(m)} \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \qquad i \in \mathbb{S},$$

and $\phi_{s_t}^{(j)} = \phi_i^{(j)}$ if $s_t = i$ ($j = 1, \ldots, m$; $i \in \mathbb{S}$); cf. Holst et al. (1994).

Another useful extension involves abandoning the assumption of homogeneity for the Markov chain $\{s_t\}$. In some applications, it is reasonable to expect that the further away from equilibrium the system is, the higher is the probability of switching from an unstable non-correcting regime to a stable error-correcting one. This suggests allowing the transition probabilities of the hidden Markov chain to depend on the extend to which the system is out of long-run equilibrium. The transition mechanism of $\{s_t\}$ may, therefore, be specified as

$$\begin{aligned} &\Pr\{s_t = i | s_{t-1} = i, z_{t-1}\} = \exp(a_i + b_i z_{t-1})/[1 + \exp(a_i + b_i z_{t-1})], \quad i \in \mathbb{S}, \\ &\Pr\{s_t = j | s_{t-1} = i, z_{t-1}\} = 1 - \Pr\{s_t = i | s_{t-1} = i, z_{t-1}\}, \quad i, j \in \mathbb{S}, \quad i \neq j, \end{aligned} \tag{9}$$

where a_i and b_i ($i \in \mathbb{S}$) are real constants.[2] Unfortunately, conditions that guarantee second-order stationarity of $\{z_t\}$ when $\{s_t\}$ is a nonhomogeneous Markov chain are currently unknown.

3 Detecting MEC Adjustment

In this section, we examine how one may detect MEC adjustment. Within a hypothesis testing framework, it would appear natural to consider testing the null hypothesis of single-regime/no-

[2] The potential applicability of an error-correction model based on the assumption in (9) is illustrated in Hall et al. (1997).

cointegration against the alternative cointegration with MEC adjustment. However, this problem is fraught with difficulties similar to those outlined by Balke and Fomby (1997) for models of threshold cointegration. Specifically, the testing problem is non-standard due to the presence of unit roots and the unidentifiability of the transition probabilities under the null hypothesis.

For this reason, we follow Balke and Fomby (1997) in suggesting using a multi-step test procedure that exploits the differences between the global and local characteristics of systems with MEC adjustment. In particular, since for a cointegrated time series with MEC adjustment the equilibrium error $\{z_t\}$ is globally stationary, conventional tests for (linear) cointegration are likely to be useful in establishing the long-run properties of the series. Once cointegration is found, then the equilibrium error or the associated error-correction model may be investigated for signs of local nonlinear Markov switching behaviour. For the latter type of analysis, tests for parameter instability or tests for nonlinearity may be employed. Alternatively, likelihood-based model selection criteria may be used to choose between models with linear and Markov switching dynamics. The usefulness of these procedures is examined in turn in the remainder of this section using Monte Carlo experimentation.

3.1 Cointegration Tests

As pointed out before, the equilibrium error $\{z_t\}$ in (1) or (8) is second-order stationary in the presence of MEC adjustment, and hence the order of integration of $\{y_t\}$ and $\{x_t\}$ in (1)–(2) is not affected. In addition, $\{z_t\}$ is known to be strong mixing with geometrically decreasing mixing coefficients (Karlsen, 1990). It follows, therefore, that standard unit root tests for $\{y_t\}$ and $\{x_t\}$ will be asymptotically valid, and should be capable of revealing the nonstationarity of the observed series. Moreover, the cointegrating vector $(1, -\alpha)$ will be super-consistently estimated by ordinary least squares, and hence conventional residual-based cointegration tests constructed under the assumption of linear adjustment towards equilibrium (cf. Phillips and Ouliaris, 1990) will still be valid, and can be expected to be able to detect the presence of an equilibrium relationship.

In our analysis below, we focus on some cointegration tests that have proved to be popular in practice. Three such tests are the residual-based Augmented Dickey–Fuller (ADF) and $\widehat{Z}_\alpha$ and $\widehat{Z}_t$ tests discussed in Phillips and Ouliaris (1990). These are all tests for the null hypothesis of no cointegration, and are based on the ordinary least-squares residuals from the static regression of y_t on x_t. In our implementation of the tests, the order of the AR(k) model on which the ADF test is based is selected by minimizing Akaike's (1973) information criterion (AIC) over $k \in \{1, 2, \ldots, \lfloor 4(T/100)^{1/4} \rfloor + 1\}$, where T is the sample size and $\lfloor \cdot \rfloor$ denotes the greatest-integer function. The $\widehat{Z}_\alpha$ and $\widehat{Z}_t$ tests are used in conjunction with a prewhitened kernel estimator for the long-run innovation variance based on the Parzen kernel function and an automatic plug-in bandwidth (see Andrews and Monahan, 1992). For all three tests, a constant term is included in the test regressions.

We also consider tests of the null hypothesis of no cointegration which are based on vector autoregressive models for $\{(y_t, x_t)^\intercal : t = 1, 2, \ldots, T\}$. These include Stock and Watson's (1988) minimum-eigenvalue test based on their statistic $q_c^\mu(2, 1)$ (denoted below by SW) and Johansen's (1991) trace and maximal-eigenvalue likelihood ratio tests (denoted by LR_{trace} and $LR_{\max}$, respectively). For the latter, the order k (say) of the vector autoregressive model used is determined by minimizing the AIC over $k \in \{1, 2, \ldots, \lfloor 4(T/100)^{1/4} \rfloor + 1\}$. The SW test is based on a corrected first-order sample autocorrelation matrix for $(y_t, x_t)^\intercal$, where the correction term is estimated nonparametrically using a prewhitened kernel estimator, the Parzen kernel function and a data-dependent automatic bandwidth (Andrews and Monahan, 1992). For both types of tests, a constant is included unrestrictedly in the vector autoregressive equations.

In our simulation experiments, the bivariate system in (1)–(2) is used as the data-generating process, with $\{s_t\}$ being an ergodic Markov chain on $\{0, 1\}$, and $\{\boldsymbol{\varepsilon}_t\}$ being independent Gaussian random vectors with $\mathsf{E}(\varepsilon_{it}) = \mathsf{E}(\varepsilon_{it}^2 - 1) = 0$ $(i = 1, 2)$ and $\mathsf{E}(\varepsilon_{1t}\varepsilon_{2t}) = \rho$. The experiments are a full factorial design of:

$$\alpha = -2, \qquad \beta \in \{-3, 0\}, \qquad \phi_0 \in \{0, 0.7\}, \qquad \phi_1 = 1, \qquad \rho \in \{0, -0.5\},$$

$$(p_{00}, p_{11}) \in \{(0.9, 0.9), (0.98, 0.98), (0.98, 0.9), (0.9, 0.98), (0.5, 0.5)\}, \qquad T \in \{50, 100, 200, 500\}.$$

The first two pairs of transition probabilities, $(0.9, 0.9)$ and $(0.98, 0.98)$, allow for symmetry in the persistence of the two regimes, the expected duration of each regime being much longer when $p_{00} = p_{11} = 0.98$. The probabilities $(p_{00}, p_{11}) = (0.98, 0.9)$, on the other hand, imply that the regime that corresponds to $s_t = 1$ is considerably less persistent than the regime that corresponds to $s_t = 0$; the opposite is true when $(p_{00}, p_{11}) = (0.9, 0.98)$. Finally, the regime indicators $\{s_t\}$ are uncorrelated when $(p_{00}, p_{11}) = (0.5, 0.5)$ so that z_t is independent of the state that prevailed at date $t - 1$. Finally, notice that setting $\beta = 0$ implies that y_t does not react to deviations from long-run equilibrium since an error-correction mechanism is only present in the equation for x_t.

In all the experiments carried out in this paper, $50 + T$ data points for (y_t, x_t) are generated according to (1)–(4) by setting $z_0 = u_0 = 0$ but, in order to attenuate the effect of the initial values, only the last T data points are used for estimation and testing purposes. Unless otherwise stated, the number of Monte Carlo replications per experiment is 2,500.

Table 1A gives Monte Carlo estimates of the Type-I error probability of 0.05-level cointegration tests in the case where $\beta = 0$ and $\rho = 0$ (the results for $\beta = -3$ are very similar and are therefore omitted in order to save space).[3] Most of the tests have empirical rejection probabilities that are close to the correct rate, the exception being the Johansen tests. The latter tend to be somewhat liberal when the sample size is small, and this must be borne in mind when interpreting results obtained under cointegration.

The empirical rejection probabilities of the tests when cointegration with MEC holds are reported in Table 1B.[4] It is evident that, although the equilibrium error follows a nonstationary

[3] The results for tests at the 0.01 and 0.10 level of significance are qualitatively similar and do not affect the conclusions about the relative merits of different tests. Critical values are taken from Phillips and Ouliaris (1990, Table IIb) for the ADF and $\widehat{Z}_t$ tests, from Phillips and Ouliaris (1990, Table Ib) for the $\widehat{Z}_\alpha$ test, from Stock and Watson (1988, Table 2) for the SW test, and from Osterwald-Lenum (1992, Table 1.1*) for the LR_{trace} and $LR_{\max}$ tests.

[4] In order to reflect empirical practice, power calculations throughout the paper are made using asymptotic critical values rather than finite-sample critical values estimated from Monte Carlo experiments where the relevant null hypothesis holds.

path occasionally, the tests are generally quite powerful to detect the presence of cointegration. This is especially true when the state indicators $\{s_t\}$ are not correlated, a finding which is perhaps unsurprising since the frequent state transitions that take place when $p_{00}+p_{11}=1$ tend to make the equilibrium error look very much like white noise with large variance. For data-generating processes with $p_{00}+p_{11}>1$, a comparison across different values of the transition probabilities reveals that the probability of correctly detecting cointegration is higher: (a) the more persistent the regime where short-run disequilibrium adjustment takes place is relative to the regime where no such adjustment occurs, and (b) the more observations correspond to the error-correcting regime $s_t=0$ (as is indeed the case when $p_{00}=0.98$ and $p_{11}=0.9$). Also, for any given pair of transition probabilities, the power of the tests rises as the strength of disequilibrium adjustment in the regime corresponding to $s_t=0$ increases (or, equivalently, as the autoregressive coefficient ϕ_0 decreases). This makes it difficult for the tests to detect cointegration with MEC when the equilibrium error is relatively persistent and the sample size is small.

Turning to the individual tests, the $\widehat{Z}_\alpha$ and $\widehat{Z}_t$ tests always have higher empirical rejection probabilities than the ADF test, presumably because the nonparametric autocorrelation corrections that the former two tests employ are more successful in accounting for Markov dynamics than the autoregressive approximations on which the ADF test is based.[5] Among system-wide tests, the SW test is generally more powerful than the LR_{trace} and $LR_{\max}$ tests, although in most cases the power differences are not very substantial.

Repeating the experiments with $\rho=-0.5$ revealed that the correlation between the innovations of the equilibrium error and the stochastic trend does not contribute to any significant changes in the power of the tests in large samples. For small and moderately-sized samples, the ADF, $\widehat{Z}_\alpha$ and $\widehat{Z}_t$ tests suffer a small decrease in power relative to the case with $\rho=0$, while the opposite is true for the LR_{trace} and $LR_{\max}$ tests.

In summary, our analysis shows that conventional cointegration tests based on the assumption of a linear adjustment process are capable of detecting the presence of a equilibrium relationship between MEC time series.

3.2 Parameter Instability Tests

As explained in Section 2, if y_t and x_t are cointegrated with MEC adjustment, the equilibrium error $\{z_t\}$ will obey a Markov switching autoregression and an error-correction model like (5)–(6) will have coefficients which are subject to Markov changes. Such time-varying behaviour raises the question of whether tests for parameter instability might be capable of revealing any evidence of nonconstancy in the coefficients of either the error-correction model or an autoregressive model for the equilibrium error. This subsection investigates the performance of some tests of this type, again using Monte Carlo simulation.

In our analysis, we consider variants of two classes of tests for parameter instability, namely tests for stochastic parameter variation and tests for a single structural break at an unknown time. A test of the former type is based on Nyblom's (1989) L statistic, which provides a

[5] These findings are similar to the results of van Dijk and Franses (1996) and Balke and Fomby (1997) who found the $\widehat{Z}_\alpha$ and $\widehat{Z}_t$ tests to be more powerful than the ADF test in the presence of threshold cointegration.

locally most powerful test for the null hypothesis of parameter stability against the alternative of martingale parameter variation (the statistic is modified as in Hansen (1992a) to achieve robustness with respect to heteroskedasticity). The second type of tests are based on functionals of the sequence of likelihood ratio statistics which test the null hypothesis of parameter stability against the alternative of an one-time break at all possible break-points in the sample. Following Andrews (1993) and Andrews and Ploberger (1994), we consider the statistics

$$SupLR = \max_{\tau \in \mathbb{T}} LR(\tau), \qquad AvgLR = (\lfloor \tau_1 T \rfloor - \lfloor \tau_0 T \rfloor + 1)^{-1} \sum_{\tau \in \mathbb{T}} LR(\tau),$$

$$ExpLR = \ln \left[(\lfloor \tau_1 T \rfloor - \lfloor \tau_0 T \rfloor + 1)^{-1} \sum_{\tau \in \mathbb{T}} \exp\{\tfrac{1}{2} LR(\tau)\} \right],$$

where $\mathbb{T} = \{\lfloor \tau_0 T \rfloor / T, (\lfloor \tau_0 T \rfloor + 1)/T, \ldots, \lfloor \tau_1 T \rfloor / T\}$ and $LR(\tau)$ denotes the likelihood ratio statistic for testing for a change at date $t = \tau T$. The tests are implemented with $\tau_0 = 1 - \tau_1 = 0.15$.

The experimental design is the same as in subsection 3.1. For computational convenience, we set $\beta = 0$ in the data-generating process so only the error-correction equation (6) is considered in the simulations. Further, in order to reflect empirical practice, we treat the cointegrating vector as unknown and estimate it from the data. Thus, in each Monte Carlo replication, we proceed according to the following two-step procedure. First, we obtain an estimate $(1, \widehat{\alpha})$ of the cointegrating vector $(1, \alpha)$ by means of the fully-modified least squares method of Phillips and Hansen (1990); long-run covariance matrices are estimated as in Andrews and Monahan (1992), using a prewhitened kernel estimator, the Parzen kernel function and a data-dependent automatic bandwidth. Second, we test for parameter instability in: (a) an AR(1) model for $\widehat{z}_t = y_t + \widehat{\alpha} x_t$ (model M_1); (b) an error-correction model like (6) with z_{t-1} replaced by $\widehat{z}_{t-1}$ (model M_2); a constant is included in both models.

The top panel in Tables 2A–2B gives Monte Carlo estimates of the empirical rejection probabilities of 0.05-level tests for models M_1 and M_2 when $\rho = 0$ and the null hypothesis of parameter constancy is true (i.e. $\phi_1 = \phi_0$, $|\phi_0| < 1$).[6] Tests in the error-correction model M_2 generally have rejection probabilities that are not very different from the 0.05 nominal value. This is not the case in model M_1 where all tests tend to be somewhat liberal even for relatively large sample sizes.

Turning to the rejection probabilities of the tests in the presence of MEC adjustment, also shown in Tables 2A–2B, it is obvious that the performance of the tests is disappointing when $p_{00} = p_{11} = 0.5$. In this case, there are frequent transitions between the two regimes, and the power of the tests suffers as a result. Fortunately, however, matters improve considerably when $p_{00} + p_{11} > 1$. In model M_1, tests other than L are capable of detecting parameter non-constancy, especially when the change in the autoregressive parameter is relatively large (i.e., $\phi_0 = 0$). The rejection probabilities of the tests are lower in the error-correction model M_2, presumably due to

[6] Results for $\rho = -0.5$ are very similar to those obtained with $\rho = 0$, and are not, therefore, reported. Asymptotic critical values are taken from Hansen (1992a, Table 1) for the L test, from Andrews (1993, Table 1) for the $SupLR$ test, from Andrews and Ploberger (1994, Table II) for the $AvgLR$ test, and from Andrews and Ploberger (1994, Table I) for the $ExpLR$ test.

the fact that the changes in the coefficient of the error-correction mechanism that are implied by our data-generating process are small (this coefficient switches between 0 and 0.5 when $\phi_0 = 0$ and between 0 and 0.15 when $\phi_0 = 0.7$). Finally, the $ExpLR$ and $SupLR$ tests are consistently more successful than the $AvgLR$ and L tests. The latter is very weak compared to the other tests, which is not perhaps surprising since the test is designed for situations where there is a relatively constant likelihood of parameter variation throughout the sample.

3.3 Tests for Neglected Nonlinearity

As demonstrated in the previous subsection, once the presence of an equilibrium relationship has been established, tests for parameter instability can be useful in detecting Markov switching behaviour in the adjustment process. Further useful information about the adequacy or otherwise of a linear adjustment process may be obtained from application of tests for neglected nonlinearity in the relevant error-correction model or in an autoregressive model for the equilibrium error.

In this subsection, we investigate the properties of such tests in the context of the linear models M_1 (AR(1) model for $\widehat{z}_t$) and M_2 (error-correction model for x_t) considered in the experiments of subsection 3.2. Each of these models is tested for neglected nonlinearity using some general tests that have been shown to have respectable power in the presence of Markov regime switching (see Psaradakis and Spagnolo, 1999). These include: (i) the modified regression equation specification error test ($RESET$) of Thursby and Schmidt (1977), with powers of regressors up to 4; (ii) the BDS test of Brock et al. (1996), with embedding dimension equal to 2 and metric bound equal to the standard deviation of the estimated residuals; (iii) the "WHITE3" dynamic information matrix test discussed in Lee et al. (1993) (denoted below by WHT); (iv) the "NEURAL2" neural network test of Lee et al. (1993), based on a logistic squashing function and the second and third largest principal components of 20 randomly generated unit signals (denoted by NNT).

Tables 3A–3B report the empirical rejection probabilities of 0.05-level nonlinearity tests for data-generating processes identical to those considered in subsection 3.2, with $\beta = \rho = 0$.[7] With the exception of the BDS test, nonlinearity tests reject at the correct rate when the null hypothesis of linearity is true ($\phi_1 = \phi_0$), even for the smaller sample sizes. In the presence of Markov cointegration, tests for neglected nonlinearity in model M_1 perform poorly when $p_{00} = p_{11} = 0.5$, with only the BDS test being capable of rejecting the linear model (although the rejection probabilities for the two smaller sample sizes is misleading since the test tends to over-reject under the null). Allowing the chain $\{s_t\}$ to be fairly persistent leads to a considerable improvement in the performance of the tests, especially when $T \geqslant 200$. In these cases, the $RESET$ and WHT tests perform best overall but test power remains low for small differences between ϕ_1 and ϕ_0.

The overall picture is the same for tests applied to model M_2. Here, however, test rejection frequencies are generally lower than those obtained for model M_1, presumably because Markov nonlinearity is less prominent in the error-correction model as a result of the shifts in coefficients

[7] Very similar results were obtained with $\rho = -0.5$.

being relatively small.

3.4 A Test for Markov Switching

The previous two subsections have demonstrated that tests for parameter instability and neglected nonlinearity can provide useful insights into the validity or otherwise of the assumption of a linear adjustment process. It should be appreciated, however, that such tests are not designed to test departures from linearity or stability in the direction of Markov switching models, and they are reasonably powerful against several types of nonlinearity and parameter nonconstancy. Thus, in the absence of additional information, rejection of the hypothesis of stability or linearity on the basis of these tests cannot be necessarily accepted as evidence in favour of Markov switching behaviour.

In the face of these difficulties, it is clearly desirable to complement parameter instability and nonlinearity tests with procedures which directly test the hypothesis of linear adjustment towards equilibrium against a Markov alternative. Unfortunately, this testing problem is nonstandard in that the transition probabilities are unidentified and scores are identically zero under the null hypothesis of linearity, thus violating conventional regularity conditions for likelihood-based inference. Hansen (1992b) proposed a general theory for testing under such non-standard conditions. By viewing the likelihood function as an empirical process of the unknown parameters, a bound for the asymptotic distribution of a suitably standardized likelihood ratio statistic can be obtained. This asymptotic distribution is generally non-standard, but an approximation to it may be obtained via simulation. The difficulty with this method is that it involves evaluation of the likelihood function across a grid of different values for the transition probabilities and for each state-dependent parameter and hence is extremely computationally intensive.[8]

The data-generating process for the simulations is similar to that used in the previous two subsections, although, due to the exceptionally high computational cost of the experiments, only the following parameter values and sample sizes are considered:

$$\alpha = -2, \qquad \beta = 0, \qquad \phi_0 \in \{0, 0.7\}, \qquad \phi_1 = 1, \qquad \rho = 0,$$

$$(p_{00}, p_{11}) \in \{(0.5, 0.5), (0.9, 0.9), (0.98, 0.9)\}, \qquad T \in \{50, 100, 200\}.$$

For each design point, we test model M_1 (i.e. one-state AR(1) model for $\widehat{z}_t$) and model M_2 (i.e. one-state error-correction model for x_t) of subsections 3.2 and 3.3 against corresponding Markov alternatives (i.e. two-state Markov switching models), using Hansen's (1992b) standardized likelihood ratio statistic. In both cases, a constant is also included in the models but, in order to reduce the computation times, it is assumed to be state-independent. For the calculations, we use for grid for the state-dependent coefficients the range $[0.01, 1.01]$ in steps of 0.1 (11 gridpoints), while the range $[0.50, 0.95]$ in steps of 0.05 (10 gridpoints) is used for the transition probabilities. The asymptotic p-values of the tests are calculated according to the method described in Hansen (1996), using 1,000 random draws from the relevant limiting Gaussian processes and bandwidth parameter $M \in \{0, 1, \ldots, 4\}$.

[8] An alternative approach to this difficult testing problem is discussed in Garcia (1998), but his method is theoretically less attractive since it overlooks the problem of identically zero scores.

Table 4 records the empirical rejection probabilities of the tests (calculated as the fraction of 500 Monte Carlo trials in which the test p-value was less than or equal to 0.05). It is clear that, when testing the AR(1) model for $\widehat{z}_t$, the likelihood ratio test is fairly powerful if the Markov chain is persistent and $\phi_0 = 0$. However, the test has virtually no power to detect Markov switching behaviour when the difference between ϕ_1 and ϕ_0 is small (the problem is, of course, exacerbated by the fact that our test procedure uses asymptotic p-values which are only an upper bound for the true p-values and hence the test tends to be conservative). A similar picture emerges when testing the error-correction model for x_t, in which case the rejection probabilities are lower than those obtained for model M_1 (due to the fact that the changes in the coefficient of the error-correction mechanism that are implied by our data-generating process are small). Finally, as with parameter instability tests, the Hansen test is more successful in detecting Markov switching the more autocorrelated the hidden regime indicators are.

3.5 A Model Selection Approach

An alternative way of distinguishing between cointegration models with linear adjustment and cointegration models with MEC adjustment is by comparing the rival models on the basis of a complexity-penalized likelihood criterion (e.g., the AIC). Procedures based on such criteria have enjoyed much popularity in statistics as a means of choosing among competing models of an empirical phenomenon and, under appropriate conditions, they are known to be capable of consistently selecting the model with lowest Kullback–Leibler divergence from the data-generating process (see Nishii, 1988; Sin and White, 1996). Furthermore, as Granger et al. (1995) point out, these methods are arguably more appropriate for model selection than procedures based on formal hypothesis testing, partly because, unlike testing, they do not favour unfairly the model chosen to be the null hypothesis. This last point is particularly important in the case of MEC adjustment since all the procedures considered in subsections 3.2–3.4 have a linear cointegration model as a null hypothesis. To make matters worse, one of the tests (i.e. the Hansen test) is conservative by construction, thus further favouring linear models.

In this subsection, the finite-sample performance of a selection procedure based on the popular AIC criterion is investigated by simulation under model (1)–(4) with $\beta = 0$. In each Monte Carlo replication, we calculate the value of the AIC for the linear models M_1 and M_2 of subsections 3.2–3.5 (which include a constant) and the corresponding Markov models (with switching intercept and slope).

The empirical probabilities of correctly selecting the Markov two-state models instead of the corresponding single-state specifications are reported in Tables 5A–5B. In the case of autoregressive models for $\widehat{z}_t$, the AIC performs extremely well when $\phi_0 = 0$ and $T \geqslant 100$. For the error-correction model for Δx_t, roughly the same picture emerges, although the AIC is somewhat less successful in selecting the right model, especially when $p_{00} + p_{11} = 1$. As with all previous procedures, difficulties are encountered when $\phi_0 = 0.7$, but even in these cases one is more likely to arrive at the correct conclusion about the presence of MEC adjustment using the AIC than the nonlinearity tests of the type discussed in the last two subsections. There appear, therefore, to be good reasons for using a model selection procedure as a further means of establishing the presence or otherwise of Markov-type nonlinearity in the adjustment process.

4 An Empirical Example

Since the influential paper of Campbell and Shiller (1987), it has been well established in the literature that US stock prices and dividends are linked by a linear long-run relationship. In the corresponding error-correction model, adjustment towards long-run equilibrium is also considered to be linear and a constant portion of the disequilibrium error throughout the sample. There are, however, numerous reasons for questioning the validity of the latter assumption. For instance, the presence of collapsing bubbles in prices (e.g., Froot and Obstfeld, 1991) may change or interrupt temporarily the adjustment towards the underlying long-run equilibrium. Other possible explanations for such departures from the standard model have been analyzed by Shiller (1989), including time-varying discount factor and fads, to mention but a couple. In this section, we investigate the possibility that the relationship between stock prices and dividends can be characterized by MEC adjustment.

In the sequel, p_t and d_t respectively denote the annual Standard and Poor's composite real stock price index and real dividends per share. It is evident from Figure 1, which plots p_t and d_t over the period 1900–1992,[9] that prices and dividends are upward trending during the sample period and that for some short periods in the sample (during the sixties, the seventies and the late eighties) the behaviour of prices does not seem to reflect the behaviour of dividends.

Table 6 reports the results of cointegration tests for p_t and d_t, along with an estimate of the cointegrating vector obtained by means of fully-modified least squares (denoted by PH).[10] The null hypothesis of no cointegration can be firmly rejected at the 5% significance level on the basis of all the tests.

Having established the "global" characteristics of the series, we now check for signs of "local" nonlinear Markov switching behaviour in the equilibrium error and the associated error-correction mechanism. To start, an AR(1) model for the equilibrium error ($z_t = p_t - 24.274d_t$) and the related error-correction model with no differenced lagged variables are estimated (both of which exhibit no signs of residual autocorrelation). These models are then subjected to the tests for parameter instability and neglected nonlinearity described in earlier sections of the paper. We also carry out Hansen's test for Markov switching and compute the value of the AIC for the single-state and corresponding two-state models. The results of the parameter instability and nonlinearity tests, shown in Table 7, are mixed with the former suggesting no parameter instability and the latter providing strong evidence in favour of neglected nonlinearity in both models. The Hansen tests, shown in Table 8, also reject the null hypothesis of linearity in favour of the Markov alternative for both models at the 10% significance level.[11] Furthermore, the AIC

[9] We discarded the last 5 observations on the available sample since for the period 1990–1997 the two series do not appear to share a long-run relationship. Furthermore, the test of Leybourne et al. (1996) rejects the null hypothesis of a fixed unit root against the alternative of a randomized root for the prices series for the period 1990–1997, but not for the period 1990–1992.

[10] The null hypothesis of a unit root in prices and dividents cannot be rejected on the basis of ADF and Phillips–Perron tests.

[11] A significance level of 10% is prefered in light of the fact that the test is conservative.

clearly selects a Markov switching specification over a linear one, both for the AR(1) model for the equilibrium error (-58.058 vs -47.113) and the error-correction model (5.070 vs 11.444). We conclude, therefore, that there is significant evidence of MEC behaviour in the two time series.

In Tables 9A and 9B, we report maximum likelihood (ML) estimates (based on the Gaussian likelihood) and associated asymptotic standard errors of the parameters of the Markov switching autoregression for the equilibrium error and of the associated error-correction model. The results are clearly consistent with a MEC relationship between prices and dividends. In the regime characterized by $s_t = 1$, the equilibrium error is 'locally' nonstationary (the t-statistic for $\varphi_1 = 1$ is –1.017) and the corresponding adjustment coefficient in the error-correction model is insignificantly different from zero (the t-ratio for ϑ_1 is –1.273). In contrast, in the regime corresponding to $s_t = 0$, the equilibrium error is 'locally' stationary and statistically significant error-correction takes place (the t-ratio for ϑ_0 is –2.387).[12]

The inferred probabilities that the system was in regime 1 at each date in the sample, based on currently available information, are shown in Figure 2, together with a time plot of the equilibrium error z_t. The unstable regime is clearly associated with the periods 1956–1973 and 1986–1992. A slightly different picture emerges from Figure 3, which shows inferred probabilities that the system was in regime 1 based on the MEC model. The unstable regime is associated with the periods 1959–1978 and 1987–1992.[13]

5 Concluding Comments

This paper has analyzed a model of cointegration in which deviations from long-run equilibrium follow a two-state Markov switching autoregressive process which is mean-reverting in one state of nature and has a unit root in the other. Thus, adjustment towards the long-run equilibrium is discontinuous and only takes place in one of the two states.

Like Balke and Fomby (1997), we have advocated a two-step approach to testing for MEC adjustment which involves first testing for cointegration under the assumption of linear adjustment and then testing for Markov switching behaviour in the dynamics of the equilibrium error or in the corresponding error-correction model. On the basis of our simulation results, we recommend starting with the $\widehat{Z}_\alpha$, $\widehat{Z}_t$ and SW cointegration tests, which have high power to detect the presence of a long-run relationship among cointegrated MEC time series. In the second step, the $ExpLR$ and $SupLR$ tests for parameter instability can be useful in revealing the invalidity of the assumption of a continuous and constant-strength adjustment process. These could be supplemented with the $RESET$ and WHT tests for neglected nonlinearity both of which have respectable power against Markov-type alternatives. Better still, if the high computational cost is acceptable, Hansen's (1992b) procedure can be used to directly test the one-state linear model

[12] It is worth mentioning that we repeated the simulation experiments of Section 3 using a data-generating process calibrated to the empirical results in Tables 9A–9B. The results were very similar to those reported for $(p_{00}, p_{11}) = (0.98, 0.9)$.

[13] The two sets of filter probabilities do not imply exactly the same classification of regimes since all parameters in the two models are allowed to be subject to Markov switching.

of interest against the corresponding Markov model. Of course, since there exist situations in which all these tests tend to have low power, it seems prudent to anyhow fit both linear and Markov switching models to the data. Then, one of the competing models may be selected by using the AIC criterion, which was found to work well in our simulations. Also, bootstrap-based techniques like those discussed in Psaradakis (1998) could be used to assess the adequacy of the rival models. As an illustration of the ideas discussed in the paper, we have analyzed the long-run relationship between stock prices and dividends in the US.

In closing, we should stress that the idea of MEC adjustment has potential applications beyond the context of modelling temporary deviations of asset prices from underlying fundamentals, especially when the class of MEC models is generalized to allow not only for the possibility of 'temporary' cointegration analyzed here but also for situations where error-correction takes place at each point in the sample but the speed of adjustment is subject to discrete changes. To give an example, most stochastic general equilibrium models incorporate intertemporal budget constraints which imply that, even though the relevant variables tend to move together in the long run, there may be substantial short-run deviations from equilibrium. Although the variables will adjust in response to such disequilibrium errors, the adjustment will not necessarily be continuous or of constant strength. One can think of situations where error correction will depend on the type of economic policy adopted by the government (e.g., disequilibrium adjustment in an imports–exports system can depend on whether the government is running current account deficits or is implementing some trade policy). Another example are studies of currency devaluations where the question of interest is the speed at which a devaluation (used as a means of improving short-run competitiveness) is transmitted to domestic prices. Such questions are typically answered by using linear error-correction models incorporating a long-run equilibrium relationship such as the PPP. It is clear, however, that inferences about the speed of adjustment made from such models will be misleading since they are based on information from two different exchange-rate regimes. A MEC model should provide a more appropriate framework for analyzing such problems.

References

[1] Akaike, H. (1973), Information theory and an extension of the maximum likelihood principle, in: B.N. Petrov and F. Czáki, eds., *Second International Symposium on Information Theory*, Budapest: Akademiai Kiadó, pp. 267–281.

[2] Anderson, H.M. (1997), Transaction costs and non-linear adjustment towards equilibrium in the US Treasury Bill market, *Oxford Bulletin of Economics and Statistics* 59, 465–484.

[3] Andrews, D.W.K. (1993), Tests for parameter instability and structural change with unknown change point, *Econometrica* 61, 821–856.

[4] Andrews, D.W.K., and Monahan, J.C. (1992), An improved heteroskedasticity and autocorrelation consistent covariance matrix estimator, *Econometrica* 60, 953–966.

[5] Andrews, D.W.K., and Ploberger, W. (1994), Optimal tests when a nuisance parameter is present only under the alternative, *Econometrica* 62, 1383–1414.

[6] Balke, N.S., and Fomby, T.B. (1997), Threshold cointegration, *International Economic Review* 38, 627–645.

[7] Brock, W.A., Dechert, W.D., Scheinkman, J.A., and LeBaron, B. (1996), A test for independence based on the correlation dimension, *Econometric Reviews* 15, 197–235.

[8] Campbell, J.Y., and Shiller, R.J. (1987), Cointegration and tests of present value models, *Journal of Political Economy* 95, 1062–1088.

[9] Engle, R.F., and Granger, C.W.J. (1987), Co-integration and error correction: representation, estimation, and testing, *Econometrica* 55, 251–276.

[10] Froot, K.A., and Obstfeld, M. (1991), Intrinsic bubbles: the case of stock prices, *American Economic Review* 81, 1189–1214.

[11] Garcia, R. (1998), Asymptotic null distribution of the likelihood ratio test in Markov switching models, *International Economic Review* 39, 763–788.

[12] Granger, C.W.J., King, M.L., and White, H. (1995), Comments on testing economic theories and the use of model selection criteria, *Journal of Econometrics* 67, 173–187.

[13] Granger, C.W.J., and Swanson, N. (1996), Future developments in the study of cointegrated variables, *Oxford Bulletin of Economics and Statistics* 58, 537–553.

[14] Haefke, C., and Helmenstein, C. (1996), Forecasting Austrian IPOs: an application of linear and neural network error-correction models, *Journal of Forecasting* 15, 237–251.

[15] Hall, S.G., Psaradakis, Z., and Sola, M. (1997), Switching error-correction models of house prices in the United Kingdom, *Economic Modelling* 14, 517–527.

[16] Hansen, B.E. (1992a), Testing for parameter instability in linear models, *Journal of Policy Modeling* 14, 517–533.

[17] Hansen, B.E. (1992b), The likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP, *Journal of Applied Econometrics* 7, S61–S82.

[18] Hansen, B.E. (1996), Erratum: the likelihood ratio test under nonstandard conditions: testing the Markov switching model of GNP, *Journal of Applied Econometrics* 11, 195–198.

[19] Hansen, G., and Kim, J.–R. (1996), Nonlinear cointegration analysis of German unemployment, manuscript, Institute of Statistics and Econometrics, Christian Albrechts University at Kiel.

[20] Holst, U., Lindgren, G., Holst, J., and Thuvesholmen, M. (1994), Recursive estimation in switching autoregressions with a Markov regime, *Journal of Time Series Analysis* 15, 489–506.

[21] Johansen, S. (1991), Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models, *Econometrica* 59, 1551–1580.

[22] Karlsen, H. (1990), A class of non-linear time series models, Ph.D. thesis, Department of Mathematics, University of Bergen.

[23] Lee, T.–H., White, H., and Granger, C.W.J. (1993), Testing for neglected nonlinearity in time series models: a comparison of neural network methods and alternative tests, *Journal of Econometrics* 56, 269–290.

[24] Leybourne, S.J., McCabe, B.P.M., and Tremayne, A.R. (1996), Can economic time series be differenced to stationarity?, *Journal of Business and Economics Statistics* 14, 435–446.

[25] Michael, P., Nobay, A.R., and Peel, D.A. (1997), Transaction costs and nonlinear adjustment in real exchange rates: an empirical investigation, *Journal of Political Economy* 105, 862–879.

[26] Nishii, R. (1988), Maximum likelihood principle and model selection when the true model is unspecified, *Journal of Multivariate Analysis* 27, 392–403.

[27] Nyblom, J. (1989), Testing for the constancy of parameters over time, *Journal of the American Statistical Association* 84, 223–230.

[28] Osterwald–Lenum, M. (1992), A note with quantiles of the asymptotic distribution of the maximum likelihood cointegration rank test statistics, *Oxford Bulletin of Economics and Statistics* 54, 461–472.

[29] Peel, D., and Davidson, J. (1998), A non-linear error correction mechanism based on the bilinear model, *Economics Letters* 58, 165–170.

[30] Phillips, P.C.B., and Hansen, B.E. (1990), Statistical inference in instrumental variables regression with I(1) processes, *Review of Economic Studies* 57, 99–125.

[31] Phillips, P.C.B., and Ouliaris, S. (1990), Asymptotic properties of residual based tests for cointegration, *Econometrica* 58, 165–193.

[32] Psaradakis, Z. (1998), Bootstrap-based evaluation of Markov-switching time series models, *Econometric Reviews* 17, 275–288.

[33] Psaradakis, Z., and Spagnolo, N. (1999), Power properties of nonlinearity tests for time series with Markov regime, Discussion Paper 24/99, Department of Economics, Birkbeck College, University of London.

[34] Shiller, R.J. (1989), *Market Volatility,* Cambridge, Mass.: MIT Press.

[35] Siklos, P.L., and Granger, C.W.J. (1997), Regime-sensitive cointegration with an application to interest-rate parity, *Macroeconomic Dynamics* 1, 640–657.

[36] Sin, C.–Y., and White, H. (1996), Information criteria for selecting possibly misspecified parametric models, *Journal of Econometrics* 71, 207–225.

[37] Stock, J.H., and Watson, M.W. (1988), Testing for common trends, *Journal of the American Statistical Association* 83, 1097–1107.

[38] Thursby, J., and Schmidt, P. (1977), Some properties of tests for specification error in a linear regression model, *Journal of the American Statistical Association* 72, 635–641.

[39] van Dijk, D.J.C., and Franses, P.H. (1996), Testing for threshold cointegration, Discussion Paper TI 96–78/7, Tinbergen Institute, Erasmus University Rotterdam.

[40] van Dijk, D.J.C., and Franses, P.H. (2000), Nonlinear error correction models for interest rates in the Netherlands, in: W.A. Barnett, D.F. Hendry, S. Hylleberg, and T. Teräsvirta, eds., *Nonlinear Econometric Modelling*, Cambridge: Cambridge Univesrity Press.

Table 1A. Rejection Probabilities (%) of Cointegration Tests
($\phi_0 = \phi_1 = 1$)

T	ADF	$\widehat{Z}_\alpha$	$\widehat{Z}_t$	SW	LR_{trace}	$LR_{\max}$
50	4.84	6.44	7.80	5.04	9.80	9.76
100	4.28	6.64	6.16	4.88	7.32	7.36
200	4.76	6.48	5.96	5.68	7.20	7.72
500	5.40	5.48	5.20	5.08	6.28	5.72

Table 1B. Rejection Probabilities (%) of Cointegration Tests ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
			$p_{00} = 0.5$,	$p_{11} = 0.5$				
ADF	44.60	8.68	92.36	25.24	99.76	81.40	100.0	100.0
$\widehat{Z}_\alpha$	75.60	15.16	99.32	42.64	100.0	93.00	100.0	100.0
$\widehat{Z}_t$	78.60	16.08	99.12	35.84	100.0	89.56	100.0	100.0
SW	83.44	12.32	99.64	40.40	100.0	93.24	100.0	100.0
LR_{trace}	56.24	16.72	94.72	30.00	99.96	74.68	100.0	100.0
$LR_{\max}$	56.92	14.64	95.76	27.60	99.96	77.80	100.0	100.0
			$p_{00} = 0.9$,	$p_{11} = 0.9$				
ADF	16.40	7.36	33.52	15.24	60.36	40.76	97.64	92.56
$\widehat{Z}_\alpha$	34.24	11.48	56.28	26.20	79.36	50.76	99.24	95.52
$\widehat{Z}_t$	38.00	12.32	55.16	21.92	78.00	46.36	99.00	93.64
SW	37.68	9.56	58.68	23.44	83.60	52.16	99.64	96.80
LR_{trace}	23.08	14.00	39.28	21.00	65.80	41.04	97.72	89.28
$LR_{\max}$	21.48	12.20	38.56	18.68	66.20	39.88	98.48	90.64
			$p_{00} = 0.98$,	$p_{11} = 0.98$				
ADF	13.24	7.64	15.76	12.20	20.32	21.20	36.48	39.32
$\widehat{Z}_\alpha$	27.40	11.32	33.28	20.16	37.92	27.72	56.08	46.04
$\widehat{Z}_t$	30.36	12.24	32.80	16.96	37.64	25.28	53.68	43.04
SW	28.60	9.32	34.28	18.68	41.40	28.60	59.48	48.08
LR_{trace}	17.40	12.52	21.84	15.48	26.80	22.84	42.28	37.36
$LR_{\max}$	17.64	12.12	21.00	14.72	25.32	21.84	42.68	37.12

Table 1C. Rejection Probabilities (%) of Cointegration Tests ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$p_{00} = 0.98, \quad p_{11} = 0.9$							
ADF	21.32	8.52	36.72	22.48	61.12	59.88	94.80	96.04
$\widehat{Z}_\alpha$	44.16	15.48	67.52	38.56	87.16	70.92	99.32	97.84
$\widehat{Z}_t$	47.92	16.68	67.60	32.92	87.16	66.84	99.12	97.28
SW	46.76	13.28	70.08	36.44	89.16	71.80	99.56	98.24
LR_{trace}	26.56	14.84	45.44	26.76	70.12	57.52	96.20	94.92
$LR_{\max}$	25.80	12.72	44.84	24.48	71.20	57.56	96.44	95.40
	$p_{00} = 0.9, \quad p_{11} = 0.98$							
ADF	8.72	5.56	9.12	6.76	12.28	9.56	35.16	19.88
$\widehat{Z}_\alpha$	16.00	7.96	18.84	10.04	21.84	11.96	45.24	23.16
$\widehat{Z}_t$	19.00	9.48	18.04	9.40	20.76	11.12	41.20	20.20
SW	16.92	6.60	19.40	8.84	24.48	11.48	52.04	24.20
LR_{trace}	15.00	11.96	15.28	11.04	18.24	13.32	37.72	20.56
$LR_{\max}$	14.12	11.72	12.88	9.32	14.76	11.04	36.88	18.40

Table 2A. Rejection Probabilities (%) of Parameter Instability Tests: Model M_1

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1 = \phi_0$							
L	6.32	2.24	7.64	3.64	9.72	4.64	12.32	6.40
$AvgLR$	7.84	8.40	8.88	8.60	10.64	7.60	12.80	8.80
$ExpLR$	8.96	11.76	9.76	9.80	11.48	8.32	12.76	8.32
$SupLR$	5.92	9.04	7.32	8.92	9.20	7.72	11.40	7.76
	$\phi_1 = 1, \quad p_{00} = 0.5, \quad p_{11} = 0.5$							
L	2.60	2.36	4.40	1.80	5.12	2.68	7.20	5.44
$AvgLR$	7.72	12.84	12.24	11.48	15.16	10.08	19.04	10.64
$ExpLR$	14.48	17.76	15.88	14.52	17.56	11.52	22.24	10.76
$SupLR$	13.56	16.08	17.04	14.84	18.24	12.80	24.60	11.08
	$\phi_1 = 1, \quad p_{00} = 0.9, \quad p_{11} = 0.9$							
L	8.12	2.64	10.60	2.80	15.00	3.68	21.48	7.68
$AvgLR$	30.12	17.92	40.04	16.64	50.60	17.28	61.32	23.48
$ExpLR$	45.44	26.20	56.72	23.68	65.60	25.44	75.92	30.80
$SupLR$	40.84	23.08	55.28	24.28	67.96	29.16	79.64	37.48
	$\phi_1 = 1, \quad p_{00} = 0.98, \quad p_{11} = 0.98$							
L	13.00	3.16	21.96	5.12	26.84	7.60	28.76	12.44
$AvgLR$	47.00	23.72	63.16	30.04	68.60	38.20	70.48	46.16
$ExpLR$	58.68	33.28	72.68	39.32	79.12	48.96	82.92	59.76
$SupLR$	53.04	29.08	70.20	37.20	79.20	49.76	84.76	65.00
	$\phi_1 = 1, \quad p_{00} = 0.98, \quad p_{11} = 0.9$							
L	14.12	2.96	25.68	3.68	39.64	8.72	58.68	23.36
$AvgLR$	41.76	19.52	66.12	22.04	80.32	31.88	92.28	48.28
$ExpLR$	54.72	26.96	75.24	27.92	87.56	35.76	96.36	56.16
$SupLR$	49.08	24.04	72.52	27.64	88.16	34.72	97.00	57.72
	$\phi_1 = 1, \quad p_{00} = 0.9, \quad p_{11} = 0.98$							
L	7.64	3.56	6.96	3.44	6.88	2.80	5.40	2.16
$AvgLR$	33.64	23.96	38.28	24.80	37.40	23.44	35.08	19.32
$ExpLR$	51.28	35.40	56.04	36.72	55.96	37.24	56.04	35.64
$SupLR$	46.56	31.92	55.44	37.76	58.64	42.64	62.04	44.08

Table 2B. Rejection Probabilities (%) of Parameter Instability Tests: Model M_2

	$T=50$		$T=100$		$T=200$		$T=500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
				$\phi_1=\phi_0$				
L	4.16	2.64	5.56	3.92	6.52	3.80	6.76	4.88
$AvgLR$	6.44	6.48	6.68	6.16	6.60	5.40	6.64	5.28
$ExpLR$	7.48	7.72	6.64	6.40	6.08	5.96	6.08	5.04
$SupLR$	5.04	5.56	4.68	5.12	4.60	5.12	5.12	4.56
				$\phi_1=1,\quad p_{00}=0.5,\quad p_{11}=0.5$				
L	2.00	2.72	3.28	2.88	3.36	2.76	4.16	4.32
$AvgLR$	5.92	7.80	7.08	6.64	8.80	6.28	10.56	6.28
$ExpLR$	8.16	9.32	8.84	7.32	10.12	6.92	12.00	6.36
$SupLR$	7.96	7.64	8.48	6.60	10.08	6.96	13.48	6.08
				$\phi_1=1,\quad p_{00}=0.9,\quad p_{11}=0.9$				
L	3.76	2.80	6.56	2.80	8.48	3.08	12.44	4.76
$AvgLR$	15.96	10.20	21.60	8.60	29.52	8.48	41.00	11.56
$ExpLR$	26.40	13.80	34.44	11.00	42.88	11.08	55.56	14.00
$SupLR$	23.32	11.40	33.32	10.28	45.44	12.88	59.96	17.32
				$\phi_1=1,\quad p_{00}=0.98,\quad p_{11}=0.98$				
L	7.52	3.36	11.96	3.76	14.92	3.60	17.68	6.16
$AvgLR$	25.28	13.76	38.00	12.64	43.52	15.24	50.60	21.32
$ExpLR$	37.44	19.28	49.80	18.64	56.96	21.52	65.32	30.64
$SupLR$	34.44	15.60	48.08	18.04	58.16	22.68	69.16	35.52
				$\phi_1=1,\quad p_{00}=0.98,\quad p_{11}=0.9$				
L	6.96	2.80	14.52	3.04	23.60	4.48	42.64	11.92
$AvgLR$	23.24	9.52	39.64	10.64	59.28	13.04	79.56	25.48
$ExpLR$	34.08	13.40	49.92	13.24	68.44	15.68	88.00	28.64
$SupLR$	29.12	11.36	46.40	12.16	69.08	15.68	89.72	29.08
				$\phi_1=1,\quad p_{00}=0.9,\quad p_{11}=0.98$				
L	3.88	3.72	3.80	2.48	3.08	2.64	2.96	2.40
$AvgLR$	16.84	12.68	19.16	12.44	18.48	10.40	15.92	7.52
$ExpLR$	31.60	20.08	34.88	18.84	35.72	17.16	31.84	13.72
$SupLR$	29.24	17.76	35.00	18.88	38.48	20.00	39.80	20.52

Table 3A. Rejection Probabilities (%) of Nonlinearity Tests: Model M_1

	$T=50$		$T=100$		$T=200$		$T=500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1=\phi_0$							
RESET	4.44	3.28	3.80	3.92	4.48	3.28	4.76	3.24
WHT	3.84	4.80	4.48	4.80	4.08	3.20	5.00	4.60
BDS	18.20	17.48	10.84	10.20	7.64	7.76	5.80	5.52
NNT	4.16	4.08	3.68	4.20	4.32	4.52	4.40	4.72
	$\phi_1=1,\quad p_{00}=0.5,\quad p_{11}=0.5$							
RESET	18.24	5.40	26.20	5.92	32.16	7.60	43.48	8.56
WHT	13.56	6.44	20.04	6.52	23.56	6.76	31.00	7.76
BDS	33.56	16.60	50.32	10.96	79.44	8.76	99.24	10.52
NNT	11.08	3.72	13.64	4.56	16.12	5.20	20.96	6.12
	$\phi_1=1,\quad p_{00}=0.9,\quad p_{11}=0.9$							
RESET	23.56	5.60	37.76	7.20	53.56	10.08	75.96	20.44
WHT	16.52	6.12	31.04	7.20	57.48	10.84	92.52	24.72
BDS	26.20	18.28	33.28	11.44	53.76	9.76	87.40	10.00
NNT	11.20	3.96	19.96	4.40	40.32	6.28	81.00	25.64
	$\phi_1=1,\quad p_{00}=0.98,\quad p_{11}=0.98$							
RESET	24.48	7.92	41.96	11.24	54.68	18.92	68.60	35.60
WHT	20.32	6.88	41.52	7.84	69.36	14.12	91.92	35.68
BDS	25.36	18.88	30.20	11.80	49.16	11.28	82.48	12.76
NNT	12.20	5.04	24.88	6.72	42.28	11.96	63.12	34.40
	$\phi_1=1,\quad p_{00}=0.98,\quad p_{11}=0.9$							
RESET	24.64	5.52	48.92	9.24	77.12	19.08	97.40	50.12
WHT	19.56	6.00	50.48	7.12	86.88	13.08	99.92	36.36
BDS	24.64	17.96	36.60	11.52	69.72	8.92	99.08	9.68
NNT	12.84	4.12	29.48	5.88	63.88	14.08	97.04	55.16
	$\phi_1=1,\quad p_{00}=0.9,\quad p_{11}=0.98$							
RESET	21.32	6.92	30.40	9.16	36.44	10.04	43.08	11.56
WHT	16.96	8.36	24.04	9.32	34.72	14.72	57.68	31.00
BDS	27.00	19.68	25.96	13.28	31.40	12.12	49.68	14.84
NNT	9.80	3.88	15.64	5.40	19.52	5.24	29.20	7.20

Table 3B. Rejection Probabilities (%) of Nonlinearity Tests: Model M_2

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
	$\phi_1 = \phi_0$							
RESET	4.20	3.56	4.04	4.48	4.28	4.92	4.20	3.76
WHT	5.16	4.12	4.68	3.44	5.20	4.96	5.08	4.56
BDS	19.92	17.68	10.48	11.60	7.84	7.76	5.92	5.84
NNT	4.00	4.24	3.80	4.96	4.76	5.20	4.40	4.36
	$\phi_1 = 1, \quad p_{00} = 0.5, \quad p_{11} = 0.5$							
RESET	12.28	5.08	17.36	5.52	21.36	6.20	29.16	7.84
WHT	9.60	4.08	11.92	4.20	13.96	5.52	19.00	5.76
BDS	19.84	17.40	16.04	12.00	23.16	7.80	45.80	6.36
NNT	9.36	4.16	10.68	4.68	11.48	5.08	15.16	5.40
	$\phi_1 = 1, \quad p_{00} = 0.9, \quad p_{11} = 0.9$							
RESET	17.00	6.40	26.28	7.00	40.16	7.44	61.12	12.88
WHT	11.36	4.56	17.88	4.76	32.72	7.24	66.52	12.16
BDS	20.76	18.44	15.84	11.20	19.28	8.32	30.20	6.48
NNT	8.72	5.12	12.20	4.68	26.72	5.04	69.08	14.24
	$\phi_1 = 1, \quad p_{00} = 0.98, \quad p_{11} = 0.98$							
RESET	19.00	6.48	29.68	8.12	42.20	11.40	58.84	21.40
WHT	13.60	6.24	24.36	6.92	50.40	10.56	84.08	19.60
BDS	22.52	18.56	18.68	12.64	23.68	10.44	40.68	9.04
NNT	8.48	4.96	16.68	5.44	29.96	7.96	52.28	20.32
	$\phi_1 = 1, \quad p_{00} = 0.98, \quad p_{11} = 0.9$							
RESET	18.60	5.56	35.20	6.92	61.00	10.28	93.68	30.04
WHT	12.92	4.48	24.96	4.76	56.56	7.44	94.36	13.92
BDS	19.84	18.00	17.20	11.00	23.48	8.48	51.16	6.96
NNT	9.28	3.88	19.16	5.24	48.40	8.04	93.28	32.96
	$\phi_1 = 1, \quad p_{00} = 0.9, \quad p_{11} = 0.98$							
RESET	18.08	7.96	23.48	8.60	29.12	8.20	34.92	9.80
WHT	12.28	6.68	18.56	8.24	25.52	12.16	39.24	19.64
BDS	22.32	18.32	19.60	13.88	16.32	10.84	19.60	10.28
NNT	8.00	4.24	9.28	4.56	12.24	4.56	19.40	6.68

Table 4A. Rejection Probabilities (%) of Hansen's Test: Model for $\widehat{z}_t$ ($\phi_1 = 1$)

	$T=50$		$T=100$		$T=200$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7
	$p_{00}=0.5,\quad p_{11}=0.5$					
$M=0$	11.20	0.40	26.00	0.80	69.40	1.20
$M=1$	11.40	0.60	26.80	0.80	69.20	1.60
$M=2$	12.60	0.80	28.40	0.80	69.40	1.40
$M=3$	14.20	1.20	29.40	1.00	69.20	1.20
$M=4$	14.60	1.20	29.60	1.00	69.20	1.40
	$p_{00}=0.9,\quad p_{11}=0.9$					
$M=0$	16.40	1.80	50.20	2.00	82.80	6.60
$M=1$	16.00	1.40	49.20	1.80	82.40	5.00
$M=2$	16.20	1.20	47.00	1.40	80.60	3.80
$M=3$	15.80	1.40	45.20	1.20	80.00	3.20
$M=4$	15.80	1.00	45.00	1.20	79.40	3.00
	$p_{00}=0.98,\quad p_{11}=0.9$					
$M=0$	19.40	0.60	52.00	2.40	87.80	10.60
$M=1$	18.40	0.60	50.20	2.00	86.40	9.40
$M=2$	17.60	0.60	46.20	1.80	85.20	8.20
$M=3$	17.00	0.60	44.60	1.80	85.00	7.60
$M=4$	16.60	0.60	42.80	1.80	83.80	7.00

Table 4B. Rejection Probabilities (%) of Hansen's Test: Model for Δx_t ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7
			$p_{00} = 0.5,\quad p_{11} = 0.5$			
$M = 0$	5.20	1.00	6.60	0.40	20.00	0.60
$M = 1$	5.40	1.40	6.60	0.40	21.20	0.70
$M = 2$	5.60	1.40	7.80	0.60	21.20	0.80
$M = 3$	6.00	1.80	8.00	0.60	21.80	0.80
$M = 4$	6.80	1.60	9.20	0.60	22.40	0.60
			$p_{00} = 0.9,\quad p_{11} = 0.9$			
$M = 0$	13.20	1.00	23.40	0.40	45.40	1.20
$M = 1$	13.00	0.80	23.00	0.20	44.40	1.00
$M = 2$	12.60	0.80	22.60	0.20	43.80	0.80
$M = 3$	12.40	1.20	21.60	0.20	43.00	0.60
$M = 4$	12.20	1.20	20.80	0.20	43.20	0.60
			$p_{00} = 0.98,\quad p_{11} = 0.9$			
$M = 0$	10.60	0.60	22.00	0.80	59.80	2.00
$M = 1$	10.40	0.60	20.80	0.80	58.60	2.00
$M = 2$	10.60	0.60	20.20	0.80	57.00	1.40
$M = 3$	11.00	0.60	19.00	0.80	54.80	1.20
$M = 4$	10.80	0.60	18.00	1.00	53.60	1.00

Table 5A. Selection Probabilities (%): Model for $\widehat{z}_t$ ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
				$p_{00} = 0.5$, $p_{11} = 0.5$				
	48.68	21.36	69.64	22.80	92.12	26.24	100.0	42.68
				$p_{00} = 0.9$, $p_{11} = 0.9$				
	57.72	23.68	83.48	28.16	98.00	42.48	100.0	75.28
				$p_{00} = 0.98$, $p_{11} = 0.98$				
	57.16	22.40	77.96	27.76	89.16	42.36	96.88	74.60
				$p_{00} = 0.98$, $p_{11} = 0.9$				
	55.88	21.84	83.60	26.84	98.12	43.56	100.0	79.60
				$p_{00} = 0.9$, $p_{11} = 0.98$				
	58.12	26.04	77.52	32.24	88.32	44.04	97.80	73.04

Table 5B. Selection Probabilities (%): Model for Δx_t ($\phi_1 = 1$)

	$T = 50$		$T = 100$		$T = 200$		$T = 500$	
ϕ_0	0.0	0.7	0.0	0.7	0.0	0.7	0.0	0.7
				$p_{00} = 0.5$, $p_{11} = 0.5$				
	31.36	16.56	42.84	17.24	65.64	19.08	95.32	23.84
				$p_{00} = 0.9$, $p_{11} = 0.9$				
	41.64	17.24	61.64	16.52	88.00	23.24	99.64	40.08
				$p_{00} = 0.98$, $p_{11} = 0.98$				
	45.80	17.24	63.16	19.44	79.44	26.80	91.28	48.88
				$p_{00} = 0.98$, $p_{11} = 0.9$				
	39.64	15.72	60.60	16.20	89.68	21.84	99.80	45.96
				$p_{00} = 0.9$, $p_{11} = 0.98$				
	51.32	21.28	66.88	26.04	81.44	32.52	94.72	53.00

Table 6. Cointegration Tests and Estimated Cointegrating Vector

ADF	–4.284 [2]	SW	–41.771
$\widehat{Z}_\alpha$	–36.201	LR_{trace}	24.997 [1]
$\widehat{Z}_t$	–4.506	$LR_{\max}$	21.673 [1]
PH	$p_t = 24.274 d_t$		

NOTES: The number of lags (in square brackets) is selected by minimizing the AIC over $k \in \{1, ..., 12\}$. For all tests, a constant and a trend are included in the test regressions.

Table 7. Parameter Instability and Nonlinearity Tests

	$z_t = \mu + \varphi z_{t-1} + e_t$		$\Delta p_t = \mu + \vartheta z_{t-1} + e_t$	
L	0.297	(0.749)	0.584	(0.749)
$AvgLR$	2.561	(4.610)	2.671	(4.610)
$ExpLR$	2.245	(3.220)	1.627	(3.220)
$SupLR$	10.938	(11.790)	5.981	(11.790)
$RESET$	5.683	[0.001]	6.389	[0.000]
WHT	8.664	[0.070]	4.569	[0.334]
BDS	3.118	[0.000]	2.192	[0.014]
NNT	11.950	[0.002]	13.760	[0.001]

NOTES: 5% critical values are in parentheses and p-values are in square brackets.

Table 8. Hansen's Test

	$z_t = \mu + \varphi z_{t-1} + e_t$	$\Delta p_t = \mu + \vartheta z_{t-1} + e_t$
$M = 0$	[0.050]	[0.035]
$M = 1$	[0.047]	[0.037]
$M = 2$	[0.049]	[0.054]
$M = 3$	[0.059]	[0.072]
$M = 4$	[0.060]	[0.089]
Standardized LR statistic	3.566	3.360

NOTE: p-values are in square brackets.

Table 9A. ML Estimates for the Equilibrium-Error Model

$z_t = \mu_{s_t} + \varphi_{s_t} z_{t-1} + \sigma_{s_t} \eta_t$		
	Estimate	Std. Error
μ_0	–0.134	0.026
μ_1	0.059	0.060
φ_0	0.406	0.112
φ_1	0.882	0.116
σ_0	0.120	0.019
σ_1	0.197	0.026
p_{00}	0.944	0.048
p_{11}	0.955	0.039
Log-likelihood	37.029	

Table 9B. ML Estimates for the Error-Correction Model

$\Delta p_t = \mu_{s_t} + \vartheta_{s_t} z_{t-1} + \sigma_{s_t} \eta_t$		
	Estimate	Std. Error
μ_0	–0.025	0.033
μ_1	0.260	0.115
ϑ_0	–0.561	0.235
ϑ_1	–0.163	0.128
σ_0	0.170	0.015
σ_1	0.328	0.044
p_{00}	0.940	0.087
p_{11}	0.969	0.025
Log-likelihood	5.465	

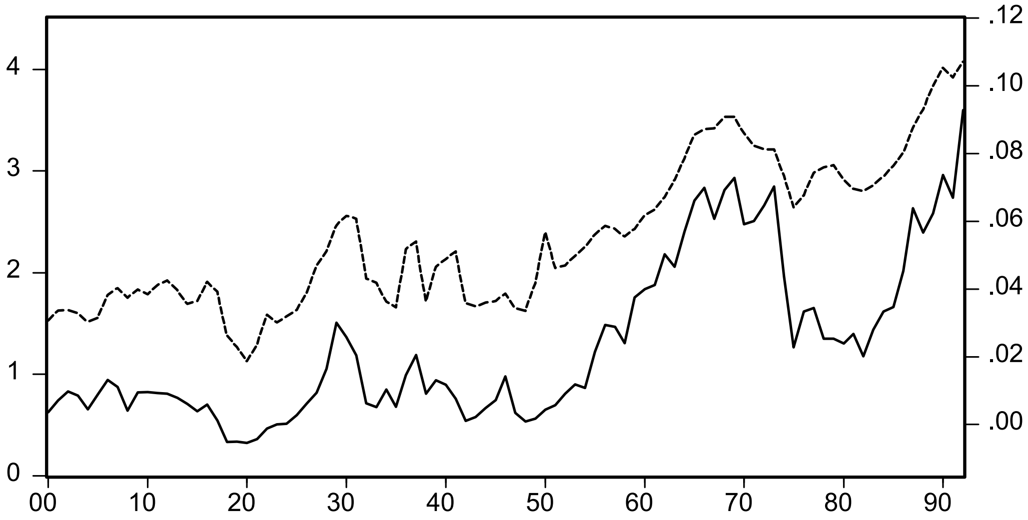

Figure 1: Stock prices (solid line) and dividends (broken line).

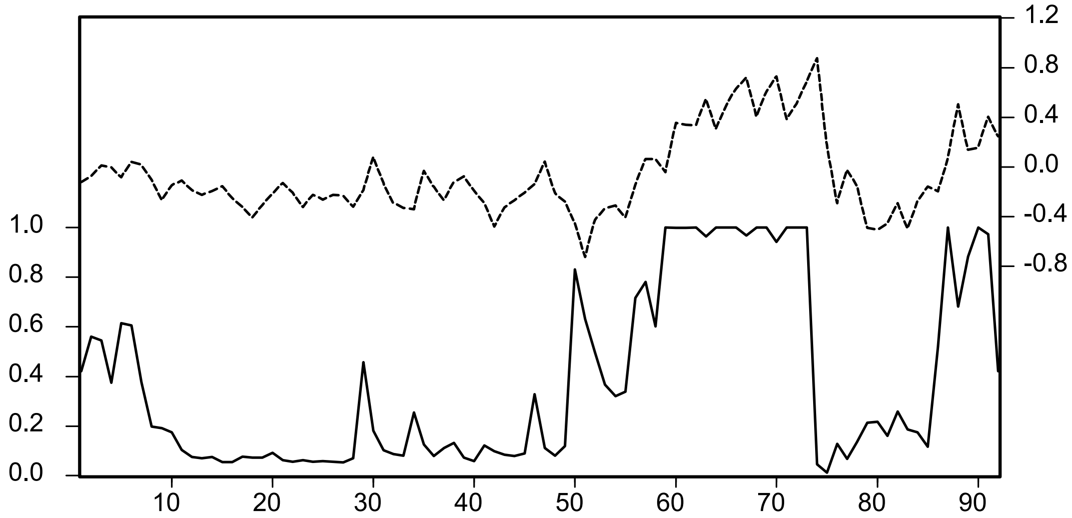

Figure 2: Inferred probability for regime 1 based on the AR(1) model (solid line) and equilibrium error (broken line).

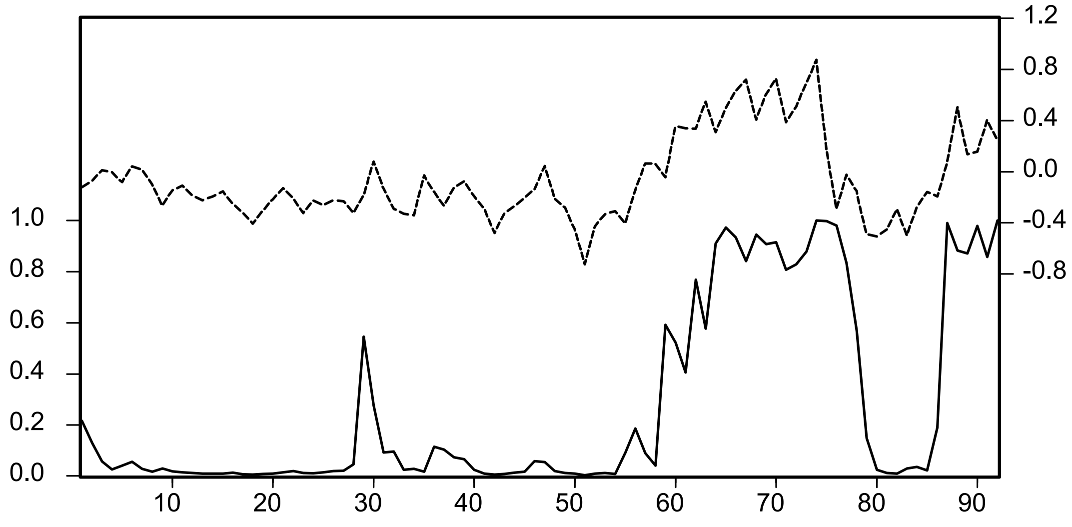

Figure 3: Inferred probability for regime 1 based on the MEC model (solid line) and equilibrium error (broken line).